ACE Answers Investigation 1

Applications

1. a. Divide 24 by 12 to see if you get a whole number. Since $12 \times 2 = 24$ or $24 \div 12 = 2$, 12 is a factor.

   b. Divide 291 by 7 to see if the answer is a whole number. Since $291 \div 7 = 41.571429...$, 7 is not a factor of 291.

2. a. 4
   b. 9
   c. 8
   d. 9

3. a. 21 has the most factors; 1 and 19 are the factors of 19; 1, 3, 7, and 21 are the factors of 21; 1 and 23 are the factors of 23; 1, 5, and 25 are the factors of 25.

   b. Answers will vary. Possible answer is 36; 1, 2, 3, 4, 6, 9, 12, 18, and 36 are the factors of 36.

   c. Answers will vary. Possible answer is 18; 1, 2, 3, 6, 9, and 18 are the factors of 18.

4. a. 6; Since the result is a whole number, 14 is a factor of 84.

   b. 5.6; Since the result is not a whole number, 15 is not a factor of 84.

5. a. Yes; $18 \div 6 = 3$

   b. No; $6 \div 18 = 0.3333...$

6. 2, 8, 16 are divisors of 64

7. a. $n = 8$

   b. $n = 12$

   c. $n = 12$

   d. $n = 20$

   e. $n = 3$

8. a. (See Figure 1.)

   b. Keiko picked 28. Cathy scored 21 points for 7 and 14, the factors of 28 that were not already circled. Here is the game board at this stage in the game: (See Figure 2.)

   c. Cathy picked 18 or 27. Keiko scored 9 points by circling 9 (the factor of 18 or 27 that is not already circled).

   d. Cathy scores 30 points for 5, 10, and 15. (the factors of 30 that are not already circled).

   e. The only numbers remaining with uncircled factors are 22 (11) and 26 (13). Cathy should choose 26 because she will score 13 more points than Keiko.

9. Check every number beginning with 1 until you begin to get the same factors over again. With each small number factor you find, you will find a second factor when you divide. Factors of 110 are: 1, 2, 5, 10, 11, 22, 55, 110. I know I have found all of the factors because I checked all the numbers from 1 to 10, and then started getting repeats.

10. a. 30 is the least possibility. Others are all multiples of 30. Common factors of any two such numbers must include 1, $2 \times 3 = 6$, $2 \times 5 = 10$, $3 \times 5 = 15$ and $2 \times 3 \times 5 = 30$ in addition to 2, 3, and 5

    b. 8 is the least possibility. Others are all multiples of 8. Common factors of any three such numbers must include 1 in addition to 2, 4, and 8.

Figure 1

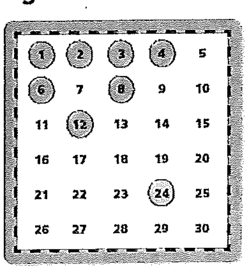
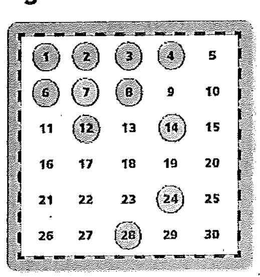


Figure 2



11. a. In the Factor Game, your opponent scores points for proper factors of the number you choose. The only proper factor of prime numbers, such as 2, 3, or 7, is 1.

b. Some numbers, such as 12, 20, and 30, have many proper factors that would give your opponent more points. (Some numbers, such as 9, 15, and 25, have fewer factors and would give you more points.)

12. a. i. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.
ii. Multiples of 3; 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30.
iii. 12, 21, 30, and 210 will all appear in the sequence, because they are all multiples of 3.

b. i. 7, 14, 21, 28, 35, 42, 49, 56, 63, 70.
ii. Multiples of 7
iii. 21 and 210 will appear in the sequence, because they are multiples of 7.

c. i. $6 \times 1 = 6$; $6 \times 2 = 12$; $6 \times 3 = 18$; $6 \times 4 = 24$; $6 \times 5 = 30$; $6 \times 6 = 36$; $6 \times 7 = 42$; $6 \times 8 = 48$; $6 \times 9 = 54$; and $6 \times 10 = 60$.
ii. Multiples of 6
iii. 12, 30, and 210 will appear in the sequence, because they are multiples of 6

13. a. (See Figure 3.)

b. 31, 37, 41, 43, and 47 are prime.

c. 36 and 49 are square numbers.

d. 47; it is the greatest prime number.

e. 48; The second player gets 76 points, which is 28 more points than the first player.

Figure 3

First Move	Proper Factors	My Score	Opponent's Score
31	1	31	1
32	1, 2, 4, 8, 16	32	31
33	1, 3, 11	33	15
34	1, 2, 17	34	20
35	1, 5, 7	35	13
36	1, 2, 3, 4, 6, 9, 12, 18	36	55
37	1	37	1
38	1, 2, 19	38	22
39	1, 3, 13	39	17
40	1, 2, 4, 5, 8, 10, 20	40	50
41	1	41	1
42	1, 2, 3, 6, 7, 14, 21	42	54
43	1	43	1
44	1, 2, 4, 11, 22	44	40
45	1, 3, 5, 9, 15	45	33
46	1, 2, 23	46	26
47	1	47	1
48	1, 2, 3, 4, 6, 8, 12, 16, 24	48	76
49	1, 7	49	8

14. **a.** Move the paper clip from 6 to make the products 5×1, 5×2, 5×3, 5×4, 5×5, 5×7, 5×8, and 5×9; move the paper clip from the 5 to make the products 6×1, 6×2, 6×3, 6×4, 6×6, 6×7, 6×8, and 6×9.

b. Moving the paper clip from the 6 to the 3, 4, or 9, makes 15, 20, or 45, respectively; moving the paper clip from the 5 to the 7 makes 42.

c. Moving the paper clip from the 5 to the 7 makes 6×7, which is 42.

d. Possible answer: Move the 5 to the 7 to get 42; this blocks the opponent and gets 3 in a row.

