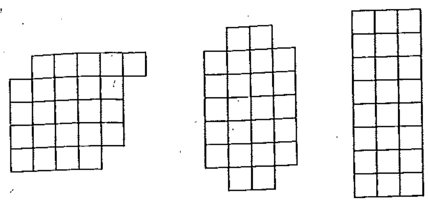
ACE Answers Investigation 1

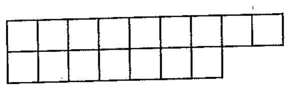
Applications

1. a. Possible answers:

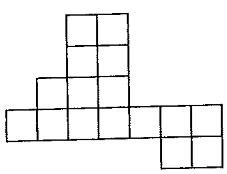


b. The bumper-car ride has an area of 24 m^2, which is the total number of square meters used to cover the floor plan of the bumper-car ride. The perimeter of 22 m is the total number of rail sections that are needed to surround the bumper-car ride.

2. Answers will vary. Maximum perimeter for whole-number dimensions is 34 units, minimum is 16 units. Possible answers:

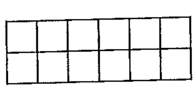


Perimeter: 22 units

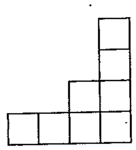


Perimeter: 26 units

3. Answers will vary. Maximum area for whole-number dimensions is 16 square units, minimum is 7 square units. Possible answers:

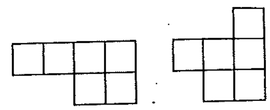


Area: 12 square units

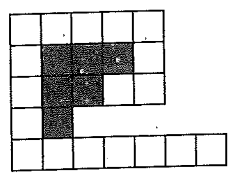


Area: 8 square units

4. Answers will vary. Possible answers:



5. Possible answers: The perimeters are not the same, because I counted the number of units around the edge of each figure and found that their perimeters were different.

6. 

Adding these six tiles reduced the perimeter of the figure. Only two of the new tiles have exposed edges, while together they cover ten previously exposed edges in the original figure.

7. $P = 4 \times 12 \text{ ft} = 48 \text{ ft}$, $A = 12 \text{ ft} \times 12 \text{ ft} = 144 \text{ ft}^2$

8. $P = 22 \times 12 \text{ ft} = 264 \text{ ft}$, $A = 144 \text{ ft}^2 \times 21 = 3{,}024 \text{ ft}^2$

9. $P = 30 \times 12 \text{ ft} = 360 \text{ ft}$, $A = 144 \text{ ft}^2 \times 26 = 3{,}744 \text{ ft}^2$

10. $P = 26 \times 12 \text{ ft} = 312 \text{ ft}$, $A = 144 \text{ ft}^2 \times 20 = 2{,}880 \text{ ft}^2$

11. $P = 16$ units, $A = 7$ units^2

12. $P = 16$ units, $A = 16$ units^2

13. $P \approx 11$ units, $A \approx 5.5$ units^2

14. $P = 40$ in., $A = 100$ in.^{2}

15. $P = 40$ m, $A = 75$ m^2

16. $P = 2\ell + 2w$, $A = \ell \cdot w$

17. Check students' sketches. (See Figure 1.)

18. $A = 65 \text{ cm}^2$, $P = 38$ cm

19. $A = 36 \text{ cm}^2$, $P = 36$ cm

20. a. The next thing that she did was she stretched out the string and measured it. She was finding the perimeter of the figure.

b. She got the same answer that she got by counting, 18 cm.

c. No, she can't because the string method measures length, not area. Instead, she must count all the squares.

21. a. $6 \text{ ft} \times 8\frac{1}{2} \text{ ft} = 51 \text{ ft}^2$

b. 29 ft of molding

c. Two walls have an area of $6 \text{ ft} \times 6 \text{ ft} = 36 \text{ ft}^2$, and two walls would have an area of $6 \text{ ft} \times 8.5 \text{ ft} = 51 \text{ ft}^2$. The total surface area would be $36 + 36 + 51 + 51 = 174 \text{ ft}^2$. You would need 4 pints of paint because $174 \text{ ft}^2 \div 50 \text{ ft}^2 = 3.48$ and you round up to 4 so that you will have enough paint.

d. Check students' work. Answers will vary, but students should use processes similar to those they used in parts (a)–(c). Students also need to make sure that they round the number of pints of paint up to the nearest whole number to make sure they have enough paint to cover the walls.

