Make a table of the measures they found for base, height, and area of each parallelogram.

- *How is the area of a parallelogram related to the area of a triangle? How are the bases and heights of the triangle and parallelogram related?*
- *If the bases and heights are the same, why is the area of the parallelogram twice the area of the triangle?*
- *How did you use b and h to write a formula for the area of a parallelogram?*

ACE

Assignment Guide for Problem 3.1

Applications: 1–9 | Connections: 39

Answers to Problem 3.1

A. **1.** Figure A: ≈ $12\frac{1}{2}$ cm; Figure B: ≈ $12\frac{1}{5}$ cm; Figure C: ≈ 21 cm; Figure D: ≈ 24 cm; Figure E: ≈ 16 cm; Figure F: ≈ $21\frac{4}{5}$ cm

2. Possible answers: add the lengths of the sides; add the lengths of the two sides that form an angle and double; or double one side length, double the other side length and total.

B. **1.** Figure A: 8 cm^2; Figure B: 8 cm^2; Figure C: 24 cm^2; Figure D: 35 cm^2; Figure E: $15\frac{3}{4}$ cm^2; Figure F: 18 cm^2

2. Possible answers: count the number of whole square centimeters and estimate how many partial square centimeters there are; cut off part of the parallelogram by cutting perpendicular to the base, rearranging to make a rectangle, and then finding the area of the rectangle.

C. **1–2.**

Parallelograms

Figure	Area (cm^2)	Height (cm)	Base (cm)
A	8	2	4
B	8	4	2
C	24	4	6
D	35	7	5
E	$15\frac{3}{4}$	$3\frac{1}{2}$	$4\frac{1}{2}$
F	18	6	3

Triangles

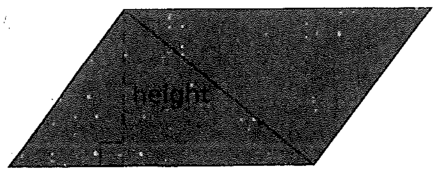
Figure	Area (cm^2)	Height (cm)	Base (cm)
A	4	2	4
B	4	4	2
C	12	4	6
D	$17\frac{1}{2}$	7	5
E	$7\frac{7}{8}$	$3\frac{1}{2}$	$4\frac{1}{2}$
F	9	6	3

Sample answer for Question C, part 1: If you multiply the base and the height, you get the area.

3. The areas of all the triangles are the same. The areas of the parallelograms are twice the area of one of the triangles. The triangles have the same bases and heights as the parallelograms. Therefore, the area of the parallelogram is 2 × area of a triangle whose base is one side of the parallelogram, and its height is the same as the height of the triangle.

D. 1. No; they will both get the same formula for finding the area of a parallelogram. Any parallelogram can be decomposed into two identical triangles each with area $\frac{1}{2} \times b \times h$. Students may see the area formula of a parallelogram as $2 \times \left(\frac{1}{2} \times b \times h\right) = b \times h$, which shows that both formulas are equivalent.

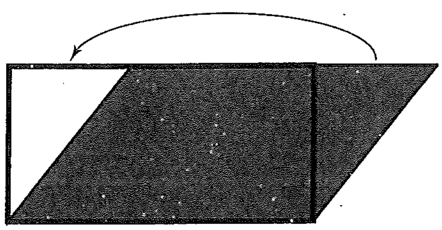
Torrin's Method



area of each triangle $= \frac{1}{2}bh$

area of parallelogram $= 2(\frac{1}{2}bh) = bh$

Maya's Method



Area of Parallelogram = Area of Rectangle
= base × height

2. a. The area of a parallelogram $= b \times h$. Students may say that this formula works because it is equivalent to the area of the two triangles that make up a parallelogram, or they may say that this formula works because it is the equivalent of the formula for the area of the rectangle that results from cutting off a triangle from one side of a parallelogram and attaching it to the other side. In both cases (decomposing into two triangles, rearranging into a rectangle), the area remains constant; nothing is lost or gained.

b. $A = 7\frac{2}{3} \times 12 = 92$ cm

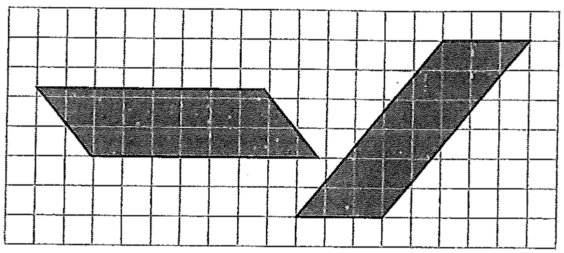
ACE

Assignment Guide for Problem 3.2

Applications: 10–21 | Connections: 40–42
Extensions: 80–87

Answers to Problem 3.2

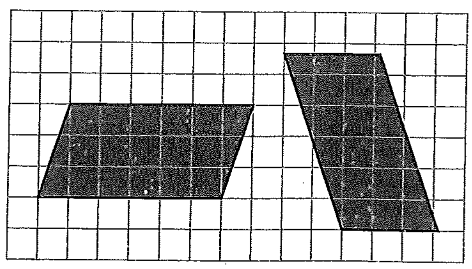
A. 1.



For parallelogram F:

Left: base ≈ 7.8 cm, height ≈ 2.3 cm

Right: base = 3 cm, height = 6 cm



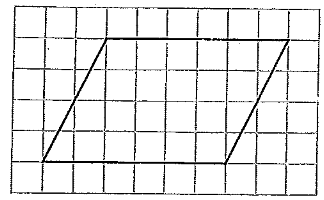
For parallelogram G:

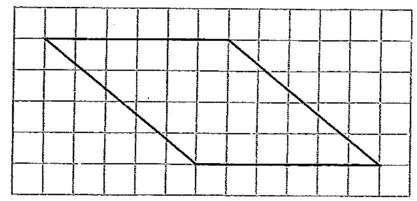
Left: base = 6 cm, height = 3 cm

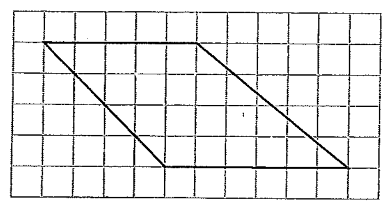
Right: base ≈ 3.2 cm, height ≈ 5.6 cm

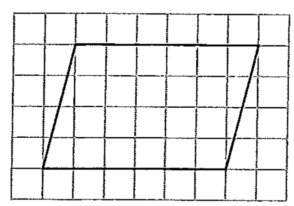
2. In each case, the base times the height is (approximately) equal to the area of the parallelogram: 18 cm^2.