15. **a.** $3 \times 1 = 3$; $3 \times 2 = 6$; $3 \times 3 = 9$; $3 \times 4 = 12$; $3 \times 5 = 15$; $3 \times 6 = 18$; $3 \times 7 = 21$; $3 \times 8 = 24$; $3 \times 9 = 27$. So you can get a 3, 6, 9, 12, 15, 18, 21, 24 and 27, which are all multiples of 3.

b. $3 \times 11 = 33$; $3 \times 13 = 39$; $3 \times 17 = 51$; $3 \times 19 = 57$; $3 \times 20 = 60$; and many others.

c. There are infinitely many multiples of 3.

16. **a.** 2 and 9; or 3 and 6

b. 1 and 18

17. **a.** 36 can be found on the product game by 6×6 or 4×9. 36 is composite.

b. 5 can be found on the product game by only 1×5. 5 is prime.

c. 7 can be found on the product game by only 1×7. 7 is prime.

d. 9 can be found on the product game by 1×9 or 3×3. 9 is composite.

18. Since the numbers on the game board are multiples of the numbers given as possible factors, you could argue in support of Sal's position. The Product Game is more specific, because it implies the result of multiplying the two numbers together, while there are infinite number multiples of two chosen numbers.

19. **a.** 2

b. No. All other even numbers have 2 as a factor 2, in addition to 1 and themselves.

20. **a.** 2, 3, and 7

b. 21 is missing.

21. **a.** 3, 5, 6, and 7

b. 25 is missing.

22. dimensions: 1×24, 2×12, 3×8, 4×6, 6×4, 8×3, 12×2, 24×1
factor pairs: 1, 24; 2, 12; 3, 8; 4, 6

23. dimensions: 1×32, 2×16, 4×8, 8×4, 16×2, 32×1
factor pairs: 1, 32; 2, 16; 4, 8

24. dimensions: 1×48, 2×24, 3×16, 4×12 6×8, 8×6, 12×4, 16×3, 24×2, 48×1
factor pairs: 1, 48; 2, 24; 3, 16; 4, 12; 6, 8

25. dimensions: 1×45, 3×15, 5×9, 9×5, 15×3, 45×1
factor pairs: 1, 45; 3, 15; 5, 9

26. dimensions: 1×60, 2×30, 3×20, 4×15 5×12, 6×10, 10×6, 12×5, 15×4, 20×3, 30×2, 60×1
factor pairs: 1, 60; 2, 30; 3, 20; 4, 15; 5, 12 6, 10

27. dimensions: 1×72, 2×36, 3×24, 4×18 6×12, 8×9, 9×8, 12×6, 18×4, 24×3, 36×2, 72×1
factor pairs: 1, 72; 2, 36; 3, 24; 4, 18; 6, 12 8, 9

28. **a.** Prime numbers have only two factors, 1 and itself. Examples: 2, 3, 5, 7, 11, . . .

b. Square numbers have odd numbers of factors. Examples: 4, 9, 16, 25, 36, . . .

c. No. Because prime numbers have two factors (from a) and square numbers have an odd number of factors (from b) you could never have a prime number that is also square.

29. two

30. B

31. His number is 25. His number must be a square number because it has an odd number of factors. 16 has five factors and 36 has nine factors.

32. 100 fans in 1 row, 50 fans in 2 rows, 25 fans in 4 rows, 20 fans in 5 rows, 10 fans in 10 rows, 5 fans in 20 rows, 4 fans in 25 rows, 2 fans in 50 rows, or 1 fan in 100 rows. Answers about which arrangement to choose will vary. Sample: I would rather have one long banner that wraps around part of the stadium, so I would choose 100 fans in one row. Sample: I would rather have a big square that you could see on TV, so I would choose ten fans in ten rows.

33. **a.** 1×64, 2×32, 4×16, 8×8, 16×4, 32×2, and 64×1.

b. Answers will vary as they did in Exercise 32.

Connections

34. $5 \text{ hours} \times 60\frac{\text{minutes}}{\text{hour}} = 300 \text{ minutes}$; $300 + 30 = 330$ minutes. $330 \div 12 = 27.5$. Therefore, they can play 27 games.

35. B; Carlos read $27 + 31 + 28 = 86$ pages the first part of the week. He had $144 - 86 = 58$ pages left for Thursday and Friday. Since he read the same number of pages each day, $58 \div 2 = 29$. Carlos read 29 pages on Thursday.

36. **a.** For $n + 3 < 50$, the possible answers of n are 0,1,2,3,4,5, . . .,45,46.

b. For $3n < 50$, the possible answers of n are 0,1,2,3,4,5, . . .,15,16.

37. 24 has many factors, so it can be divided into many equal parts. Since 23 is prime, it cannot be subdivided. The only proper factors of 25 are 1 and 5, so it can only be subdivided into 5 groups of 5.

38. Because 60 has many factors, and 59 and 61 do not.

39. **a.** Various answers; for example, group sizes 1, 2, 4, 5, 6, 3, 2, 2, 3, and 2. The goal is to find 10 numbers whose sum is 30.

b. Group sizes 3, 3, 3, 3, 3, 3, 3, 3, 3, and 3. If she does not have ten groups, she could have 1 group of 30 students, 2 groups of 15 students each, 5 groups of 6 students each, 6 groups of 5 students each, 10 groups of 3 students each, 15 groups of 2 students each, or 30 groups of 1 student each.

c. In part (a), the sum of the numbers in the ten groups must be 30. In part (b), we are considering the factors of 30.