22. a. Since $40 \times 120 = 4{,}800$, the cost of this model is $4{,}800 \times \$95 = \$456{,}000$

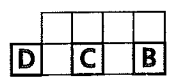
b. $4{,}800 \div 100 = 48$ cars

23. Designs will vary and costs are dependent on the number of tiles and rail sections used. Two possible answers:
7 m by 5 m: area = 35 m^2, perimeter = 24 m, cost = \$1,650.00

6 m by 6 m: area = 36 m^2, perimeter = 24 m, cost = \$1,680.00

Students will have to make a guess to get started and then alter the guess to increase or decrease three inter-related variables. Look for ways that students proceeded from their first guesses.

24. A



25. A 4 ft-by-4 ft square requires the least amount of material for the sides: 16 ft of board.

Figure 1

Rectangle	Length (in.)	Width (in.)	Area (square in.)	Perimeter (in.)
A	5	6	30	22
B	4	13	52	34
C	$6\frac{1}{2}$	8	52	29

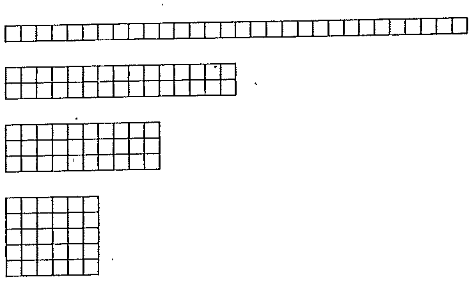
26. a.

Length (ft)	Width (ft)
1	240
2	120
3	80
4	60
5	48
6	40
8	30
10	24
12	20
15	16

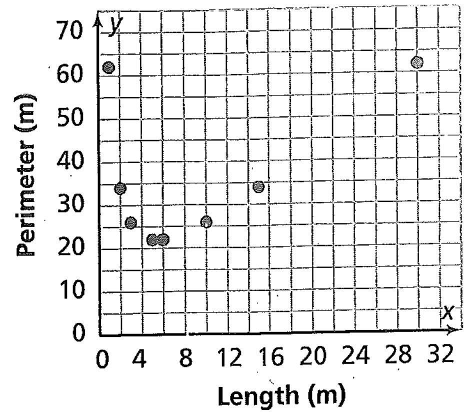
b. Possible answer: A car needs at least 8 ft for the length, so the 8 ft-by-30 ft design would probably be too snug. The 10 ft-by-24 ft, 12 ft-by-20 ft, and 15 ft-by-16 ft designs would all be appropriate as garages.

27. a.

Length (m)	Width (m)	Area (m^2)	Perimeter (m)
1	30	30	62
2	15	30	34
3	10	30	26
5	6	30	22
6	5	30	22
10	3	30	26
15	2	30	34
30	1	30	62



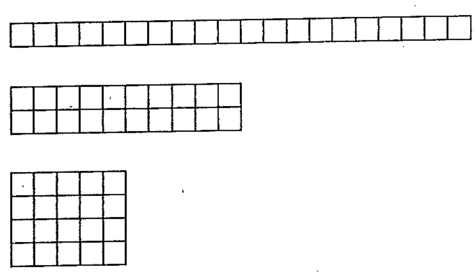
b.



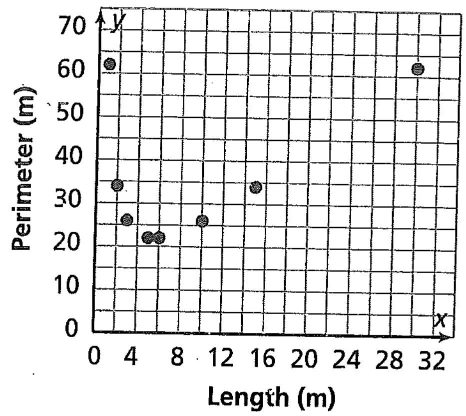
c. On the table, look for the greatest (least) number in the perimeter column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read left to the perimeter axis to get the perimeter. The greatest perimeter is 62 meters (a 1 m × 30 m rectangle). The least perimeter is 22 meters (a 5 m × 6 m rectangle).

28. a.

Length (m)	Width (m)	Area (m^2)	Perimeter (m)
1	20	20	42
2	10	20	24
4	5	20	18
5	4	20	18
10	2	20	24
20	1	20	42



b.



