

Make a table of the measures they found for base, height, and area of each parallelogram.

- How is the area of a parallelogram related to the area of a triangle? How are the bases and heights of the triangle and parallelogram related?
- If the bases and heights are the same, why is the area of the parallelogram twice the area of the triangle?
- How did you use  $b$  and  $h$  to write a formula for the area of a parallelogram?



*Assignment Guide for Problem 3.1*

Applications: 1–9 | Connections: 39

### Answers to Problem 3.1

- A.**
- Figure A:  $\approx 12\frac{1}{2}$  cm; Figure B:  $\approx 12\frac{1}{5}$  cm;  
Figure C:  $\approx 21$  cm; Figure D:  $\approx 24$  cm;  
Figure E:  $\approx 16$  cm; Figure F:  $\approx 21\frac{4}{5}$  cm
  - Possible answers: add the lengths of the sides; add the lengths of the two sides that form an angle and double; or double one side length, double the other side length and total.
- B.**
- Figure A:  $8 \text{ cm}^2$ ; Figure B:  $8 \text{ cm}^2$ ;  
Figure C:  $24 \text{ cm}^2$ ; Figure D:  $35 \text{ cm}^2$ ;  
Figure E:  $15\frac{3}{4} \text{ cm}^2$ ; Figure F:  $18 \text{ cm}^2$
  - Possible answers: count the number of whole square centimeters and estimate how many partial square centimeters there are; cut off part of the parallelogram by cutting perpendicular to the base, rearranging to make a rectangle, and then finding the area of the rectangle.

C. 1–2.

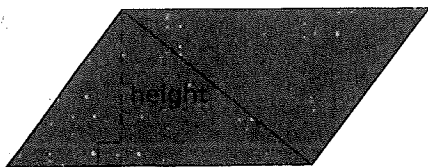
Parallelograms			
Figure	Area (cm <sup>2</sup> )	Height (cm)	Base (cm)
A	8	2	4
B	8	4	2
C	24	4	6
D	35	7	5
E	$15\frac{3}{4}$	$3\frac{1}{2}$	$4\frac{1}{2}$
F	18	6	3
Triangles			
Figure	Area (cm <sup>2</sup> )	Height (cm)	Base (cm)
A	4	2	4
B	4	4	2
C	12	4	6
D	$17\frac{1}{2}$	7	5
E	$7\frac{7}{8}$	$3\frac{1}{2}$	$4\frac{1}{2}$
F	9	6	3

Sample answer for Question C, part 1:  
If you multiply the base and the height, you get the area.

- The areas of all the triangles are the same. The areas of the parallelograms are twice the area of one of the triangles. The triangles have the same bases and heights as the parallelograms. Therefore, the area of the parallelogram is  $2 \times$  area of a triangle whose base is one side of the parallelogram, and its height is the same as the height of the triangle.

- D. 1. No; they will both get the same formula for finding the area of a parallelogram. Any parallelogram can be decomposed into two identical triangles each with area  $\frac{1}{2} \times b \times h$ . Students may see the area formula of a parallelogram as  $2 \times (\frac{1}{2} \times b \times h) = b \times h$ , which shows that both formulas are equivalent.

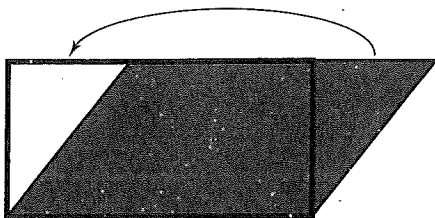
**Torrin's Method**



area of each triangle =  $\frac{1}{2}bh$

area of parallelogram =  $2(\frac{1}{2}bh) = bh$

**Maya's Method**



Area of Parallelogram = Area of Rectangle  
= base  $\times$  height

2. a. The area of a parallelogram =  $b \times h$ . Students may say that this formula works because it is equivalent to the area of the two triangles that make up a parallelogram, or they may say that this formula works because it is the equivalent of the formula for the area of the rectangle that results from cutting off a triangle from one side of a parallelogram and attaching it to the other side. In both cases (decomposing into two triangles, rearranging into a rectangle), the area remains constant; nothing is lost or gained.