B. 1. Solutions will vary. Four examples are shown below.









2. The area of each parallelogram is 24 cm^2. As with a family of triangles, each member of this family of parallelograms has the same base, height, and area.

3. All parallelograms with the same base and height will have the same area.

Summarize

You can start the summary by asking students to think about the Problem in general.

- *What kinds of constraints make drawing a figure easy? What kinds of constraints make drawing a figure difficult?*
- *Were there any questions for which you could make only one figure that fit the constraints?*
- *For which descriptions of a parallelogram was it possible to make more than one shape?*
- *Which measures are necessary for determining area and perimeter of parallelograms? Of triangles?*

Question C is the case in which base and height are held constant. There are infinitely many parallelograms that can be drawn with the same base and height. These parallelograms form a "family" with the same base, height, and area.

- *Do these parallelograms have the same area? Why?*
- *Do the parallelograms have the same perimeter? Why?*

Question D presents a situation in which side lengths are fixed.

- *What is the same and what is different with this set of parallelograms?*
- *Why does the area change?*

ACE

Assignment Guide for Problem 3.3

Applications: 22–33

Answers to Problem 3.3

A. Drawings will vary. The most common drawings will be 1 unit-by-18 unit, 2 unit-by-9 unit, and 3 unit-by-6 unit rectangles. The rectangles do not have the same perimeter.

B. It is not possible to draw two different rectangles. Some students may draw the same rectangle twice with different orientations.

C. Drawings will vary. All the parallelograms will have an area of 28 cm^2. A geoboard is an excellent tool for demonstrating that you can keep the same base and height but move the side parallel to the base to get different parallelograms with the same area.

D. Drawings will vary. The areas of the parallelograms can vary from almost 0 to 36 cm^2.

E. Drawings will vary. The base times the height of all the parallelograms will be 30 cm^2. The perimeters of the parallelograms will vary.

Assignment Guide for Problem 3.4

Applications: 34–38 | Connections: 43
Extensions: 44–48

Answers to Problem 3.4

A. 1. Figure 1: parallelogram
Figure 2: trapezoid
Figure 3: right triangle
Figure 4: isosceles triangle

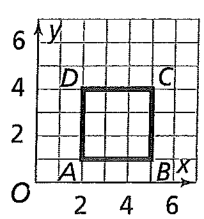
2. Figure 1: (2, 8) (3, 11) (7, 11) (6, 8)
Figure 2: (7, 7) (8, 9) (10, 9) (11, 7)
Figure 3: (8, 1) (11, 6) (11, 1)
Figure 4: (2, 2) (2, 6) (7, 4

3. It is possible to use the coordinates to find the lengths of sides when the sides are horizontal or vertical. Figure 1: Horizontal sides (top and bottom) are 4 units long each; Figure 2: Top is 2 units, bottom is 4 units; Figure 3: Bottom is 3 units, right side is 5 units; Figure 4: Left side is 4 units.

4. Figure 1: base = 4 units, height = 3 units, area = 12 units^2; Figure 2: base$_1$ = 4 units, base$_2$ = 2 units, height = 2 units, area = 6 units^2; Figure 3: base = 3 units, height = 5 units, area = 7.5 units^2; Figure 4: base = 4 units, height = 5 units, area = 10 units^2

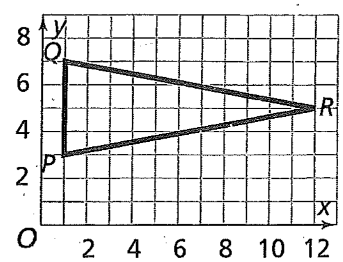
5. The coordinates of the points will change, because the locations of the points are different. The area, perimeter, side lengths, base, and height will remain the same because the shapes themselves are not stretching or shrinking, only their locations are changing.

B. 1. Point *C*: (5, 4); Point *D*: (2, 4); area = 9 units^2

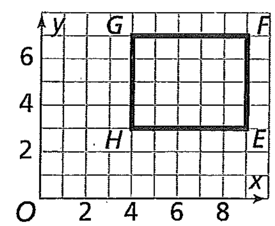


Note: There is a second possible answer that involves coordinates in the fourth quadrant. Point *C* (5, −2); Point *D*: (2, −2).

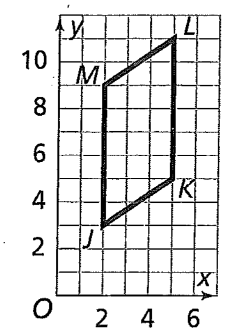
2. Point *R*: (12, 5); area = 22 units^2



3. Point *H*: (4, 3); area = 20 units^2



4. Point *M*: (3, 9); area = 21 units^2



C. 1. Rectangle, top left: $x = 4$
Right triangle: $x = 10$
Parallelogram: $x = 14$
Rectangle, bottom right: $x = 12$

2. No. The top left rectangle and the parallelogram have the same area (24 square units), but the right triangle has half the area of each of the quadrilaterals (12 square units). The bottom right rectangle has an area of 72 square units, which is three times the area of the top left rectangle.

3. No. The parallelogram has a greater perimeter than the top left rectangle because the slanted sides of the parallelogram are longer than the vertical sides of the rectangle. The top left rectangle has a greater perimeter than the right triangle because the base and height of the rectangle are longer (combined) than the hypotenuse of the right triangle. The bottom right rectangle has the greatest perimeter. The base of the bottom right rectangle has the same length as the base of the parallelogram, so students just have to compare the nonbase sides of the other shapes. The bottom right rectangle's vertical sides are longer than the parallelogram's slanted sides.

Note: Students will not be able to definitively calculate the slanted side of the parallelogram, but an estimated answer such as the one provided should suffice.

D. Some students may count the squares. They should be encouraged to look for another method.

1. The area of each polygon is 16 cm^2. Some students may decompose the polygons into triangles and rectangles as shown below. They can then find the area of each triangle and rectangle and add the areas together to find the area of the entire polygon.

   For the first polygon, the area of upper and lower triangles is 2 cm $\times$ 2 cm $\times$ $\frac{1}{2}$ = cm^2 and the area of the rectangle is $2 \times 6 = 12$ cm^2. So the area of the first polygon is 16 cm^2 (i.e., 2 cm^2 (upper triangle) + 2 cm^2 (lower triangle) + 12 cm^2 (rectangle) = 16 cm^2).