40. 500 days

Extensions

41. **a.** $1 + 2 + 4 + 5 + 10 + 20 + 25 + 50 = 117$

b. $1 + 3 + 9 + 11 + 33 = 57$

c. 97, because it is the largest prime less than 100.

42. The numbers that have two odd digits (Clue 2) and give a remainder of 4 when divided by 5 (Clue 1) are 19, 39, 59, 79, and 99. Of these numbers, 19 is the only one with digits that add to 10 (Clue 3). The number is 19.

ACE Answers Investigation 2

Applications

1. 24, 48, 72, and 96; the LCM is 24.
2. 15, 30, 45, 60, 75, and 90; the LCM is 15.
3. 77; the LCM is 77.
4. 90; the LCM is 90.
5. 72; the LCM is 72.
6. 100; the LCM is 100.
7. 42, 84; the LCM is 42
8. 60; the LCM is 60.
9. **a.** Possible answers: 3, 5; 8, 9; 7, 11

   b. They have no common factors except 1.
10. Possible answers: 2, 5; 1, 10
11. Possible answers: 4, 9; 18, 36
12. Possible answers: 4, 15; 12, 5
13. Possible answers: 3, 35; 7, 15
14. **a.** Twenty-four 1-hour shifts; twelve 2-hour shifts; eight 3-hour shifts; six 4-hour shifts; four 6-hour shifts; three 8-hour shifts; two 12-hour shifts, and one 24-hour shift. These are all factors of 24.

    b. 45 seconds, which is the LCM of 9 and 15
15. 24 days
16. 1, 2, 3, and 6; the GCF is 6.
17. 1; the GCF is 1.
18. 1, 3, 5, and 15; the GCF is 15.
19. 1; the GCF is 1.
20. 1, 7; the GCF is 7.
21. 1, 5; the GCF is 5.
22. 1, 2; the GCF is 2.
23. 1, 3, 7, 21; the GCF is 21.
24. D
25. F
26. D
27. **a.** 2 packages of hot dogs and 3 packages of buns; 1 hot dog and 1 bun

    b. 10 packages of hot dogs and 15 packages of buns; 4 hot dogs and 4 buns
28. 20 members; each member gets 1 cookie and 2 carrot sticks.
    10 members; each member gets 2 cookies and 4 carrot sticks.
    5 members; each member gets 4 cookies and 8 carrot sticks.
    4 members; each member gets 5 cookies and 10 carrot sticks.
    2 members; each member gets 10 cookies and 20 carrot sticks.
    1 member; the member gets all 20 cookies and 40 carrot sticks.
29. **a.** Answers will vary. Sample: The Morgan family buys a 12-pack of bottled water and a 24-pack of boxes of raisins. Each person in the family gets the same number of bottles of water and the same number of boxes of raisins. How many people could the Morgan family have?

    b. Answers will vary. Sample: John eats an apple once a week. Ruth eats an apple every third day. If they both eat an apple today, when will John and Ruth next eat an apple on the same day?

    c. The Morgan family could have 1, 2, 3, 4, 6, or 12 people; these numbers are common factors of 12 and 24. John and Ruth will next eat an apple on the same day in 21 days; this problem involves overlapping cycles, so it can be solved with common multiples.
30. Students need to be able to reason proportionally (without knowing that vocabulary) to move from 20 minutes in 1 day to 1 hour in 3 days to 12 hours in 36 days. Julio's watch gains 12 hours in 36 days. Mario's watch gains 12 hours in 12 days. Since 12 is a factor of 36, the watches will next show the correct time together 36 days after Julio and Mario set their watches, or at 9:00 A.M. on the 6th Tuesday.

31. a. 30 students; each student receives 4 cans of juice and 3 packs of crackers because $120 = 30 \times 4$ and $90 = 30 \times 3$.

b. 8 students; each student receives 15 cans of juice and 11 packs of crackers because $120 = 8 \times 15$ and $88 = 8 \times 11$.

32. 7, 14, 21, and 42 ($42 = 2 \times 3 \times 7$ and $6 = 2 \times 3$.)

33. any odd multiple of 3

34. a. Aaron's method does not work for any pair of numbers. For example, the LCM of 4 and 8 is 8, which does not equal 4×8, or 32. Ruth's method does not work for any pair of numbers. For example, the LCM of 2 and 7 is 14, which does not equal 7. Walter's method does not work for any pair of numbers. For example, the LCM of 3 and 5 is 15, which does not equal $3 \times \frac{5}{2}$, or $\frac{15}{2}$.

b. Aaron's method works when the two numbers in pair don't have any common factors except 1; Ruth's method works when the greater of the two numbers in pair is a multiple of the lesser number; Walter's method works when 2 is the greatest common factor of the two numbers in pair.

Connections

35. 7 is a factor of 63. 9 is a factor of 63. 7 is a divisor of 63. 9 is a divisor of 63. 63 is a multiple of 7. 63 is a multiple of 9. The product of 7 and 9 is 63. 63 is divisible by 7. 63 is divisible by 9.

36. 4; $12 \times 4 = 48$

37. 10; $11 \times 10 = 110$

38. 8; $6 \times 8 = 48$

39. 11; $11 \times 11 = 121$

40. 4,995,000; multiply 4,995 by 1,000 for the three factors of 10.