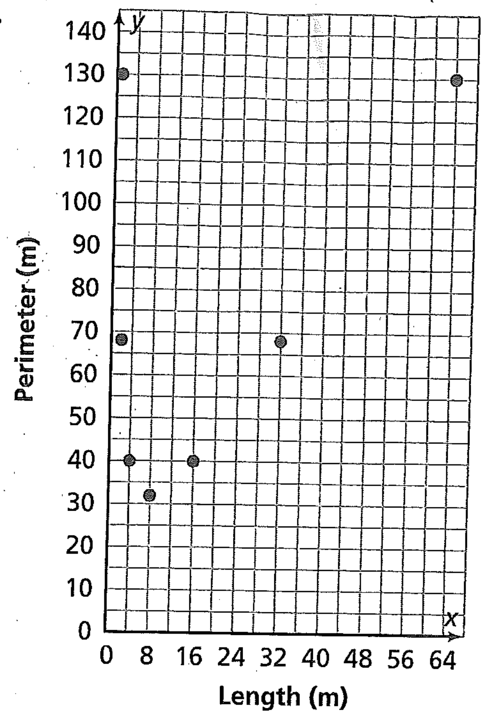
c. On the table, look for the greatest (least) number in the perimeter column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read left to the perimeter axis to get the perimeter. The greatest perimeter is 42 meters (a 1 m × 20 m rectangle). The least perimeter is 18 meters (a 4 m × 5 m rectangle).

29. a.

Length (m)	Width (m)	Area (m^2)	Perimeter (m)
1	64	64	130
2	32	64	68
4	16	64	40
8	8	64	32
16	4	64	40
32	2	64	68
64	1	64	130

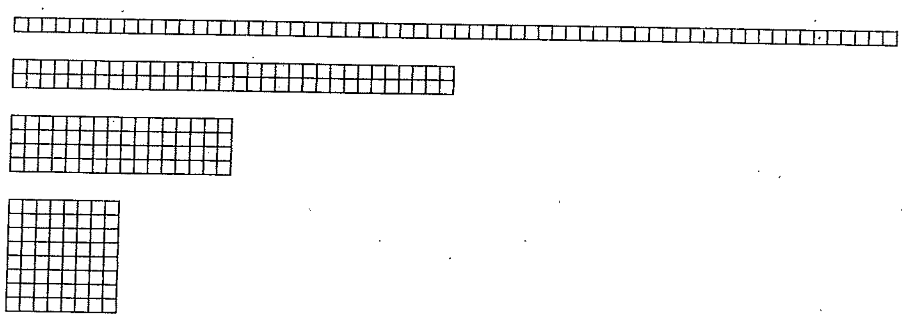
(See Figure 2.)

b.



c. On the table, look for the greatest (least) number in the perimeter column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read left to the perimeter axis to get the perimeter. The greatest perimeter is 130 meters (a 1 m × 64 m rectangle). The least perimeter is 32 meters (a 8 m × 8 m rectangle).

Figure 2



30. **a.** 32 m, 14 m

b. It is a 1 m-by-28 m rectangle. It is long and skinny.

c. It is a 4 m-by-7 m rectangle. It is more compact, or closer to a square.

d. The fixed area is 28 m^2. This is the area of the two rectangles in parts (b) and (c).

31. **a.** 24 m^2, 12 m

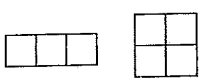
b. It is a 7 m-by-7 m square.

c. This rectangle is long and skinny; 1 m by 13 m.

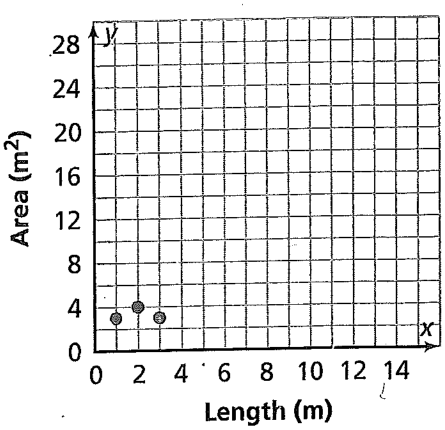
d. The fixed perimeter is 28 m. This is the perimeter of the two rectangles in parts (b) and (c).

32. **a.**

Length (m)	Width (m)	Area (m^2)	Perimeter (m)
1	3	3	8
2	2	4	8
3	1	3	8



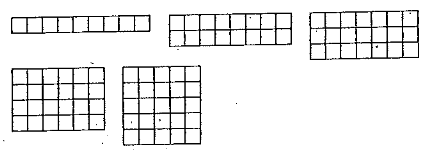
b.