   For the second polygon, the area of the upper and lower triangles is 4 cm $\times$ 1 cm $\times \frac{1}{2} = 2$ cm^2 and the area of the rectangle is $3 \times 4 = 12$ cm^2. So the area of the second polygon is 16 cm^2 (i.e., 2 cm^2 (upper triangle) + 2 cm^2 (lower triangle) + 12 cm^2 (rectangle) = 16 cm^2).

   For the third polygon, the area of upper triangle is 4 cm $\times$ 2 cm $\times \frac{1}{2} = 4$ cm^2 and the area of the rectangle is $3 \times 4 = 12$ cm^2. So the area of the third polygon is 16 cm^2 (i.e., 4 cm^2 (upper triangle) + 12 cm^2 (rectangle) = 16 cm^2). (See Figure 1.)

   Some students may enclose each polygon in a rectangle, find the area of the rectangle, and then subtract off the areas of the regions that are in the rectangle but not in the polygon. This may require finding areas of triangles or counting.

2. Angie is correct. There are two ways to explain why she is correct. Explanation 1: If you count the number of unit squares, there are 16 in each polygon. Thus, all the polygons have the same area. Explanation 2: For each polygon, if you measure the area of the shaded rectangle and the area of the triangle(s) and add them, you get the area of the entire polygon. This comes out to 16 cm^2 for each polygon, so all the polygons have the same area.

3. Answers will vary. Possible answers are shown below.

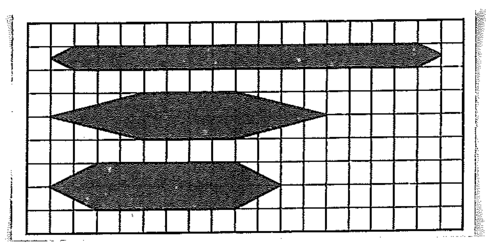
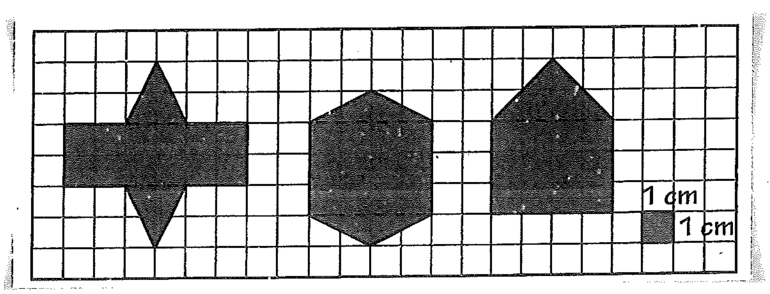


Figure 1



ACE Answers Investigation 3

Applications

1. Area = 20 cm^2 (base = 5 cm, height = 4 cm), perimeter = 20 cm (Each side is 5 cm.)

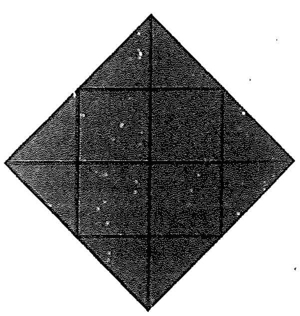
2. Area = 9 cm^2 (base = 3 cm, height = 3 cm), perimeter ≈ 12.4 cm (The side lengths are 3 cm and ~ 3.2 cm.) Explanations will vary.

3. Area = 30 cm^2 (base = 6 cm, height = 5 cm), perimeter ≈ 25 cm (The side lengths are 6 cm and ~ 6.5 cm.)

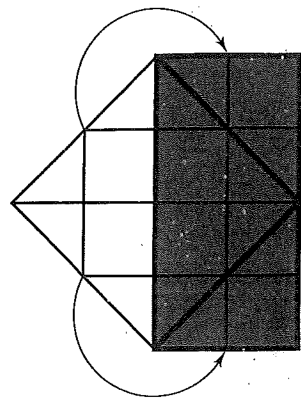
4. Area = 8 cm^2 (base = 2 cm, height = 4 cm), perimeter ≈ 15.4 cm (The side lengths are 2 cm and ~ 5.7 cm.)

5. Area = 6 cm^2 (base = 6 cm, height = 1 cm), perimeter ≈ 20.2 cm (The side lengths are 6 cm and 4.1 cm.)

6. Area = 8 cm^2, perimeter ≈ 11.2 cm (Each side length is about 2.8 cm.)
Note: The area of this square can be found easily. The length of the sides, from the Pythagorean Theorem, are $\sqrt{8} \approx$ 2.8284271. We don't expect students will know this, but some families who help their children may offer this as an answer. Some students may say that the length of the sides is 3 cm, and thus the perimeter is 12 cm. Although this is close, you will want to discuss why it is not possible for the length to be 3 cm. If it were, the area would be 9 cm^2, and they can easily see from the drawing that this is not the case. The figure can be split into 4 pieces.



These pieces can be rearranged to form a rectangle with an area 8 cm^2.



7. Area = 20 cm^2 (base = 5 cm, height = 4 cm), perimeter ~ 19 cm (The side lengths are 5 cm and about 4.5 cm.) Explanations will vary.

8. a. The base is 4 units, the height is 5 units, and the area of each parallelogram is 20 square units.
   b. The bases, heights, and areas are the same.
   c. The parallelograms are a family because the bases, heights, and areas are the same.

9. a. $109\frac{1}{4}$ cm^2; $9\frac{1}{2} \times 11\frac{1}{2} = 109\frac{1}{4}$
   b. $109\frac{1}{4}$ cm^2; $9\frac{1}{2} \times 11\frac{1}{2} = 109\frac{1}{4}$
   c. Usually, the perimeter of a parallelogram will be greater than the perimeter of a rectangle when both have the same base and height. The only times the perimeters will be equal is when the parallelogram is itself a rectangle.

10. $A = 24$ cm^2, $P = 22$ cm

11. $A = 30$ cm^2, $P = 30$ cm

12. $A = 29\frac{3}{4}$ cm^2, $P = 25\frac{7}{10}$ cm

13. $A = 72$ in.^{2}, $P =$ 35 in.

14–19.