41. a. In 2 hours, the jet will travel $12 \times 2 \times 60 = 1{,}440$ kilometers. In 6 hours, the jet will travel $1{,}440 \times 3 = 4{,}320$ kilometers.

b. In 6 hours, the jet will travel $4{,}320 - 1{,}440 = 2{,}880$ kilometers more than in 2 hours.

c. In 4 hours, the jet would travel twice as many miles as in 2 hours, or 2,880 kilometers.

42. This question also asks students to reason proportionally. (Note: This provides preparation for the next Unit, *Comparing Bits and Pieces.*)

a. $9 \times 5 \times 7$ will be 3 times as great as $3 \times 5 \times 7$, or 315.

b. $3 \times 5 \times 14$ will be twice as great as $3 \times 5 \times 7$, or 210.

c. $3 \times 50 \times 7$ will be 10 times as great as $3 \times 5 \times 7$, or 1,050.

d. $3 \times 25 \times 7$ will be 5 times as great as $3 \times 5 \times 7$, or 525.

43. a. composite and square ($5 \times 5 = 25$)

b. prime

c. composite ($3 \times 17 = 51$)

d. square ($1 \times 1 = 1$)

Applications

1. The path is $7 \times 5 \times 2 \times 3 \times 4$.
2. The path is $3 \times 4 \times 5 \times 6$.
3. Mazes will vary.
4. He can give each child 3 cookies, and he will have 12 left for himself.
5. $2 \times 2 \times 3 \times 3$
6. $2 \times 2 \times 3 \times 3 \times 5$
7. $3 \times 5 \times 5 \times 7$
8. $3 \times 5 \times 11$
9. 293
10. $2 \times 2 \times 2 \times 5 \times 19$
11. $2 \times 2 \times 2 \times 3 \times 3 \times 3$
12. $3 \times 7 \times 11$
13. $2 \times 2 \times 2 \times 3 \times 13$
14. $36 = 2^2 \times 3^2$, $180 = 2^2 \times 3^2 \times 5$, $525 = 3 \times 5^2 \times 7$, $165 = 3 \times 5 \times 11$, $293 = 293$, $760 = 2^3 \times 5 \times 19$, $216 = 2^3 \times 3^3$, $231 = 3 \times 7 \times 11$, and $312 = 2^3 \times 3 \times 13$
15. D
16. Jamahl is correct. Possible answer: Consider 216. It has six prime factors: $2 \times 2 \times 2 \times 3 \times 3 \times 3$. Then consider 231. It has three prime factors: $3 \times 7 \times 11$. 231 is greater than 216, even though it has fewer prime factors.
17. 10, 20, 30, 40, 50, 60, 70, 80, and 90. They are all multiples of 10.
18. H; $2 \times 3 \times 5$
19. $2 \times 3 \times 5 = 30$, $2 \times 3 \times 7 = 42$, $2 \times 3 \times 11 = 66$, $2 \times 3 \times 13 = 78$, $2 \times 5 \times 7 = 70$
20. a. $8 \times 8 = 64$, so Mr. and Mrs. Fisk have 64 grandchildren.
    b. $64 \times 8 = 512$, so they have 512 great-grandchildren.
    c. Expressions may vary. Sample: $8^3 = 512$.
21. GCF = 9, LCM = 180
22. GCF = 15, LCM = 150
23. GCF = 26, LCM = 312
24. GCF = 15, LCM = 60
25. GCF = 1, LCM = 1,440
26. GCF = 1, LCM = 444
27. a. $N = 45$, 90, or 180. Any other factor of 180 has a common multiple with 12 that is less than 180.
    b. $M = 14$ or any multiple of 14 that doesn't have a factor of 3 or 5.
28. 16, 81, and 625 (the numbers that are a prime number to the 4th power)
29. a. Locker 15
    b. Locker 60
    c. Locker 504
    d. Locker 630
30. a. Only Student 1 touched both lockers because the locker numbers are relatively prime.
    b. Students 1, 2, 5, 7, 10, 14, 35, and 70 touched both lockers 140 and 210. These are the common factors.
    c. Students 1, 3, 5, 11, 15, 33, 55, and 165 touched both lockers 165 and 330. These are the common factors.
    d. Students 1, 2, 7, 14, 49, and 98 touched both lockers 196 and 294. These are the common factors.

Connections

31. Rosa is correct because the number 1 is not a prime number. Tyee is correct that his string is a longer string of factors, but it is not a string of prime factors for 30.

32. Mathematicians have determined that it is important for a number to be able to be identified by its longest string of factors. If the number 1 were prime, the prime factorization for a number would have to include 1 and could include 1 as a factor any number of times. So a prime factorization would not be the same as the longest string possible without using 1.

33. **a.** $100 = 2^2 \times 5^2$, which has nine factors (1, 2, 4, 5, 10, 20, 25, 50, 100).

    b. 101 is prime and has only two factors (1, 101).

    c. $102 = 2 \times 3 \times 17$, which has eight factors (1, 2, 3, 6, 17, 24, 51, 102).

    d. 103 is prime and has only two factors (1, 103).

    e. Answers may vary. Students may notice that for four consecutive numbers, 2 is a factor of two of the numbers; 3 is a factor of at least one of the numbers; 4 must be a factor of one of the numbers; larger numbers do not necessarily have more factors.

34. These numbers are all multiples of 6.

35. These numbers are all multiples of 15.

36. 1, 2, 3, 5, 15, and 30 are also common factors.

