ACE Answers Investigation 2

Applications

1. 24, 48, 72, and 96; the LCM is 24.
2. 15, 30, 45, 60, 75, and 90; the LCM is 15.
3. 77; the LCM is 77.
4. 90; the LCM is 90.
5. 72; the LCM is 72.
6. 100; the LCM is 100.
7. 42, 84; the LCM is 42
8. 60; the LCM is 60.
9. a. Possible answers: 3, 5; 8, 9; 7, 11
   b. They have no common factors except 1.
10. Possible answers: 2, 5; 1, 10
11. Possible answers: 4, 9; 18, 36
12. Possible answers: 4, 15; 12, 5
13. Possible answers: 3, 35; 7, 15
14. a. Twenty-four 1-hour shifts; twelve 2-hour shifts; eight 3-hour shifts; six 4-hour shifts; four 6-hour shifts; three 8-hour shifts; two 12-hour shifts, and one 24-hour shift. These are all factors of 24.
    b. 45 seconds, which is the LCM of 9 and 15
15. 24 days
16. 1, 2, 3, and 6; the GCF is 6.
17. 1; the GCF is 1.
18. 1, 3, 5, and 15; the GCF is 15.
19. 1; the GCF is 1.
20. 1, 7; the GCF is 7.
21. 1, 5; the GCF is 5.
22. 1, 2; the GCF is 2.
23. 1, 3, 7, 21; the GCF is 21.
24. D
25. F
26. D
27. a. 2 packages of hot dogs and 3 packages of buns; 1 hot dog and 1 bun
    b. 10 packages of hot dogs and 15 packages of buns; 4 hot dogs and 4 buns
28. 20 members; each member gets 1 cookie and 2 carrot sticks.
    10 members; each member gets 2 cookies and 4 carrot sticks.
    5 members; each member gets 4 cookies and 8 carrot sticks.
    4 members; each member gets 5 cookies and 10 carrot sticks.
    2 members; each member gets 10 cookies and 20 carrot sticks.
    1 member; the member gets all 20 cookies and 40 carrot sticks.
29. a. Answers will vary. Sample: The Morgan family buys a 12-pack of bottled water and a 24-pack of boxes of raisins. Each person in the family gets the same number of bottles of water and the same number of boxes of raisins. How many people could the Morgan family have?
    b. Answers will vary. Sample: John eats an apple once a week. Ruth eats an apple every third day. If they both eat an apple today, when will John and Ruth next eat an apple on the same day?
    c. The Morgan family could have 1, 2, 3, 4, 6, or 12 people; these numbers are common factors of 12 and 24. John and Ruth will next eat an apple on the same day in 21 days; this problem involves overlapping cycles, so it can be solved with common multiples.
30. Students need to be able to reason proportionally (without knowing that vocabulary) to move from 20 minutes in 1 day to 1 hour in 3 days to 12 hours in 36 days. Julio's watch gains 12 hours in 36 days. Mario's watch gains 12 hours in 12 days. Since 12 is a factor of 36, the watches will next show the correct time together 36 days after Julio and Mario set their watches, or at 9:00 A.M. on the 6th Tuesday.

31. a. 30 students; each student receives 4 cans of juice and 3 packs of crackers because $120 = 30 \times 4$ and $90 = 30 \times 3$.

  b. 8 students; each student receives 15 cans of juice and 11 packs of crackers because $120 = 8 \times 15$ and $88 = 8 \times 11$.

32. 7, 14, 21, and 42 ($42 = 2 \times 3 \times 7$ and $6 = 2 \times 3$.)

33. any odd multiple of 3

34. a. Aaron's method does not work for any pair of numbers. For example, the LCM of 4 and 8 is 8, which does not equal 4×8, or 32. Ruth's method does not work for any pair of numbers. For example, the LCM of 2 and 7 is 14, which does not equal 7. Walter's method does not work for any pair of numbers. For example, the LCM of 3 and 5 is 15, which does not equal $3 \times \frac{5}{2}$, or $\frac{15}{2}$.

  b. Aaron's method works when the two numbers in pair don't have any common factors except 1; Ruth's method works when the greater of the two numbers in pair is a multiple of the lesser number; Walter's method works when 2 is the greatest common factor of the two numbers in pair.

Connections

35. 7 is a factor of 63. 9 is a factor of 63. 7 is a divisor of 63. 9 is a divisor of 63. 63 is a multiple of 7. 63 is a multiple of 9. The product of 7 and 9 is 63. 63 is divisible by 7. 63 is divisible by 9.