Exercise	Approximate Area (cm^2)	Approximate Perimeter (cm)
14	$8(b = 2, h = 4)$	12
15	$4.5(b = 3, h = 3)$	$10\frac{1}{4}$
16	$15(b = 5, h = 3)$	$16\frac{4}{5}$
17	$18(b = 4.5, h = 4)$	$17\frac{1}{5}$
18	$3(b = 3, h = 2)$	$12\frac{3}{10}$
19	$18(15 + 3)$	18

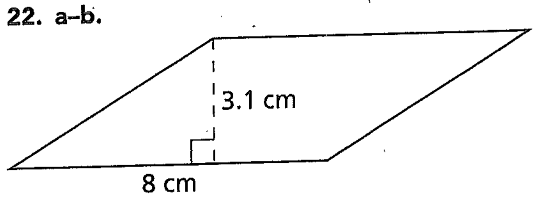
20. **a.** The area of Tennessee is approximately 41,800 mi^2.

b. The estimate is slightly smaller than the actual area because Tennessee is not quite a parallelogram. There are chunks of land attached to the north and west sides of the state beyond the shape of a parallelogram. There is also a chunk of land in the southeast that is not part of Tennessee but was included in the parallelogram estimate. The area of this land that was included in the estimate is greater than the chunks to the north and west that were left out of the estimate.

21. These parallelograms all have the same area because the first two have a base of 4 and a height of 3, and the last parallelogram has a base of 3 and a height of 4. Since area is base times height, the areas are all the same.

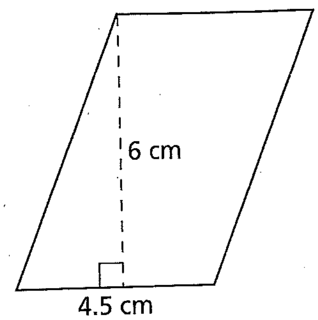
For Exercises 22–27, answers will vary. Possible answers are listed.

22. a–b.



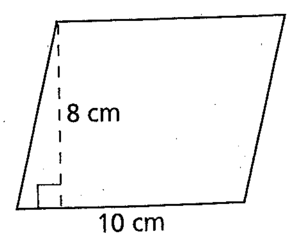
c. Yes, you can draw more than one parallelogram with base 8 and perimeter 28. Rotating the side segments around the two vertices on the base will keep the same side lengths and base. Keeping the side lengths the same while changing the measure of the interior angles will result in a parallelogram with the same perimeter but a different shape and a different area.

23. a–b.



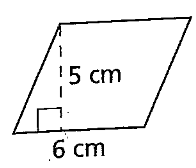
c. Yes, you can draw more than one parallelogram with base $4\frac{1}{2}$ and area 27. You just slide the top vertices over, keeping the same base and height. The area remains the same, but the perimeter changes.

24. a–b.



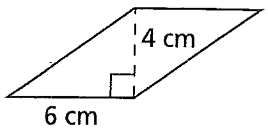
c. Yes, you can draw more than one parallelogram with base 10 and height 8. You just slide the top vertices over, keeping the same base and height. The area remains the same, but the perimeter changes.

25. a–b.



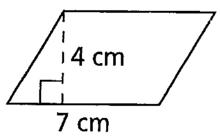
c. Yes, you can draw more than one parallelogram with base 6 and area of 30. Use a height of 5, and slide the top vertices over.

26. a–b.



c. Yes, you could have some of these combinations: $b = 1$, $h = 24$; $b = 2$, $h = 12$; $b = 3$, $h = 8$; $b = 4$, $h = 6$. They would all have an area of 24 cm^2.

27. a–b.



c. Yes, you could have some of these side lengths: $b = 1$, $s = 11$; $b = 2$, $s = 10$; $b = 3$, $s = 9$; $b = 4$, $s = 8$; $b = 5$, $s = 7$; $b = 6$, $s = 6$, where b = base length, and s = nonbase side length. All the parallelograms would have a perimeter of 24 cm^2.

28. a. There are three parallelograms. Each consists of two small triangles.

b. Yes

c. 8 square units

29. a. 4 ft^2

b. 24 ft^2

30. Mr. Lee will need 72 tiles. One possible method: $24 \div 3 = 8$ tiles along the length, $18 \div 2 = 9$ tiles along the width, $8 \times 9 = 72$ tiles. Another method: $18 \div 3 = 6$ tiles along the length, $24 \div 2 = 12$ tiles along the width, $6 \times 12 = 72$ tiles. Another method: $24 \times 18 = 432$ ft^2 is the area for the whole ceiling divided by the area of 1 tile, which is 6 ft^2, to find the total number of tiles, 72.

31. The area of the lot minus the area of the house is the area left for grass.
$20{,}000 - 2{,}250 = 17{,}750$ ft^2

32. a. $\frac{4}{9}$ of the park will be used for skateboarding.
$(\frac{2}{3}\ell \times \frac{2}{3}w = \frac{4}{9}(\ell \times w) = \frac{4}{9}A)$

b. The dimensions of the playground area are 20 ft by 70 ft, giving an area of 1,400 ft^2 and a perimeter of 180 ft.

33. a. 6 $in.^2$ of fabric for each nonsquare parallelogram and 4 $in.^2$ of fabric for each square

b. 20 $in.^2$ of fabric for all the squares together

c. 56 $in.^2$ of gray fabric will be visible.
$100 - (40 + 16) = 100 - 56 = 44$

34. a. 1: (2, 8), (2.5, 11), (4.5, 8), (5, 11)
2: (7, 8), (7, 11), (12, 11)
3: (8, 2), (8, 6), (11, 1), (11, 7)
4: (2, 2), (4.5, 2), (2, 6), (4.5, 6)

b. 1: horizontal = 2.5 units, 2.5 units
2: horizontal = 5 units, vertical = 3 units
3: vertical = 4 units, 6 units
4: horizontal = 2.5 units, 2.5 units, vertical = 4 units, 4 units

c. 1: parallelogram
2: right triangle
3: trapezoid
4: rectangle

35. a. 1: (1, 7), (5, 11), (5,7)

2: (7, 8), (8, 10), (10, 10), (11, 8)

3: (6, 1), (6, 4), (10, 0), (10, 3)

4: (1, 2), (1, 5), (3, 2), (3, 5)

b. 1: horizontal = 4 units, vertical = 5 units
2: horizontal = 2 units, 4 units
3: vertical = 3 units, 3 units
4: horizontal = 2 units, 2 units, vertical = 3 units, 3 units

c. 1: right triangle
2: trapezoid
3: parallelogram
4: rectangle

36. a. $x = 1$, $m = 5$, $n = 2$
b. $y = 3$
c. $x = 3$, $y = 4$
d. $x = 3$, $y = 1$

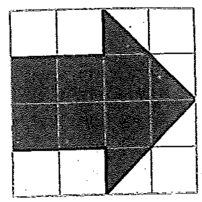
37. a. 1: horizontal = 4, vertical = 4
2: horizontal = 4, vertical = 4
3: horizontal = 4, vertical = 5
4: vertical = 10

b. 1: $A = 4 \times 4 = 16$
2: $A = 4 \times 4 \times \frac{1}{2} = 8$
3: $A = 4 \times 5 = 20$
4: $A = 10 \times 2 = 20$