37. **a.** 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, and 99

    b. 21, 42, 63, and 84

    c. 63

    d. 126

38. Factor to find that $184 = 8 \times 23$ and $207 = 9 \times 23$. 23 is the only common facto other than 1. If they only earned \$1 a day, they would have to work longer than one month to earn \$184 or more. Therefore, Tomas worked 8 days at \$23 per day and Sharina worked 9 days at \$23 per day.

39. Answers will vary. You might want to have students post some of their stories around the room. **Note:** The prime factorization o 648 is $648 = 2^3 \times 3^4$.

40. 53

41. Yes. Numbers that end in 0 are multiples o 10. Numbers that are multiples of 10 have both 2 and 5 as factors.

42. 70. Since the number is a multiple of 2 and 7 (Clue 1), which are relatively prime, the number must be a multiple of 14. The multiples of 14 between 50 and 100 (Clue 2) are $56 = 2 \times 2 \times 2 \times 7$, $70 = 2 \times 5 \times 7$, $84 = 2 \times 2 \times 3 \times 7$, and $98 = 2 \times 7 \times 7$. Of these numbers, only 70 is the product of three different prime numbers (Clue 3).

43. The factors of 32 are 1, 2, 4, 8, 16, and 32 (Clue 3). Of these numbers, only 1, 16 and 32 have digits that sum to odd numbers (Clue 4). 1 and 16 are square numbers (Clue 1). Of these two numbers, only 16 has 2 in its prime factorization (Clue 2). Th number is 16.

44. A number that is a multiple of 3 (Clue 2) and of 5 (Clue 1) must be a multiple of 15. The multiples of 15 that are less than 50 are 15, 30, and 45. Only 30 has 8 factors (Clue 3).

45. Multiples of 5 that don't end in 5 are multiples of 10 (Clue 1). The factor string is three numbers long (Clue 2), and two of these are 2 and 5. Since two of the numbers in the factor string are the same (Clue 3), the number is $2 \times 2 \times 5 = 20$ or $2 \times 5 \times 5 = 50$. The number is greater than the seventh prime, 49, so 50 is the number.

46. Answers will vary. You may want to post some of the best student responses for a Problem of the Week.

47. Only Locker 42 meets all three criteria. A good way to approach the problem is to list all the even numbers 50 or less and cross out those that fail to meet each of the other criteria. A student might also start by listing only the numbers divisible by 7 and then apply the other two criteria to those numbers.

48. a. 2, 4, 8, 16, 32, 64, 128, 256, 512

b. 1,024

49. This is the only pair of primes that are consecutive numbers. We know that this is the only such pair because 2 is the only even prime number.

50. There are (infinitely) more odd prime numbers. The only even prime is 2.

Extensions

51. a. Possible answers:
$1994 = 2 \times 997$, $1995 = 3 \times 5 \times 7 \times 19$,
$1996 = 2 \times 2 \times 499$, $1997 = 1997$,
$1998 = 2 \times 3 \times 3 \times 3 \times 37$,
$1999 = 1999$,
$2000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$,
$2001 = 3 \times 23 \times 29$,
$2002 = 2 \times 7 \times 11 \times 13$, $2003 = 2003$,
$2004 = 2 \times 2 \times 3 \times 167$,
$2005 = 5 \times 401$, $2006 = 2 \times 17 \times 59$,
$2007 = 3 \times 3 \times 223$,
$2008 = 2 \times 2 \times 2 \times 251$,
$2009 = 7 \times 7 \times 41$,
$2010 = 2 \times 3 \times 5 \times 67$

b. Answers will vary. For example, 1996 is not square; it is a multiple of 4 and of 998. It is not prime, and it is even.

52. a. 52 weeks, with 1 extra day if it is not a leap year and 2 extra days if it is a leap year.

b. January 8, 15, and 22

c. Monday

d. Tuesday

e. Your birthday will fall one day later in the week each year, except when leap day (February 29) falls between your birthdays. In that case, your birthday will be two days later in the week. If your birthday is February 29, your birthday will be five days later in the week each time it occurs.

53. Answers will vary. Sample: If a number is the least common multiple of several prime numbers, its prime factorization will include only those primes, and no others.

54. The common multiples of 2, 3, 4, 5, and 6 are 60, 120, 180, ... If we add the clue that the box contains fewer than 100 books, the only answer would be 60.

55. a. Barry is correct. The LCM$(a,b) = a \times b \div$ GCF(a,b). When you multiply a and b together, you will get each of the overlapping prime factors twice. Therefore, if you divide by the GCF, you will get only one set of overlapping factors, along with the factors that do not overlap.

b. The product of the LCM and the GCF is equal to the product of the numbers.

Applications

1. An even number minus an even number will be even. Students may use examples, tiles, the idea of "groups of two," or the inverse relationship between addition and subtraction.
   - Using an example: 16 − 4 is 12.
   - Using tiles: For example, if you take away one rectangle with a height of 2 from another rectangle with a height of 2, you will still have a rectangle with a height of 2.
   - Using groups of 2: If you have an even number of objects, you can bundle the number of objects into groups of 2. If you take away some bundles of 2 from a group of bundles of 2, you are still left with bundles of 2.
   - Using the inverse relationship between addition and subtraction: Students may know that if $a + b = c$, then $a = c - b$. In this way, the question is asking "If c and b are even, is a even or odd?" In the equation $a + b = c$, if the values of b and c are even, then the value of a must also be even, because even + even = even.
2. An odd number minus an odd number is even. If you have a rectangle with one extra square and you take away a rectangle with one extra square, you have taken away the extra square, and you are left with a rectangle with a height of 2.
3. An even number minus an odd number is odd. If you have a rectangle with a height of 2 and you subtract a rectangle with one extra square, you have broken up a pair of squares on the original rectangle and are left with another rectangle with an extra square.
4. An odd number minus an even number is odd. If you have a rectangle with one extra square and you subtract a rectangle with a height of 2, you are left with a rectangle with an extra square.
5. Evens have ones digits of 0, 2, 4, 6, or 8, and they are divisible by 2. Odds have ones digits of 1, 3, 5, 7, or 9, and they are not divisible by 2.
6. A sum is even if all of the addends are even, or if there is an even number of odd addends. Otherwise, the number is odd.
7. $4 \times (3 + 6)$ and $(4 \times 3) + (4 \times 6)$, total area 36
8. $(4 + 2) \times 7$ and $(4 \times 7) + (2 \times 7)$, total area 42
9. $5 \times (3 + 6 + 2)$ and $(5 \times 3) + (5 \times 6) + (5 \times 2)$, total area 55