36. 4; $12 \times 4 = 48$

37. 10; $11 \times 10 = 110$

38. 8; $6 \times 8 = 48$

39. 11; $11 \times 11 = 121$

40. 4,995,000; multiply 4,995 by 1,000 for the three factors of 10.

41. a. In 2 hours, the jet will travel $12 \times 2 \times 60 = 1{,}440$ kilometers. In 6 hours, the jet will travel $1{,}440 \times 3 = 4{,}320$ kilometers.

  b. In 6 hours, the jet will travel $4{,}320 - 1{,}440 = 2{,}880$ kilometers more than in 2 hours.

  c. In 4 hours, the jet would travel twice as many miles as in 2 hours, or 2,880 kilometers.

42. This question also asks students to reason proportionally. (Note: This provides preparation for the next Unit, *Comparing Bits and Pieces*.)

  a. $9 \times 5 \times 7$ will be 3 times as great as $3 \times 5 \times 7$, or 315.

  b. $3 \times 5 \times 14$ will be twice as great as $3 \times 5 \times 7$, or 210.

  c. $3 \times 50 \times 7$ will be 10 times as great as $3 \times 5 \times 7$, or 1,050.

  d. $3 \times 25 \times 7$ will be 5 times as great as $3 \times 5 \times 7$, or 525.

43. a. composite and square ($5 \times 5 = 25$)

  b. prime

  c. composite ($3 \times 17 = 51$)

  d. square ($1 \times 1 = 1$)

Extensions

44. $3 \times 4 \times 5 + 1 = 61$ (Find the LCM of 2, 3, 4, 5, and 6, and add 1.)

45. 14 and 35

46. sometimes; GCF(2, 2) = 2 and GCF(4, 6) = 2, but GCF(6, 12) = 6, GCF(8, 20) = 4.

47. always; every prime number has only two factors, 1 and itself. Therefore, any two different prime numbers have no common factors other than 1.

48. sometimes; it is only true when the two numbers in the pair are both 1.

49. always; the greatest factor of any number n is itself.

50. always; the multiples of n are $n \times 1$, $n \times 2$, $n \times 3$, . . . The multiples of 1 are 1×1, 1×2, 1×3, . . ., $1 \times n$, . . . Therefore, LCM(n, 1) = n.

51. never; the LCM of any number n greater than 1 and itself is n. Note that the statement is true for $n = 1$.

52. sometimes; it is only true when the number doesn't have 3 as a factor.

53. always; the multiples of p are $p \times 1$, $p \times 2$, $p \times 3$, . . . The multiples of 1 are 1×1, 1×2, 1×3, . . ., $1 \times p$, . . . Therefore, LCM(p, 1) = p.

54. Answers will vary. Example: The GCF of any composite number c and any prime number p is p; sometimes; it is only true when the composite number in the pair has the factor p.

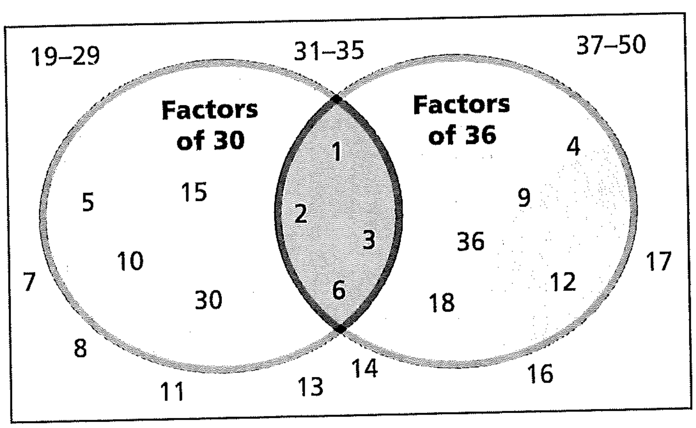
55. Factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30. Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, and 36.
(See Figure 1.)

a. They are common factors of both 30 and 36 (also factors of 6).

b. The numbers in the intersection are 1, 2, 3, and 6. The greatest of these is 6, so 6 is the GCF of 30 and 36.

c. The lowest common factor is 1. 1 is a factor of all whole numbers, so it is in the intersection of Venn diagrams for factors.

Figure 1



56. Factors of 20: 1, 2, 4, 5, 10, and 20.
Factors of 27: 1, 3, 9, and 27.
(See Figure 2.)

a. There is only one number in the intersection: 1. It is a factor of both 20 and 27.

b. The only number in the intersection is 1. It is the only common factor of 20 and 27. Since the only factor in common is 1, it is the GCF.

c. In the Venn diagram for 30 and 36, there are several common factors. In the Venn diagram for 20 and 27, only 1 is in the intersection since 20 and 27 have no common factors other than 1.