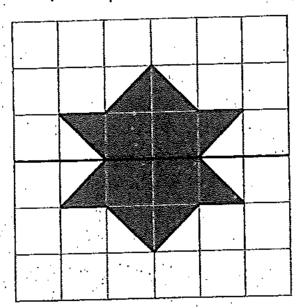
38. **a.** All polygons and the fox shape have same area: 8 cm^2. There are three ways to measure the area of these shapes:

Count inside units

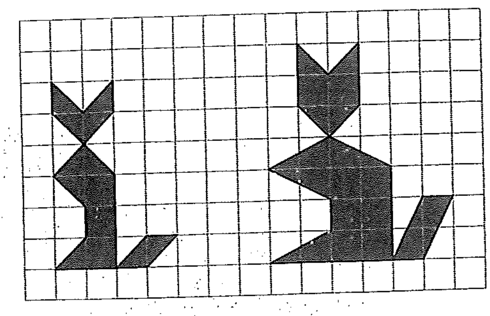
Draw a rectangle around the shape and subtract the pieces not used



Count half the inside squares and double (except the fox shape)



b. Answers will vary. Some students may double the area but not maintain the fox shape. Possible answer:



Connections

39. D

40. J

41. **a.** The angles will change, area will decrease, and perimeter will stay the same.

b. Opposite sides have equal length and are parallel. Opposite angles have equal measure.

42. Answers will vary. Possible examples: two sheets of notebook paper or two speed limit signs

43. **a.** 240 rectangular floor pieces are needed. One possible method: $120 \div 5 = 24$ pieces along the length, $40 \div 4 = 10$ pieces along the width, $24 \times 10 = 240$ pieces.

b. 240 tiles $\times$ \$20 per tile = \$4,800 for the floor. Answers for number of bumper cars will vary. Sample: 30 bumper cars $\times$ \$10 per car = \$300. The total cost is \$5,100.

Begin to discuss Question B by asking the following questions.

- *Explain how you decided where to fold each net.*
- *How can you find the dimensions from the nets? From the box?*

Encourage students to share strategies for finding surface area.

- *What features of the box do you see that might make it easier to find the surface area?*
- *What is the surface area of each box? How did you find it?*
- *Suppose the sides were not drawn on grid paper. How might you find the surface area?*

Have the class save the boxes from Question B to use in the next Problem.

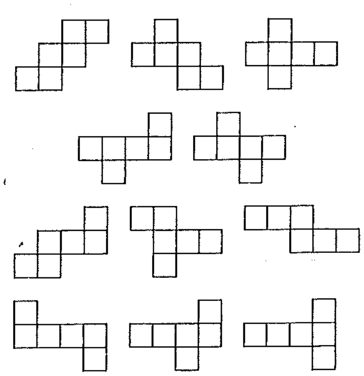
ACE

Assignment Guide for Problem 4.1

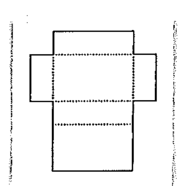
Applications: 1–14 | Connections: 47–51

Answers to Problem 4.1

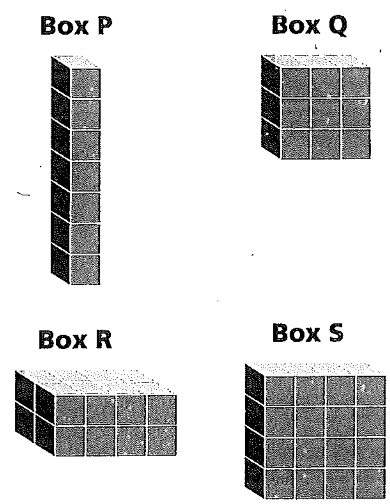
A. 1. There are 35 different nets that can be made with six squares (these are called hexominos), but only the 11 shown below will fold into a cube. The total area of each net is 6 square units. A unit cube has 6 faces, each of which has an area of 1 square unit.



2. Possible net:



B.



1. Box P: 1 cm × 1 cm × 6 cm
   Box Q: 1 cm × 3 cm × 3 cm
   Box R: 2 cm × 2 cm × 4 cm
   Box S: 1 cm × 4 cm × 4 cm
2. Each combination of two dimensions results in the dimensions of a pair of congruent faces.
3. Box P: 26 cm^2; Box Q: 30 cm^2; Box R: 40 cm^2; Box S: 48 cm^2
4. Box P: 6 unit cubes; Box Q; 9 unit cubes; Box R: 16 unit cubes; Box S: 16 unit cubes
5. Answers will vary. The box should hold 6 unit cubes and have dimensions 1 cm × 2 cm × 3 cm.

- *If the length of a side of a cube is 15 centimeters, use each of the three formulas, Kurt's, Natasha's, and Dushane's, to compute the volume.*

For Question C, discuss the difference between half of a unit cube and a $\frac{1}{2}$-unit cube.

- *Are two $\frac{1}{2}$-unit cubes the same volume as a 1-unit cube? Why or why not?*
- *How many $\frac{1}{2}$-unit cubes fit inside a 1-unit cube?*

Question D encourages students to move from concrete situations to more abstract situations. Again, notice which formulas they tend to use.

- *What is being measured when you determine how much tape is needed?*
- *What is your process for finding how much tape is needed for each prism?*

Note that, unlike with surface area, if students add the perimeters of all the faces, they do not get the total edge length. They actually get twice the total edge length.

Summarize

Repeat some of the questions from Question B in the Explore.

- *What is a formula for finding the volume using Kurt's method?*
- *What is a formula for finding the volume using Natasha's method?*
- *How can you find the area of the base?*
- *Is there a relationship between the two formulas?*

Use one of the boxes from Problem 4.1 and ask different students to demonstrate Kurt's strategy and Natasha's strategy and explain why they work.

- *Dushane said that we should use the formula ℓ^3. Would this would work for rectangular prisms?*
- *How does having a fractional dimension affect how you find the volume of a rectangular prism?*

As a final check, you could hold up a juice or cereal box and ask students how they could find the volume.

ACE

Assignment Guide for Problem 4.2

Applications: 15–30 | Extensions: 56–65

Answers to Problem 4.2

A. 1. Prism I: length = 5 cm, width = 4 cm, height = 1 cm; Prism II: length = 5 cm, width = 4 cm, height = 2 cm; Prism III: length = 5 cm, width = 4 cm, height = 5 cm

2. Prism I: volume = 20 cm^3
Prism II: volume = 40 cm^3
Prism III: volume = 100 cm^3

Students might have found the volume by multiplying length × width × height, or by finding the number of cubes in a layer and then multiplying the number of layers by the number of cubes in each layer.