For Exercises 10–12, the area of the largest rectangle is the sum of the areas of the two smaller rectangles. To find the dimensions of each rectangle, first find a common factor of each pair of numbers. Each Exercise has multiple possible dimensions.

10. (See Figure 1.)

Figure 1

Dimensions of 39 Square Unit Rectangles and Partitions

	Small	Medium	Large
Possible Rectangle 1	3×4	3×9	3×13
Possible Rectangle 2	1×12	1×27	1×39

11. (See Figure 2.)

12. (See Figure 3.)

13. $3 \times (4 + 6) = 3 \times 10$ or
$(3 \times 4) + (3 \times 6) = 12 + 18$

14. $3 \times (5 + 1 + 3) = 3 \times 9$ or
$(3 \times 5) + (3 \times 1) + (3 \times 3) = 15 + 3 + 9$
(See Figure 4.)

Figure 2

Dimensions of 49 Square Unit Rectangles and Partitions

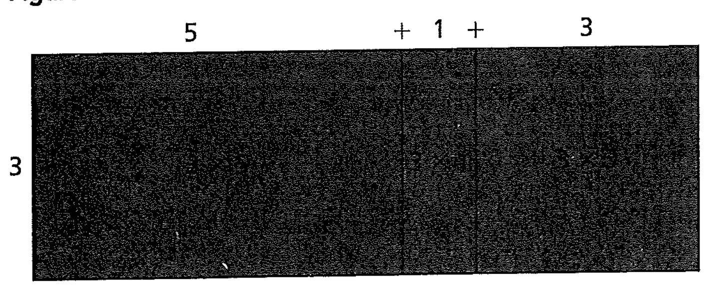
	Small	Medium	Large
Possible Rectangle 1	7×3	7×4	7×7
Possible Rectangle 2	1×21	1×28	1×49

Figure 3

Dimensions of 48 Square Unit Rectangles and Partitions

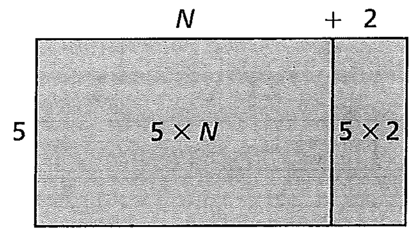
	Small	Medium	Large
Possible Rectangle 1	1×18	1×30	1×48
Possible Rectangle 2	2×9	2×15	2×24
Possible Rectangle 3	3×6	3×10	3×16
Possible Rectangle 4	6×3	6×5	6×8

Figure 4



15. $N \times (2 + 6) = 8N$ or
$(N \times 2) + (N \times 6) = 2N + 6N$
(See Figure 5.)

16. $5 \times (N + 2)$ or $(5 \times N) + (5 \times 2)$, $5N + 10$



17. $9 \times (30 + 4) = (9 \times 30) + (9 \times 4)$
$= 270 + 36 = 306$
(See Figure 6.)

Figure 5

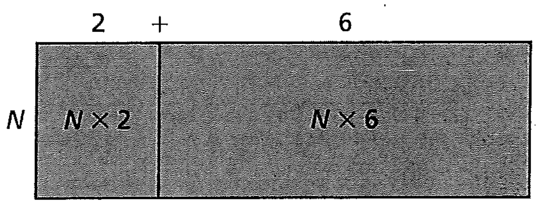
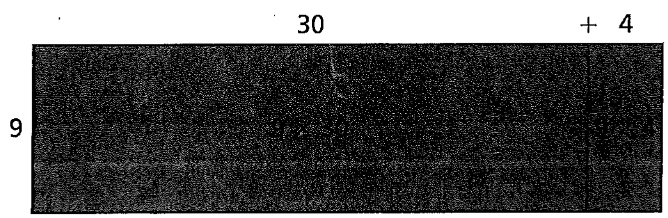
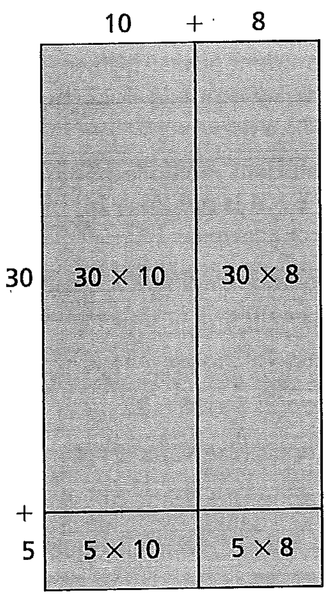


