

11. a. In the Factor Game, your opponent scores points for proper factors of the number you choose. The only proper factor of prime numbers, such as 2, 3, or 7, is 1.
- b. Some numbers, such as 12, 20, and 30, have many proper factors that would give your opponent more points. (Some numbers, such as 9, 15, and 25, have fewer factors and would give you more points.)
12. a. i. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.
 ii. Multiples of 3; 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30.
 iii. 12, 21, 30, and 210 will all appear in the sequence, because they are all multiples of 3.
- b. i. 7, 14, 21, 28, 35, 42, 49, 56, 63, 70.
 ii. Multiples of 7
- iii. 21 and 210 will appear in the sequence, because they are multiples of 7.
- c. i. $6 \times 1 = 6$; $6 \times 2 = 12$; $6 \times 3 = 18$;
 $6 \times 4 = 24$; $6 \times 5 = 30$; $6 \times 6 = 36$;
 $6 \times 7 = 42$; $6 \times 8 = 48$; $6 \times 9 = 54$;
 and $6 \times 10 = 60$.
 ii. Multiples of 6
 iii. 12, 30, and 210 will appear in the sequence, because they are multiples of 6
13. a. (See Figure 3.)
- b. 31, 37, 41, 43, and 47 are prime.
- c. 36 and 49 are square numbers.
- d. 47; it is the greatest prime number.
- e. 48; The second player gets 76 points, which is 28 more points than the first player.

Figure 3

First Move	Proper Factors	My Score	Opponent's Score
31	1	31	1
32	1, 2, 4, 8, 16	32	31
33	1, 3, 11	33	15
34	1, 2, 17	34	20
35	1, 5, 7	35	13
36	1, 2, 3, 4, 6, 9, 12, 18	36	55
37	1	37	1
38	1, 2, 19	38	22
39	1, 3, 13	39	17
40	1, 2, 4, 5, 8, 10, 20	40	50
41	1	41	1
42	1, 2, 3, 6, 7, 14, 21	42	54
43	1	43	1
44	1, 2, 4, 11, 22	44	40
45	1, 3, 5, 9, 15	45	33
46	1, 2, 23	46	26
47	1	47	1
48	1, 2, 3, 4, 6, 8, 12, 16, 24	48	76
49	1, 7	49	8

14. a. Move the paper clip from 6 to make the products 5×1 , 5×2 , 5×3 , 5×4 , 5×5 , 5×7 , 5×8 , and 5×9 ; move the paper clip from the 5 to make the products 6×1 , 6×2 , 6×3 , 6×4 , 6×6 , 6×7 , 6×8 , and 6×9 .
- b. Moving the paper clip from the 6 to the 3, 4, or 9, makes 15, 20, or 45, respectively; moving the paper clip from the 5 to the 7 makes 42.
- c. Moving the paper clip from the 5 to the 7 makes 6×7 , which is 42.
- d. Possible answer: Move the 5 to the 7 to get 42; this blocks the opponent and gets 3 in a row.
15. a. $3 \times 1 = 3$; $3 \times 2 = 6$; $3 \times 3 = 9$; $3 \times 4 = 12$; $3 \times 5 = 15$; $3 \times 6 = 18$; $3 \times 7 = 21$; $3 \times 8 = 24$; $3 \times 9 = 27$. So you can get a 3, 6, 9, 12, 15, 18, 21, 24 and 27, which are all multiples of 3.
- b. $3 \times 11 = 33$; $3 \times 13 = 39$; $3 \times 17 = 51$; $3 \times 19 = 57$; $3 \times 20 = 60$; and many others.
- c. There are infinitely many multiples of 3.
16. a. 2 and 9; or 3 and 6
- b. 1 and 18
17. a. 36 can be found on the product game by 6×6 or 4×9 . 36 is composite.
- b. 5 can be found on the product game by only 1×5 . 5 is prime.
- c. 7 can be found on the product game by only 1×7 . 7 is prime.
- d. 9 can be found on the product game by 1×9 or 3×3 . 9 is composite.
18. Since the numbers on the game board are multiples of the numbers given as possible factors, you could argue in support of Sal's position. The Product Game is more specific, because it implies the result of multiplying the two numbers together, while there are infinite number multiples of two chosen numbers.
19. a. 2
- b. No. All other even numbers have 2 as a factor 2, in addition to 1 and themselves.
20. a. 2, 3, and 7
- b. 21 is missing.
21. a. 3, 5, 6, and 7
- b. 25 is missing.
22. dimensions: 1×24 , 2×12 , 3×8 , 4×6 , 6×4 , 8×3 , 12×2 , 24×1
factor pairs: 1, 24; 2, 12; 3, 8; 4, 6
23. dimensions: 1×32 , 2×16 , 4×8 , 8×4 , 16×2 , 32×1
factor pairs: 1, 32; 2, 16; 4, 8
24. dimensions: 1×48 , 2×24 , 3×16 , 4×12 , 6×8 , 8×6 , 12×4 , 16×3 , 24×2 , 48×1
factor pairs: 1, 48; 2, 24; 3, 16; 4, 12; 6, 8
25. dimensions: 1×45 , 3×15 , 5×9 , 9×5 , 15×3 , 45×1
factor pairs: 1, 45; 3, 15; 5, 9
26. dimensions: 1×60 , 2×30 , 3×20 , 4×15 , 5×12 , 6×10 , 10×6 , 12×5 , 15×4 , 20×3 , 30×2 , 60×1
factor pairs: 1, 60; 2, 30; 3, 20; 4, 15; 5, 12, 6, 10
27. dimensions: 1×72 , 2×36 , 3×24 , 4×18 , 6×12 , 8×9 , 9×8 , 12×6 , 18×4 , 24×3 , 36×2 , 72×1
factor pairs: 1, 72; 2, 36; 3, 24; 4, 18; 6, 12, 8, 9
28. a. Prime numbers have only two factors, 1 and itself. Examples: 2, 3, 5, 7, 11, ...
- b. Square numbers have odd numbers of factors. Examples: 4, 9, 16, 25, 36, ...
- c. No. Because prime numbers have two factors (from a) and square numbers have an odd number of factors (from b) you could never have a prime number that is also square.
29. two
30. B