57. The multiples of 5 up to 40 are 5, 10, 15, 20, 25, 30, 35, and 40. The multiples of 4 up to 40 are 4, 8, 12, 16, 20, 24, 28, 32, 36 and 40.
(See Figure 3.)

a. They are multiples of both 5 and 4.

b. The least number in the intersection is 20. So, 20 is the LCM of 5 and 4.

c. 60, 80, 100, 120, 140, 160, etc. These numbers are all multiples of the LCM, which is 20. There is no greatest common multiple.

Figure 2

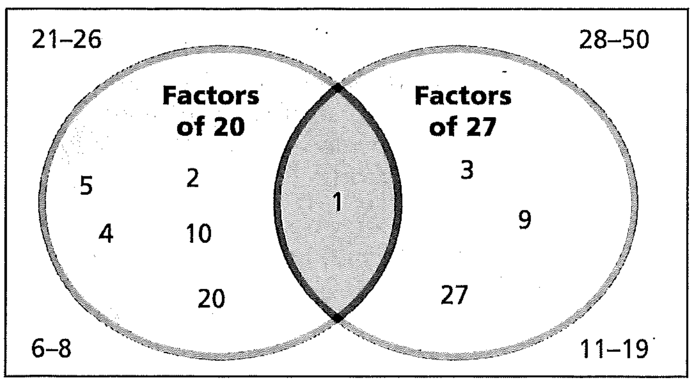
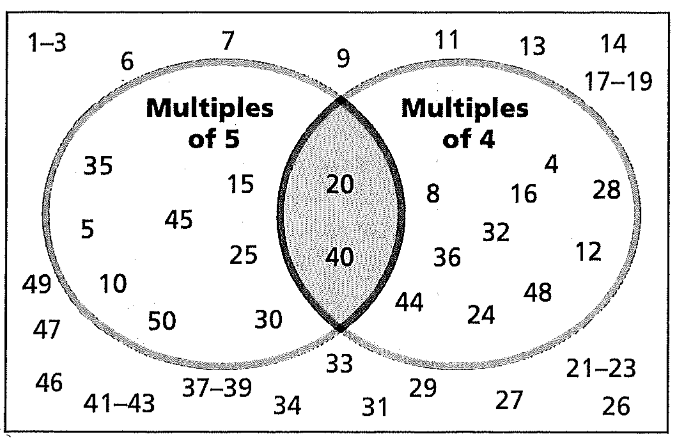


Figure 3



58. The multiples of 6 up to 48 are 6, 12, 18, 24, 30, 36, 42, and 48. The multiples of 8 up to 48 are 8, 16, 24, 32, 40, and 48. (See Figure 4.)

- **a.** They are multiples of both 6 and of 8.
- **b.** The least number in the intersection is 24. So, 24 is the LCM of 6 and 8.
- **c.** In the Venn diagram for 5 and 4, only multiples of 5×4 are in the intersection. In the Venn diagram for 6 and 8, 24 is in the intersection, but it is not a multiple of 6×8.

59. 90 years

60. **a.** 36

- **b.** 0
- **c.** Eric forgot that multiplication is commutative, e.g., $3 \times 4 = 4 \times 3$. He only needs to know 28 different products. Also, zero times anything is zero, and 13 of the 49 computations involve zero as a factor. (**Note:** There are only 19 different answers possible using two factors from 0 through 6.)

61. **a.** 12-year cicadas would meet 2-year predators either every time they emerge or never. The 13-year cicadas would encounter predators every other time they emerge, so they could be better or worse off depending on whether the predator came out on odd or even years.

- **b.** The 12-year cicadas would meet one or both types of predators every time they emerge. The 13-year cicadas would meet the 2-year predators every other time they emerge, and the 3-year predators every third time they emerge. This means that it would be 6 cycles, or 78 years, before the 13-year cicadas would have to face both predators again. They are better off than the 12-year cicadas.

62. 4 and 6; GCF(4, 6) = 2 and LCM(4, 6) = 12

63. 2 and 4; LCM(2, 4) = 4.

64. 4 and 8; GCF(4, 8) = 4.

65. 2 and 2; GCF(2, 2) = 2 and LCM(2, 2) = 2. In general, the GCF and LCM are equal if and only if the two numbers in pair are the same.

66. 24 and 36; GCF(24, 36) = 12 and LCM (24, 36) = 72.

67. 5, 10 and 15; GCF(5, 10, 15) = 5.

68. 3, 6 and 8; LCM(3, 6, 8) = 24.

69. 2, 3, 5 and 7; GCF(2, 3, 5, 7) = 1.

Figure 4