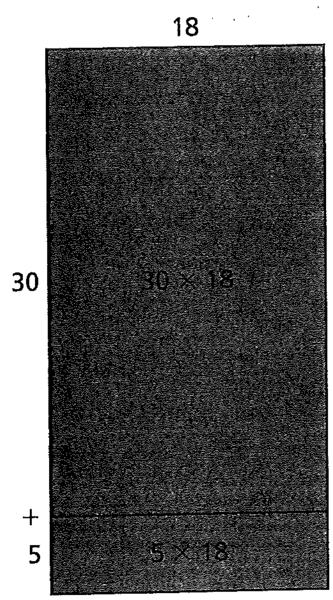
Figure 6



18. $35 \times 18 = (30 + 5) \times (10 + 8)$
$= (30 \times 10) + (5 \times 10) + (30 \times 8) + (5 \times 8)$
$= 300 + 50 + 240 + 40$
$= 630$



$(30 + 5) \times 18 = (30 \times 18) + (5 \times 18)$
$= 540 + 90$
$= 630$



Note: Some students may write $35 \times (20 - 2) = 700 - 70 = 630$, although this is not connected to the typical multiplication algorithm. The arithmetic may be easier with these numbers.

19. **a.** Answers will vary. Possible answer: $30 + 30$

b. Answers will vary. Possible answer: 6×10

c. Answers will vary. Possible answer:
$6 \times 10 = 6 \times (5 + 5)$
$= (5 \times 6) + (5 \times 6)$
$= 30 + 30$

20. **a.** $90 = 20 + 70 = 10(2 + 7)$

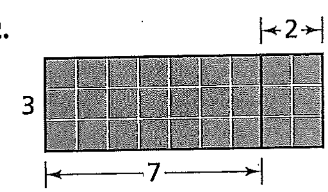
b. $90 = 36 + 54 = 9(4 + 6)$

21. **a.** The black number in the lower-right-hand square is the sum of the red numbers in the right-most column and also the sum of the red numbers in the bottom row.

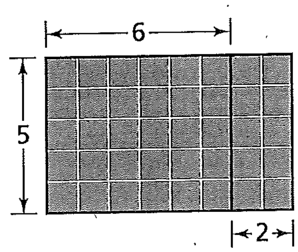
b. The same relationship will hold for any four numbers in the border squares. The black number is always the sum of the red numbers in the right-hand column and the red numbers in the bottom row.

c. Shalala is correct. The sum of the bottom row is $6(2 + 8) + 3(2 + 8)$. The sum of the last column is $2(6 + 3) + 8(6 + 3)$. From the first expression, you can factor $(2 + 8)$ to get $(2 + 8)(6 + 3)$. If you factor $(6 + 3)$ from the second expression, you get $(6 + 3)(2 + 8)$. By the Commutative Property of Multiplication, these two products are equal.

22.



23.



24. $m = 3$

25. $m = 10$

26. m = 1

27. m = 6

28. $(3 + 4) \times 2$

29. $12 \div 6 \times 2$

30. $11 \times 2 + 1$

31. $3^2 \cdot 3^2$

32. $2 + 5 \times 3 = 17$

33. $2 \times 5 + 3 = 13$

34. $2 \times 5 \times 3 = 30$

35. $2 \times 5 - 3 = 7$

36. Answers will vary. $3 + 2 + 4(1) = 9$ or $3 + (2 + 4)(1) = 9$ or $(3 + 2 + 4)(1) = 9$.

37. $3 + (2)(4 + 1) = 13$

38. $(3)(2 + 4 + 1) = 21$

39. $3 + (2)(4) + 1 = 12$

40. $3 + 2 + 4 + 1 = 10$

41. **a.** $4 + 3(6 + 1) = 25$, which is a multiple of 5.

b. $4(3) + 6(1) = 18$, which is a factor of 36.

42. **a.** Answers will vary. Possible answer:

$$\begin{aligned} 21 &= 3 \times 7 \\ &= 3 \times (5 + 2) \\ &= 3 \times 5 + 3 \times 2 \\ &= 15 + 6 \end{aligned}$$

b. Answers will vary. Possible answer:

$$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times (10 + 2) \\ &= 2 \times 10 + 2 \times 2 \\ &= 20 + 4 \end{aligned}$$

c. Answers will vary. Possible answer:

$$\begin{aligned} 55 &= 5 \times 11 \\ &= 5 \times (10 + 1) \\ &= 5 \times 10 + 5 \times 1 \\ &= 50 + 5 \end{aligned}$$

d. Answers will vary. Possible answer:

$$\begin{aligned} 48 &= 2 \times 24 \\ &= 2 \times (20 + 4) \\ &= 2 \times 20 + 2 \times 4 \\ &= 40 + 8 \end{aligned}$$

43. The student interpreted exponents as multiplying the two numbers. 3^2 is not 6, and 3^3 is not 9. The correct answer is 9.

44. The student performed multiplication before exponentiation; 2×3^2 is not 6^2, but 18. The correct answer is 26.

45. The student added before he subtracted; $18 - 6 + 6$ is not $18 - 12$, but $12 + 6$. The correct answer is 18.

46. The student multiplied before he divided; $24 \div 6 \times 4$ is not $24 \div 24$, but 4×4. The correct answer is 16.

47. Any number will work. Explanations will vary. Sample:

Step 1: Choose 7.

Step 2: $7 + 15 = 22$

Step 3: $(7 + 15) \times 2 = 44$

Step 4: $(7 + 15) \times 2 - 30 = 14$

$14 = 2 \times 7$, which is double the original number.

Alternatively,

$$\begin{aligned} (n + 15) \times 2 - 30 &= n \times 2 + 30 - 30 \\ &= n \times 2 \end{aligned}$$

which is double the original number.