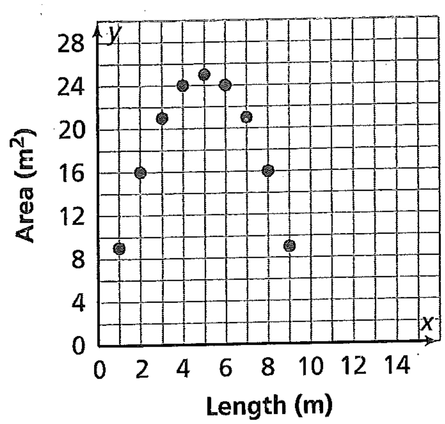
c. On the table, look for the greatest (least) number in the area column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read down to the length axis to get the length. Divide the area by the length to get the width. The greatest area is 4 square meters (a 2 m × 2 m rectangle). The least area is 3 square meters (a 1 m × 3 m rectangle).

33. **a.**

Length	Width	Area	Perimeter
1	9	9	20
2	8	16	20
3	7	21	20
4	6	24	20
5	5	25	20
6	4	24	20
7	3	21	20
8	2	16	20
9	1	9	20



b.



c. On the table, look for the greatest (least) number in the area column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read down to the length axis to get the length. Divide the area by the length to get the width. The greatest area is 25 square meters (a 5 m × 5 m rectangle). The least area is 9 square meters (a 1 m × 9 m rectangle).

d. If they make 20 brownies, then each could be 2 in. by $2\frac{1}{2}$ in. The area of the bottom of the brownie is 5 in.^{2}.

e. If they make 30 brownies, then each could be 2 in. by $1\frac{2}{3}$ in. The area of the bottom of the brownie is $3\frac{1}{3}$ in.^{2}.

53. **a.** $A = 86{,}250$ ft^2, $P = 1{,}210$ ft

b. $A = 86{,}250 \div 9 = 9{,}583\frac{1}{3}$ yd^2, $P = 403\frac{1}{3}$ yd

c. 15 ft × 25 ft = 375 ft^2, 86,250 ÷ 375 = 230 classrooms

Note: The shape of the classroom is not necessarily maintained.

54. **a.** 38.25 square feet

b. Both students are correct. The area of any rectangle can be found by multiplying the length and the width, regardless of whether the values are whole numbers or fractions. Nathan is using the partial products method, finding the area of the four regions then finding the sum. He is using the Distributive Property that was developed in *Prime Time*.

55. The area of a square is side × side and all sides of a square are equal. Therefore, the side length is $\sqrt{169} = 13$ and the perimeter is $4 \times 13 = 52$.

56. No. There are many rectangles that have an area of 120 cm^2, such as 5 × 24, 2 × 60, 40 × 3, etc.

57. F

58. The 36 card tables should be arranged in a straight line, seating 74 people.

59. **a.** 1 by 60, 2 by 30, 3 by 20, 4 by 15, 5 by 12, and 6 by 10.

b. 1 by 61

c. 1 by 62 and 2 by 31

d. The factors of a number and the dimensions of the rectangles that can be made from that number of tiles are the same. For example, the factors of 62 are 1, 2, 31, and 62.

60. This is always true, because $E + E + E + E = E$, $O + O + O + O = E$, and $E + E + O + O = E$, where E stands for an even number, O for an odd number.

Alternatively, the formula $2(\ell + w)$ shows that 2 is a factor of any perimeter with whole-number length and width.

61. All of them are correct. A rectangle has four sides: two lengths and two widths.

62. $2 \times (5 + 7.5) = 25$

63. By using Stella's formula, $2 \times (50 + w) = 196$, the width is 48 cm.

64. Matt is correct because a square has four sides and all the sides (two lengths and two widths) are equal.

65. $A = 121$ in.^{2}, $P = 44$ in.

66. $A = 156.25$ in.^{2}, $P = 50$ in.

67. $\sqrt{144} = 12$ cm

68. **a.** Possible answer:
Largest rectangle: $\frac{5.6 \text{ cm}}{3.5 \text{ cm}} = 1.6$
Second-largest rectangle: $\frac{3.5 \text{ cm}}{2.15 \text{ cm}} \approx 1.63$
Third-largest rectangle: $\frac{2.15 \text{ cm}}{1.3 \text{ cm}} \approx 1.65$

b. Possible answer: The Nautilus shell is so popular because the dimensions of its spiral shape are close to the golden ratio, which makes it visually appealing.