3. Prism I: surface area = 58 cm^2
Prism II: surface area = 76 cm^2
Prism III: surface area = 130 cm^2

Students can use many methods. They can find the surface area by drawing a net and finding the area of the net, or by finding the area of the three faces they can see and doubling that area.

B. 1. Both strategies are correct. Since the base of a rectangular prism is a rectangle and its area can be found by multiplying length × width, the two methods are the same.

2. Kurt's: $V = \ell \times w \times h$, Natasha's: $V = B \times h$, where V = volume, ℓ = length, w = width, h = height, and B = area of base. The formulas are the same since $V = (\ell \times w) \times h = (B) \times h$.

Note that often an upper-case "B" is used to represent the area of a base, whereas a lower-case "b" is used to identify the base segment in a triangle or parallelogram.

3. 840 cm^3

4. The formula, volume $= \ell^3$, is the same as the other two formulas. The height, width, and length of a cube are the same, so they can be represented by the same letter or variable in the volume formula.

C. 1. Durian is correct that he cannot fit unit centimeter cubes exactly into the box, but he is incorrect in thinking that none of the formulas for volume work. Durian could find the correct volume by using a different sized cube to fill the box, or he could use the volume formula by multiplying the fraction edge lengths. Suppose the unit is a centimeter cube. Students can argue that they can either use parts of a 1-centimeter cube, or they can change the size of the unit cube to be $\frac{1}{4}$-centimeter cube or $\frac{1}{2}$-centimeter cube. This is an example of proportional reasoning, which is discussed in more depth in Grade 7 during *Stretching and Shrinking* and *Filling and Wrapping*.

2. 63.75 cm^3. If they use 1-centimeter cubes, the volume is 63.75 cm^3. If they use $\frac{1}{2}$-centimeter cubes, the volume is 8(63.75), or 510, $\frac{1}{2}$-centimeter cubes (which is equal to 63.75 cm^3 since there are eight $\frac{1}{2}$-centimeter cubes in one 1-centimeter cube). If they use $\frac{1}{4}$-centimeter cubes, the volume is 64(63.75), or 4,080, $\frac{1}{4}$-centimeter cubes (which is equal to 63.75 cm^3 since there are sixty-four $\frac{1}{4}$-centimeter cubes in one 1-centimeter cube).

D. 1. Prism I: volume = 32 $in.^3$
Prism II: volume = 326.4 cm^3
Prism III: volume = 67.5 $in.^3$

Methods will vary. One common method will be to multiply length, width, and height.

2. Prism I: surface area = 64 $in.^2$
Prism II: surface area = 308.8 cm^2
Prism III: surface area = 133.5 $in.^2$

Students may use different strategies for finding surface area. One common strategy is to find the area of the three unique faces of the rectangular prism, add those three faces' areas, then multiply that total by 2.

3. Prism I: tape = 40 in.
Prism II: tape = 88.8 cm
Prism III: tape = 64 in.

Find the perimeter of the base of the prism and multiply that perimeter by 2. This will be the amount of tape needed to cover the edges of the top and bottom bases. Then, multiply the height by 4. This will be the amount of tape needed to cover the vertical edges. Add two numbers (the doubled perimeter and the quadrupled height) together to find the total amount of tape needed to cover all the edges of the prism. Another way to find it is by recognizing that there are 4 edges with measure h units, 4 with measure ℓ units, and 4 with measure w units. Therefore, the amount of tape needed is $4(h + \ell + w)$.

Assignment Guide for Problem 4.3

Applications: 31–46 | Connections: 52–55
Extensions: 66–71

Answers to Problem 4.3

A. 1. Box 1: Rectangular prism
Faces are rectangles
Dimensions: four faces—1 cm × 8 cm; two faces—1 cm × 1 cm

Box 2: Pyramid (tetrahedron)
Faces are equilateral triangles
Dimensions: four faces—each edge = 4 cm, height is about 3.5 cm

Box 3: Triangular Prism
Faces are isosceles triangles and rectangles
Dimensions: two isosceles triangles—
base = 6 cm,
height = 4 cm;
one rectangular face—2 cm × 6 cm;
two rectangular faces—2 cm × 5 cm

Box 4: Rectangular prism.
Faces are rectangles
Dimensions: two faces—3 cm × 4 cm;
two faces—2 cm × 3 cm;
two faces—2 cm × 4 cm

2. Box 1: 34 cm^2; Box 2: about 28 cm^2; Box 3: 56 cm^2; Box 4: 52 cm^2

3. One method for finding the surface area of each box is to find the area of each face of the three-dimensional figure. Another method is to find the area of the net, either by finding the area of each face or by finding areas of larger combined shapes (e.g., for Box #3, calculate the combined area of the three rectangles, then double the area of one of the triangular faces, then add those amounts together).

4. Box 1: 40 cm; Box 2: 24 cm; Box 3: 38 cm; Box 4: 36 cm

Method #1: Fold up the net and measure each edge using grid paper or a centimeter ruler.

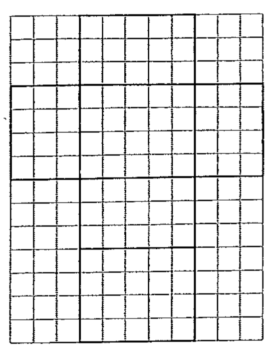
Method #2: Mark each segment that is an outside edge, but be careful not to double-count edges. Then, mark each segment in the interior of the net. Add the lengths of the marked edges.

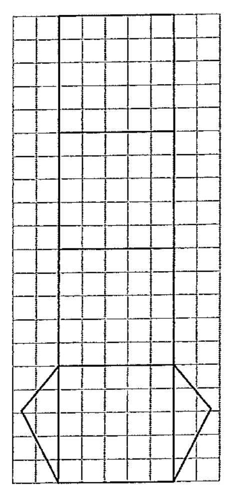
Method #3: Find the perimeter of each face. Add all the perimeters, and then divide by 2. Every edge on the object will be the edge of exactly two shapes of the net, so adding all the perimeters is the same as adding the lengths of all the edges twice. Dividing by 2 gives the amount of tape needed.

5. Answers will vary. Students may state advantages or disadvantages related to appearance, volume, how many boxes fit on a single sheet of grid paper (for example, many pyramids can be made from the same sheet), how much material is needed to construct the box, the strength of the box's structure, etc.