48. Choose N. Then,

$$\begin{aligned} ((N \times 2 + 6) - 3) &= (N \times 2 \div 2) + (6 \div 2) - 3 \\ &= N + 3 - 3 \\ &= N \end{aligned}$$

49. This can be solved algebraically. An area model works as well.

Let $N =$ the area of a rectangle.

N	$= N$

Double it.

N	N	$= 2N$

Add 6.
(See Figure 7.)

Divide by 2.
(See Figure 8.)

Subtract 3.

$\frac{1}{2}N$	$\frac{1}{2}N$	$= N$

50. 6

51. 2

In Exercises 52–57, each case could be explained by the Distributive Property and knowledge of place value.

52. True. $432 = 400 + 32$. So $50 \times 432 = 50(400 + 32)$ and $50(400 + 32) = 50 \times 400 + 50 \times 32$.

53. True. $50 \times 368 = 50(400 - 32)$
$= 50 \times 400 - 50 \times 32$

54. False. If the equation involved subtraction instead of addition, then it would be true. $50 \times 800 = (50 \times 1{,}000) - (50 \times 200)$, since $800 = 1{,}000 - 200$.

55. False. $90 \times 70 = (90 \times 30) + (90 \times 40)$;
$90 \times 30 > 70 \times 20$ and
$90 \times 40 > 50 \times 20$, so
$(90 \times 30) + (90 \times 40) > (70 \times 20) + (50 \times 2($
Alternatively,
$90 \times 70 = 9 \times 7 \times 100 = 9 \times 7 \times 5 \times 20$.
$9 \times 7 \times 5 \neq 70 + 50$, though, because $70 + 50$ is even and $9 \times 7 \times 5$ is odd.

56. False. 50 is not multiplied by the sum $(400 + 32)$. It is added to the product of 400 and 32.

57. True. $6 \times 17 = 6(20 - 3) = 6(20) - 6(3)$. Each expression is 102.

58. Yes; each expression has a value of 12.

59. **a.** Mrs. Lee is correct. Because you do multiplication before subtraction, Mrs. Lee's expression will calculate the area of the yard and swing set first, then take the difference of those areas to find the remaining area.

b. Mr. Lee is correct. Because you operate in parenthesis first, Mr. Lee's expressior will calculate the difference in length first to find the length of the lawn, then multiply by the width to find the area.

Figure 7

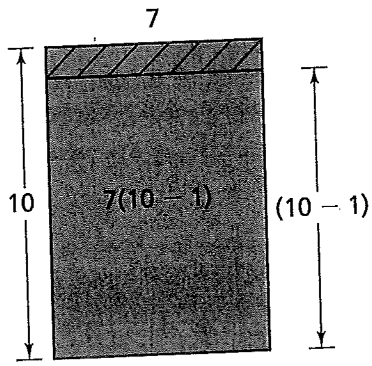
N	N	6	$= 2N + 6$

Figure 8

[redacted]

$\frac{1}{2}N$	$\frac{1}{2}N$	3	$= N + 3$

60.



The expression in expanded form is $7 \times 10 - 7 \times 1$.
If we simplify within parentheses first, we find the expression is equal to 63:
$7(10 - 1) = 7 \times 9 = 63$.
If we distribute the 7, we find that the expression is still equal to 63:
$7(10 - 1) = 7 \times 10 - 7 \times 1 = 70 - 7 = 63$.

61. There are $36 \cdot 12 = 432$ trading cards and $36 \cdot 2 = 72$ stickers.

62. $30(12 - 3) = 30 \cdot 9 = 270$, or \$270. Alternatively, students might find the cost for all students $30 \cdot 12 = 360$ and then subtract the total discount $30 \cdot 3 = 90$ for a total of 270, or \$270.

63. Tuesday's high temperature is 3 degrees colder than Sunday's high temperature. Students could use a variable, n, to represent Sunday's temperature. Then Tuesday's temperature can be represented by $n + 5 - 8$, which simplifies to $n - 3$, so Tuesday's high temperature is 3 degrees colder than n, Sunday's temperature.

Another method is to choose a few examples to see the relationship. Suppose Sunday's high temperature is 60 degrees. Then Monday's high temperature is 65 degrees, and Tuesday's high temperature is 57 degrees. For any starting amount (Sunday's high temperature), Tuesday's high temperature will be 3 degrees colder.

64. Elijah collected \$264, \$192 for the school and \$72 for his homeroom. Students might calculate the two parts first: $24 \cdot 8 = 192$ (school) and $24 \cdot 3 = 72$ (homeroom). Solving it this way uses the Distributive Property, because $24(11) = 24(8 + 3) = 24(8) + 24(3)$.

65. \$360. One way to solve this is to multiply $15(6)(4) = 360$. Another number sentence is $15(1 + 3 + 2)4 = 360$. In the second equation, students could distribute either the 15 or the 4 to each addend inside the parentheses.

Connections

66. A

67. 3,500

68. 1,750

69. 100

70. 6,000

71. 938

72. 3,200

73. 100

74. 900

75. $4 \times 5 = 4 \times (3 + 2)$
$= (4 \times 3) + (4 \times 2)$

76. a. 32×12 is the area of a rectangle with dimensions 32 and 12. The sum of the areas of the bottom two rectangles in Jim's figure is $32 \times 2 = (30 + 2) \times 2 = 30 \times 2 + 2 \times 2 = 60 + 4 = 64$, the first partial product in the example. The sum of the areas of the upper two rectangles is $32 \times 10 = (30 + 2) \times 10 = 30 \times 10 + 2 \times 10 = 300 + 20 = 320$, the second partial product. By adding these areas together, we get the final product, 384