69. **a.** Possible answer:
Largest rectangle: $\frac{7.6 \text{ cm}}{4.7 \text{ cm}} \approx 1.617$
Second-largest rectangle: $\frac{2.5 \text{ cm}}{1.4 \text{ cm}} \approx 1.786$
Third-largest rectangle: $\frac{1.1 \text{ cm}}{0.7 \text{ cm}} \approx 1.571$

Some of the ratios are less than the golden ratio, and others are greater. Overall, the ratios are all close in value to each other and to the golden ratio.

c. About 104 ft; you can approximate its width by using the golden ratio, 1 : 1.62.

Extensions

70. You may want to ask students to write their answers as fractions because the patterns are more obvious.

a. $\frac{1}{4}$ m

b. Rectangle: side lengths are $\frac{1}{4}$ m, $\frac{1}{8}$ m, $\frac{1}{4}$ m, $\frac{1}{8}$ m; perimeter is $\frac{3}{4}$ m.

c. Rectangle: side lengths are $\frac{1}{4}$ m, $\frac{3}{16}$ m, $\frac{1}{4}$ m, $\frac{3}{16}$ m; perimeter is $\frac{7}{8}$ m.

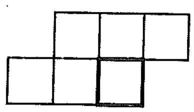
d. Rectangle: side lengths are $\frac{1}{4}$ m, $\frac{7}{32}$ m, $\frac{1}{4}$ m, $\frac{7}{32}$ m; perimeter is $\frac{15}{16}$ m.

e. Perimeter $= \frac{31}{32}$ m

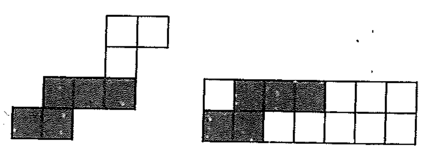
71. 3 cm $\times$ 6 cm rectangle

72. No; there are many different combinations of lengths and widths that could add up to a certain perimeter. A square, on the other hand, has four equal sides, so you can find the length of one of the sides by dividing the perimeter by 4.

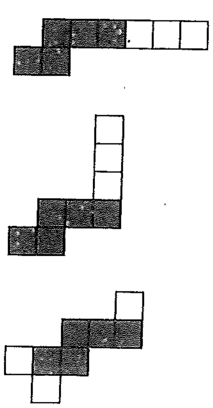
73. a. Yes. See diagram below.



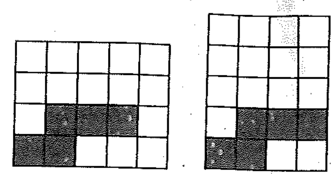
b. Answers will vary. Possible answers:



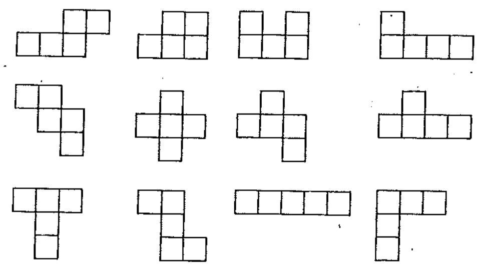
c. 3; each of the three tiles must touch only one edge as they are added. Possible answers:



d. 15 tiles can be added. Each figure must enclose the pentomino in a 4-by-5 rectangle. Possible answers:

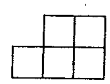


74. a.



b. Possible answer: I conducted a systematic search.

c. This pentomino has the least perimeter, 10 units, because four of the tiles have two edges joined. All of the other pentominos have a perimeter of 12 units.



75. a. The area of Loon Lake is 38–42 square units (380,000–420,000 square meters). The area of Ghost Lake is 34–37 square units (340,000–370,000 square meters).

b. Answers will vary, but one possibility is to use a grid such as a 0.5 with smaller units.

76. a. You could wrap a string around the lake on the grid and then measure the string.

b. The perimeter of Loon Lake is 25–26 units (2,500–2,600 m). The area of Ghost Lake is 45–50 units (4,500–5,000 m).

77. a. Ghost Lake. Ghost Lake would also make a better Nature Preserve since it has more shoreline for bird nests, a variety of vegetation, etc.

b. Loon Lake has more room to cruise.

c. Loon Lake is better for swimming, boating, and fishing.

d. Ghost Lake has more shoreline for campsites.

78. a. approximately 31 cm^2

b. approximately 40 cm

c. The amount of rubber in the sole is related to the area of the foot. The amount of thread required to stitch the sole to the rest of the shoe is related to the perimeter (although you would have to ignore the part of the perimeter between the toes!).