31. His number is 25. His number must be a square number because it has an odd number of factors. 16 has five factors and 36 has nine factors.

32. 100 fans in 1 row, 50 fans in 2 rows, 25 fans in 4 rows, 20 fans in 5 rows, 10 fans in 10 rows, 5 fans in 20 rows, 4 fans in 25 rows, 2 fans in 50 rows, or 1 fan in 100 rows. Answers about which arrangement to choose will vary. Sample: I would rather have one long banner that wraps around part of the stadium, so I would choose 100 fans in one row. Sample: I would rather have a big square that you could see on TV, so I would choose ten fans in ten rows.

33. a. 1×64 , 2×32 , 4×16 , 8×8 , 16×4 , 32×2 , and 64×1 .

b. Answers will vary as they did in Exercise 32.

Connections

34. $5 \text{ hours} \times 60 \frac{\text{minutes}}{\text{hour}} = 300 \text{ minutes}$;
 $300 + 30 = 330 \text{ minutes}$. $330 \div 12 = 27.5$.
 Therefore, they can play 27 games.

35. B; Carlos read $27 + 31 + 28 = 86$ pages the first part of the week. He had $144 - 86 = 58$ pages left for Thursday and Friday. Since he read the same number of pages each day, $58 \div 2 = 29$. Carlos read 29 pages on Thursday.

36. a. For $n + 3 < 50$, the possible answers of n are 0, 1, 2, 3, 4, 5, . . . , 45, 46.

b. For $3n < 50$, the possible answers of n are 0, 1, 2, 3, 4, 5, . . . , 15, 16.

37. 24 has many factors, so it can be divided into many equal parts. Since 23 is prime, it cannot be subdivided. The only proper factors of 25 are 1 and 5, so it can only be subdivided into 5 groups of 5.

38. Because 60 has many factors, and 59 and 61 do not.

39. a. Various answers; for example, group sizes 1, 2, 4, 5, 6, 3, 2, 2, 3, and 2. The goal is to find 10 numbers whose sum is 30.

b. Group sizes 3, 3, 3, 3, 3, 3, 3, 3, 3, and 3. If she does not have ten groups, she could have 1 group of 30 students, 2 groups of 15 students each, 5 groups of 6 students each, 6 groups of 5 students each, 10 groups of 3 students each, 15 groups of 2 students each, or 30 groups of 1 student each.

c. In part (a), the sum of the numbers in the ten groups must be 30. In part (b), we are considering the factors of 30.

40. 500 days

Extensions

41. a. $1 + 2 + 4 + 5 + 10 + 20 + 25 + 50 = 117$

b. $1 + 3 + 9 + 11 + 33 = 57$

c. 97, because it is the largest prime less than 100.

42. The numbers that have two odd digits (Clue 2) and give a remainder of 4 when divided by 5 (Clue 1) are 19, 39, 59, 79, and 99. Of these numbers, 19 is the only one with digits that add to 10 (Clue 3). The number is 19.