b.  $A = 7\frac{2}{3} \times 12 = 92 \text{ cm}$



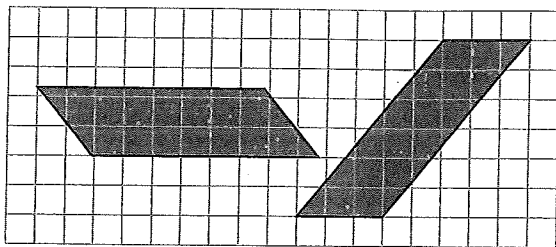
Assignment Guide for Problem 3.2

Applications: 10–21 | Connections: 40–42

Extensions: 80–87

Answers to Problem 3.2

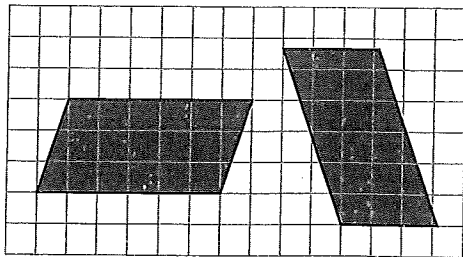
A. 1.



For parallelogram F:

Left: base  $\approx 7.8$  cm, height  $\approx 2.3$  cm

Right: base = 3 cm, height = 6 cm



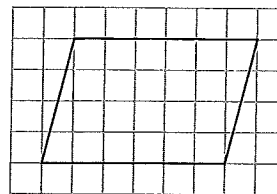
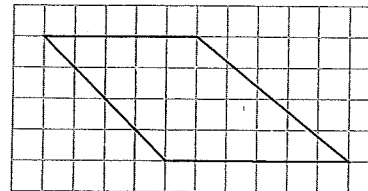
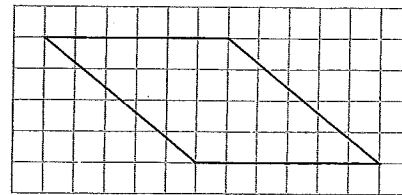
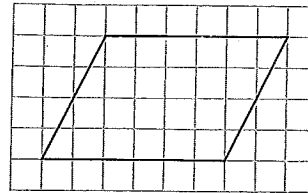
For parallelogram G:

Left: base = 6 cm, height = 3 cm

Right: base  $\approx 3.2$  cm, height  $\approx 5.6$  cm

- In each case, the base times the height is (approximately) equal to the area of the parallelogram:  $18 \text{ cm}^2$ .

- Solutions will vary. Four examples are shown below.



- The area of each parallelogram is  $24 \text{ cm}^2$ . As with a family of triangles, each member of this family of parallelograms has the same base, height, and area.
- All parallelograms with the same base and height will have the same area.

## Summarize

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You can start the summary by asking students to think about the Problem in general.

- What kinds of constraints make drawing a figure easy? What kinds of constraints make drawing a figure difficult?
- Were there any questions for which you could make only one figure that fit the constraints?
- For which descriptions of a parallelogram was it possible to make more than one shape?
- Which measures are necessary for determining area and perimeter of parallelograms? Of triangles?

Question C is the case in which base and height are held constant. There are infinitely many parallelograms that can be drawn with the same base and height. These parallelograms form a "family" with the same base, height, and area.

- Do these parallelograms have the same area? Why?
- Do the parallelograms have the same perimeter? Why?

Question D presents a situation in which side lengths are fixed.