B. 1. To find the surface area, students might draw a net, and then find the area of the net. Another way students might find the surface area is to find the area of each of the faces separately and then add the areas together. The surface area of the right rectangular prism (on the left) is 94 cm^2. The surface of the right triangular prism is 72 cm^2.

2. Nets will vary. Samples:





ACE Answers Investigation 4

Applications

1–4. Patterns 2 and 4 *can* fold to form closed boxes. Patterns 1 and 3 *cannot* fold to form closed boxes.

5. a. Figures 1 and 2 *can* be folded to form a closed box. Pattern C *cannot.*

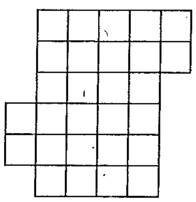
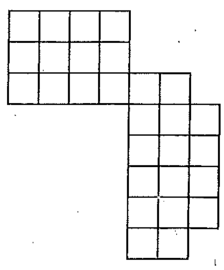
b. Figure 1: 1 unit × 1 unit × 4 units
Figure 2: 1 unit × 2 units × 4 units

c. Figure 1: 18 sq. units
Figure 2: 28 sq. units

d. Figure 1: 4 cubes
Figure 2: 8 cubes

6. a. 2 cm × 4 cm × 1 cm

b. Possible answers:

c. All nets for this box have an area of 28 cm^2.

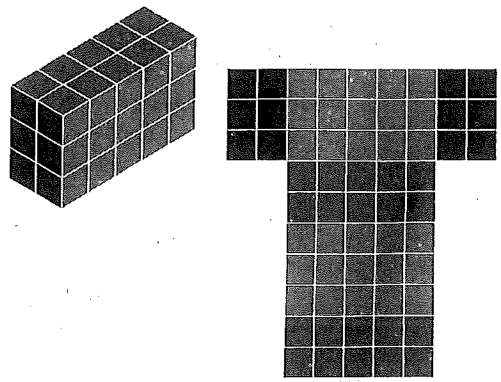
d. There are two faces with an area of 8 cm^2, two with an area 2 cm^2, and two with an area of 4 cm^2 for a total of 28 cm^2. This is the same as the area of the net.

7. Figures 1, 3, 4 and 5 *will not* fold into a box, 2 and 6 *will.* Figures 2 and 6 fold to form boxes because they have edges that will match up fully and evenly when folded. The other figures will not fold to form boxes because edges that are supposed to line up with one another have different lengths, so there will either be overlaps or spaces.

8. a, b, e, f

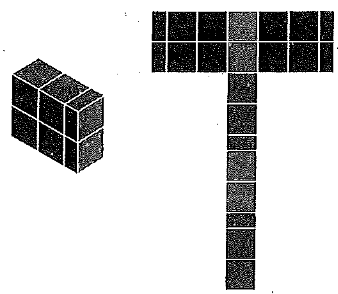
9. This net *will* fold into an open cubic box. The two triangles will meet to become one end of the box.

10. Sketch of box and possible net:



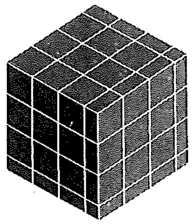
There are two of each of these faces:
2 cm × 3 cm (area is 6 cm^2);
2 cm × 5 cm (area is 10 cm^2);
3 cm × 5 cm (area is 15 cm^2).
The sum of the area of the faces is 62 cm^2.

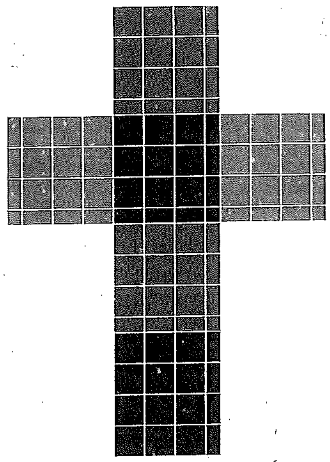
11. Sketch of box and possible net:



There are two of each of these faces:
2 cm × 1 cm (area is 2 cm^2)
2 cm × $2\frac{1}{2}$ cm (area is 5 cm^2)
1 cm × $2\frac{1}{2}$ cm (area is $2\frac{1}{2}$ cm^2).
The sum of the areas of the faces is 19 cm^2.

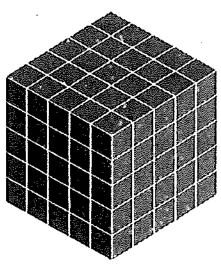
12. Sketch of box and possible net:

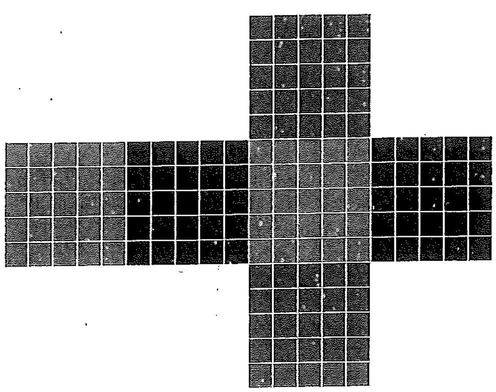




There are six faces. Each is $3\frac{2}{3}$ in. $\times$ $3\frac{2}{3}$ in. (each face has area $13\frac{4}{9}$ in.^{2}). The sum of the areas of the faces is $80\frac{2}{3}$ in.^{2}.

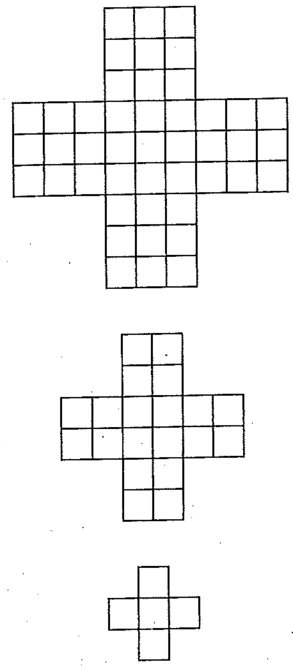
13. Sketch of box and possible net:



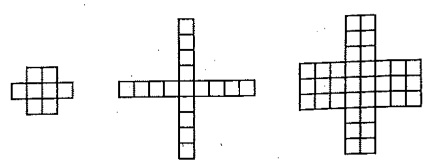


There are six faces. Each is 5 cm $\times$ 5 cm (area is 25 cm^2). The sum of the areas of the faces is 150 cm^2.