Applications

1. 24, 48, 72, and 96; the LCM is 24.
2. 15, 30, 45, 60, 75, and 90; the LCM is 15.
3. 77; the LCM is 77.
4. 90; the LCM is 90.
5. 72; the LCM is 72.
6. 100; the LCM is 100.
7. 42, 84; the LCM is 24.
8. 60; the LCM is 60.
9. a. Possible answers: 3, 5; 8, 9; 7, 11
b. They have no common factors except 1.
10. Possible answers: 2, 5; 1, 10
11. Possible answers: 4, 9; 18, 36
12. Possible answers: 4, 15; 12, 5
13. Possible answers: 3, 35; 7, 15
14. a. Twenty-four 1-hour shifts; twelve 2-hour shifts; eight 3-hour shifts; six 4-hour shifts; four 6-hour shifts; three 8-hour shifts; two 12-hour shifts, and one 24-hour shift. These are all factors of 24.
b. 45 seconds, which is the LCM of 9 and 15
15. 24 days
16. 1, 2, 3, and 6; the GCF is 6.
17. 1; the GCF is 1.
18. 1, 3, 5, and 15; the GCF is 15.
19. 1; the GCF is 1.
20. 1, 7; the GCF is 7.
21. 1, 5; the GCF is 5.
22. 1, 2; the GCF is 2.
23. 1, 3, 7, 21; the GCF is 21.
24. D
25. F
26. D
27. a. 2 packages of hot dogs and 3 packages of buns; 1 hot dog and 1 bun
b. 10 packages of hot dogs and 15 packages of buns; 4 hot dogs and 4 buns
28. 20 members; each member gets 1 cookie and 2 carrot sticks.
10 members; each member gets 2 cookies and 4 carrot sticks.
5 members; each member gets 4 cookies and 8 carrot sticks.
4 members; each member gets 5 cookies and 10 carrot sticks.
2 members; each member gets 10 cookies and 20 carrot sticks.
1 member; the member gets all 20 cookies and 40 carrot sticks.
29. a. Answers will vary. Sample: The Morgan family buys a 12-pack of bottled water and a 24-pack of boxes of raisins. Each person in the family gets the same number of bottles of water and the same number of boxes of raisins. How many people could the Morgan family have?
b. Answers will vary. Sample: John eats an apple once a week. Ruth eats an apple every third day. If they both eat an apple today, when will John and Ruth next eat an apple on the same day?
c. The Morgan family could have 1, 2, 3, 4, 6, or 12 people; these numbers are common factors of 12 and 24. John and Ruth will next eat an apple on the same day in 21 days; this problem involves overlapping cycles, so it can be solved with common multiples.
30. Students need to be able to reason proportionally (without knowing that vocabulary) to move from 20 minutes in 1 day to 1 hour in 3 days to 12 hours in 36 days. Julio's watch gains 12 hours in 36 days. Mario's watch gains 12 hours in 12 days. Since 12 is a factor of 36, the watches will next show the correct time together 36 days after Julio and Mario set their watches, or at 9:00 A.M. on the 6th Tuesday.

31. a. 30 students; each student receives 4 cans of juice and 3 packs of crackers because $120 = 30 \times 4$ and $90 = 30 \times 3$.
- b. 8 students; each student receives 15 cans of juice and 11 packs of crackers because $120 = 8 \times 15$ and $88 = 8 \times 11$.
32. 7, 14, 21, and 42 ($42 = 2 \times 3 \times 7$ and $6 = 2 \times 3$.)
33. any odd multiple of 3
34. a. Aaron's method does not work for any pair of numbers. For example, the LCM of 4 and 8 is 8, which does not equal 4×8 , or 32. Ruth's method does not work for any pair of numbers. For example, the LCM of 2 and 7 is 14, which does not equal 7. Walter's method does not work for any pair of numbers. For example, the LCM of 3 and 5 is 15, which does not equal $3 \times \frac{5}{2}$, or $\frac{15}{2}$.
- b. Aaron's method works when the two numbers in pair don't have any common factors except 1; Ruth's method works when the greater of the two numbers in pair is a multiple of the lesser number; Walter's method works when 2 is the greatest common factor of the two numbers in pair.

Connections

35. 7 is a factor of 63. 9 is a factor of 63. 7 is a divisor of 63. 9 is a divisor of 63. 63 is a multiple of 7. 63 is a multiple of 9. The product of 7 and 9 is 63. 63 is divisible by 7. 63 is divisible by 9.
36. 4; $12 \times 4 = 48$
37. 10; $11 \times 10 = 110$
38. 8; $6 \times 8 = 48$
39. 11; $11 \times 11 = 121$
40. 4,995,000; multiply 4,995 by 1,000 for the three factors of 10.
41. a. In 2 hours, the jet will travel $12 \times 2 \times 60 = 1,440$ kilometers. In 6 hours, the jet will travel $1,440 \times 3 = 4,320$ kilometers.
- b. In 6 hours, the jet will travel $4,320 - 1,440 = 2,880$ kilometers more than in 2 hours.
- c. In 4 hours, the jet would travel twice as many miles as in 2 hours, or 2,880 kilometers.
42. This question also asks students to reason proportionally. (Note: This provides preparation for the next Unit, *Comparing Bits and Pieces*.)
- a. $9 \times 5 \times 7$ will be 3 times as great as $3 \times 5 \times 7$, or 315.
- b. $3 \times 5 \times 14$ will be twice as great as $3 \times 5 \times 7$, or 210.
- c. $3 \times 50 \times 7$ will be 10 times as great as $3 \times 5 \times 7$, or 1,050.
- d. $3 \times 25 \times 7$ will be 5 times as great as $3 \times 5 \times 7$, or 525.
43. a. composite and square ($5 \times 5 = 25$)
- b. prime
- c. composite ($3 \times 17 = 51$)
- d. square ($1 \times 1 = 1$)