- What is the same and what is different with this set of parallelograms?
  - Why does the area change?
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### Assignment Guide for Problem 3.3

Applications: 22-33

## Answers to Problem 3.3

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- A.** Drawings will vary. The most common drawings will be 1 unit-by-18 unit, 2 unit-by-9 unit, and 3 unit-by-6 unit rectangles. The rectangles do not have the same perimeter.
- B.** It is not possible to draw two different rectangles. Some students may draw the same rectangle twice with different orientations.
- C.** Drawings will vary. All the parallelograms will have an area of  $28 \text{ cm}^2$ . A geoboard is an excellent tool for demonstrating that you can keep the same base and height but move the side parallel to the base to get different parallelograms with the same area.
- D.** Drawings will vary. The areas of the parallelograms can vary from almost 0 to  $36 \text{ cm}^2$ .
- E.** Drawings will vary. The base times the height of all the parallelograms will be  $30 \text{ cm}^2$ . The perimeters of the parallelograms will vary.



### Assignment Guide for Problem 3.4

Applications: 34–38 | Connections: 43

Extensions: 44–48

## Answers to Problem 3.4

A. 1. Figure 1: parallelogram

Figure 2: trapezoid

Figure 3: right triangle

Figure 4: isosceles triangle

2. Figure 1: (2, 8) (3, 11) (7, 11) (6, 8)

Figure 2: (7, 7) (8, 9) (10, 9) (11, 7)

Figure 3: (8, 1) (11, 6) (11, 1)

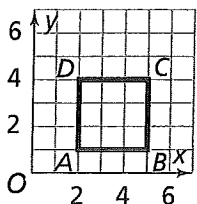
Figure 4: (2, 2) (2, 6) (7, 4)

3. It is possible to use the coordinates to find the lengths of sides when the sides are horizontal or vertical. Figure 1: Horizontal sides (top and bottom) are 4 units long each; Figure 2: Top is 2 units, bottom is 4 units; Figure 3: Bottom is 3 units, right side is 5 units; Figure 4: Left side is 4 units.

4. Figure 1: base = 4 units, height = 3 units, area = 12 units<sup>2</sup>; Figure 2: base<sub>1</sub> = 4 units, base<sub>2</sub> = 2 units, height = 2 units, area = 6 units<sup>2</sup>; Figure 3: base = 3 units, height = 5 units, area = 7.5 units<sup>2</sup>; Figure 4: base = 4 units, height = 5 units, area = 10 units<sup>2</sup>

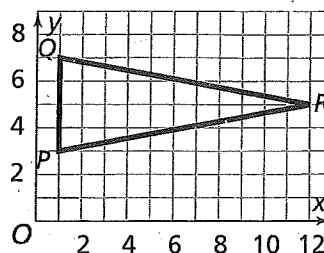
5. The coordinates of the points will change, because the locations of the points are different. The area, perimeter, side lengths, base, and height will remain the same because the shapes themselves are not stretching or shrinking, only their locations are changing.

B. 1. Point C: (5, 4); Point D: (2, 4); area = 9 units<sup>2</sup>

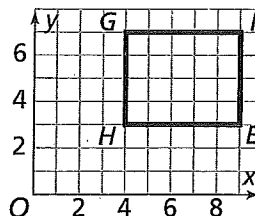


**Note:** There is a second possible answer that involves coordinates in the fourth quadrant. Point C (5, -2); Point D: (2, -2).

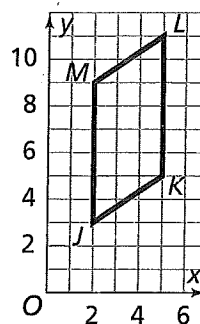
2. Point R: (12, 5); area = 22 units<sup>2</sup>



3. Point H: (4, 3); area = 20 units<sup>2</sup>



4. Point M: (3, 9); area = 21 units<sup>2</sup>



C. 1. Rectangle, top left: x = 4

Right triangle: x = 10

Parallelogram: x = 14

Rectangle, bottom right: x = 12

2. No. The top left rectangle and the parallelogram have the same area (24 square units), but the right triangle has half the area of each of the quadrilaterals (12 square units). The bottom right rectangle has an area of 72 square units, which is three times the area of the top left rectangle.

3. No. The parallelogram has a greater perimeter than the top left rectangle because the slanted sides of the parallelogram are longer than the vertical