14. a. Possible nets:



b. Possible nets:



c. Answers will depend on answers to parts (a) and (b). For the examples given above, the areas (in order) are: (a) 45 square units, 20 square units, 5 square units, and (b) 8 square units, 17 square units, and 36 square units.

15. $\ell = \frac{1}{2}$ in., $w = 2$ in., $h = \frac{1}{2}$ in.

volume = 12 cubic $\frac{1}{2}$-inches

volume = $1\frac{1}{2}$ cubic inches

surface area = $9\frac{1}{2}$ in.^{2}

16. $\ell = 1\frac{1}{2}$ in., $w = 2\frac{1}{2}$ in., $h = 1\frac{1}{2}$ in.

volume = 45 cubic $\frac{1}{2}$-inches

volume = $5\frac{5}{8}$ cubic inches.

surface area = $19\frac{1}{2}$ in.^{2}

17. $\ell = 2\frac{1}{2}$ in., $w = 2\frac{1}{2}$ in., $h = 3\frac{1}{2}$ in.

volume = 175 cubic $\frac{1}{2}$-inches

volume = $21\frac{7}{8}$ cubic inches

surface area = $47\frac{1}{2}$ in.^{2}

Note: There are eight cubes measuring $\frac{1}{2}$ inch on each side (8 cubic $\frac{1}{2}$-inches) in 1in.^{3}. Therefore, if you multiply the volume in cubic inches by 8, you will get the volume in cubic $\frac{1}{2}$-inches.

18. No, Keira does not have enough paper. The surface area of the package is 792 in.^{2}, which is greater than the amount of wrapping paper she has.

19. **a.** square

b. No, you do not have enough information to find the surface area of the pyramid, because the height of the triangular faces is not known.

20. S.A. = 288 m^2; V = 256 m^3

21. S.A. = 864 cm^2; V = 1,728 cm^3

22. S.A. = 301 in.^{2}; V = 294 in.^{3}

23. S.A. = $234\frac{3}{8}$ ft^2; $V = 235\frac{1}{8}$ ft^3

24. Brenda is incorrect, because the units are different for height. She could find the volume in cubic inches by multiplying $36 \times 48 \times 3 = 5{,}184$ in.^{3}. Alternatively, she could find the volume in cubic feet by multiplying $3 \times 4 \times \frac{1}{4} = 3$ ft^3.

25. **a.** 144 in.^{3}

b. 12 board-feet = 1 ft^3. Stacking 12 board-feet together would result in a stack of wood that is 1 ft by 1 ft by 12 in.

c. 6 in. × 6 in. × 48 in. = 1,728 in.^{3}, which is greater than a board-foot (which has a volume of 144 in.^{3}). The difference between the two volumes is 1,728 − 144 = 1,584 in.^{3}.

Note: If students multiply 6 × 6 × 4, they will get 144, but the unit will not be cubic inches, since they have not converted the measurement of 4 ft to 48 in.

26. **a.** 576 in.^{3}

b. Some possible dimensions include: 1 in. × 1 in. × 576 in.; 2 in. × 3 in. × 96 in.; 24 in. × 3 in. × 8 in.; 18 in. × 2 in. × 16 in.; any factor triple of 576 works.

c. No, the surface areas are different. In general, objects with different dimensions typically have different surface areas.

27. **a.** Andrea can use $\frac{1}{2}$-inch blocks or $\frac{1}{4}$-inch blocks.

b. For $\frac{1}{2}$-inch blocks, Andrea would need 5 blocks in a row, 7 blocks in a column, and 8 in a stack. In total, she would need 280 $\frac{1}{2}$-inch blocks. For $\frac{1}{4}$-inch blocks, Andrea would need 10 blocks in a row, 14 blocks in a column, and 16 in a stack. In total, she would need 2,240 quarter-inch blocks.

c. Yes, it is possible to describe the volume as 35 in.^{3}.

28. A (volume of A = 35 in.^{3}; volume of B = 27 in.^{3}; volume of C = 32 in.^{3}; volume of D = 27 in.^{3})

29. One method is to calculate the volumes of all the prisms. A second method is to notice whether or not there is one common dimension in all prisms. That way, you can find the greatest product of the dissimilar dimensions.

30. **a.** 8 blocks

b. $2\frac{1}{4}$ in. long; $\frac{3}{4}$ in. tall; 2 in. wide

c. $2\frac{1}{4}$ in. $\times \frac{3}{4}$ in. $\times$ 2 in. $= 3\frac{3}{8}$ in.^{3}

d. Possible solutions: 36 blocks by 1 block; 18 blocks by 2 blocks; 12 blocks by 3 blocks; 9 blocks by 4 blocks; 6 blocks by 6 blocks

31. **a.** Net 1: S.A. ≈ 21.3 square units
Net 2: S.A. = 48 square units
Net 3: S.A. = 32 square units
Net 4: S.A. = 104 square units

b. Only Nets 3 and 4 fold up to be a right rectangular prism.

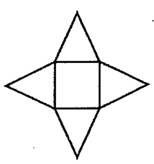
32. A and E; B and F; C and D

33. 6 rectangles

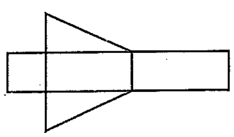
34. 3 rectangles, 2 triangles

35. 1 square, 4 triangles

36.



37.



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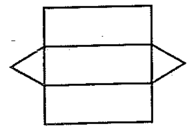

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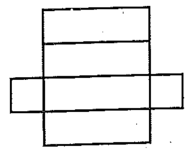


40. 96 in.^{2}

41. 184 m^2

42. 243 cm^2

43. 144 ft^2

44. 60 ft

45. **a.** Two faces are 4 ft × 6 ft, two faces are 4 ft × 12 ft, and two faces are 6 ft × 12 ft.

b. 24 ft^2; 48 ft^2; 72 ft^2

c. 288 ft^2

46. **a.** Z

b. T

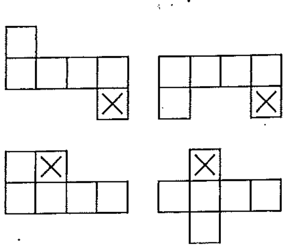
c. M

Connections

47. A, B, C, and E all have a perimeter of 14 units; D a has perimeter of 12 units.

48. B and E

49. Any of hexominos B, C, D, or E can have one square removed to form a net for an open cubic box. Examples:



50. Hexominos B, C, D, and E can all have one square added without changing the perimeter. The perimeter does not change if you add the new square to a corner—the square covers two units of perimeter while adding two new units. In the examples below, the shaded square has been added.