

You might extend students' understanding by asking them to evaluate the expression for the dependent variable for a given value of the independent variable.

Have students use the equations they wrote to answer each of the following questions:

- In Question A, what is the discounted price if the regular price is \$166?
- In Question B, what is the sales tax on a tour that costs \$200?
- In Question C, how far can you bike at 20 miles per hour if you travel 7.5 hours?
- In Question D, how long will it take to drive 350 miles if the van is traveling at 55 miles per hour?

Because students will probably not give sophisticated answers to the Focus Question, you might actually pose the Focus Question in more concrete form with a specific value of k . For example, you might say, "What can you tell about how two variables are related if the equation $y = x + 3$ is given?" and expect answers like, "To get the value of y you always add 3 to the value of x " or "The value of y is always 3 more than the value of x ."



Assignment Guide for Problem 3.1

Applications: 1 | Connections: 22–25

Answers to Problem 3.1

- A.**
1. With a \$50 discount, the new prices would be \$350, \$450, and \$600 respectively.
 2. $D = P - 50$
- B.**
1. With 6% sales tax rate, the taxes would be \$24, \$30, and \$39 respectively. (Note: It might be useful to point out that sales tax calculations that do not yield exact dollars and cents (e.g. 6% of \$3.05 = \$.183) are generally rounded to the next higher penny, not the nearest penny.)
 2. $T = 0.06P$
- C.**
1. Travelling at 20 miles per hour, the cyclist would cover 40 miles in 2 hours, 60 miles in 3 hours, and 70 miles in 3.5 hours.
 2. There is a constant rate of increase in the distance traveled depending on time. An equation is $d = 20t$.
 3. As time increases by 1 hour, distance increases by 20 miles.
- D.**
1. Time will be 8.75 hours (8 hours and 45 minutes) at 40 miles per hour, 7 hours at 50 miles per hour, and 5.8333... hours (5 hours and 50 minutes) at 60 miles per hour.
 2. $t = \frac{350}{s}$ or $t = 350 \div s$. It might be useful to point out the use of fraction notation for division as in $t = \frac{350}{s}$.

Ask the following questions for Question C.

- Use the equation from Question C to find the number of inches that correspond to 10 centimeters and the number of centimeters that correspond to 10 inches. How do these values relate to the data in a table or graph?



Assignment Guide for Problem 3.2

Applications: 2–5 | Connections: 26–30
Extensions: 36

Answers to Problem 3.2

- A. 1. (See Figure 1.)
2. Equations that show how distance d and time t are related would be $d = 50t$, $d = 55t$, and $d = 60t$.
3. (See Figure 2.)
4. a. The pattern relating distance and time in the table shows constant hourly increases in distance equal to the
- b. Yes, Theo is correct. Average speed provides information about the change in distance for every 1-hour increase. These coefficients can also be considered as unit rate because they indicate change in distance for every 1 hour.
5. Table: The pattern relating distance and time in the table shows constant hourly increases in distance equal to the speed. You can add twice the rate of change to the last entry in order to find the distance for 6 hours, or you can add two more rows for $t = 5$ hours and $t = 6$ hours.

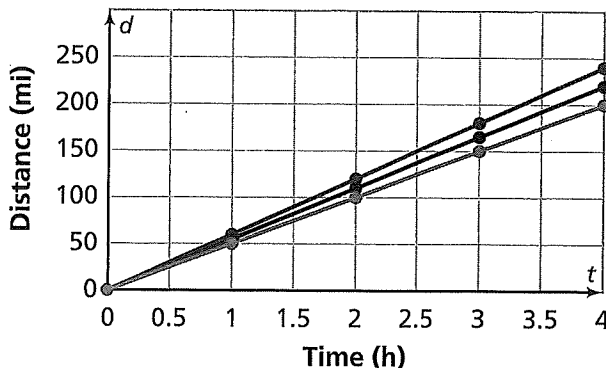
Figure 1

Distance Traveled at Different Average Speeds

Time (h)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0	0	0
1	50	55	60
2	100	110	120
3	150	165	180
4	200	220	240

Figure 2

Distance Traveled at Different Average Speeds



Graph: You can expand the graph to $x = 6$ and read the corresponding y -value.

- a. Equation: You can substitute 6 for t in each equation.

(See Figure 3.)

If you substitute 6 for t in $d = 50t$, you get $d = 50(6) = 300$ miles.

If you substitute 6 for t in $d = 55t$, you get $d = 55(6) = 330$ miles.

If you substitute 6 for t in $d = 60t$, you get $d = 60(6) = 360$ miles.

- b. Equation: You can substitute 275 for d in each equation and use fact families or guess-and-check to find t .

For $d = 50t$, when $d = 275$, $t = 5.5$.

For $d = 55t$, when $d = 275$, $t = 5$.

For $d = 60t$, when $d = 275$, $t \approx 4.58$.

B. 1. a.

Smartphone Monthly Charges

Number of Text Messages	Cost
0	\$0
500	\$15
1,000	\$30
1,500	\$45
2,000	\$60
2,500	\$75

- b. The cost for 1,000 messages is \$30.

For 1,725 messages, the cost is the cost for 1,500 messages, \$45, plus $\$.03 \times 225 = \6.75 . So the cost of 1,725 messages is \$51.75.

- c. Using the table, the charge for 2,500 messages is \$75. The charge for 2,000 messages is \$60. \$18 is more than the charge for 500 messages and less than the charge for 1,000 messages. The charge for 600 messages is \$18.

Smartphone Monthly Charges

Number of Text Messages	Cost
500	\$15
600	\$18
700	\$21
800	\$24
900	\$27
1,000	\$30

2. a. The monthly charge B is related to the number of text messages n by the equation $B = 0.03n$.

- b. For 1,250 text messages, the cost is $B = \$.03 \times 1,250 = \37.50 .

(See Figure 3.)

Figure 3

Distance Traveled at Different Average Speeds

Time (h)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0	0	0
1	50	55	60
2	100	110	120
3	150	165	180
4	200	220	240
5	250	275	300
6	300	330	360

3. a. (See Figure 4.)

b. You can use the graph by finding 1,000 (or 1,725 or 1,250) on the x-axis, finding the corresponding point on the line, and then looking at the y-axis for the cost.

c. 1. 5 inches \approx 12.5 centimeters,
12 inches \approx 30 centimeters, and
7.5 inches \approx 18.75 centimeters.

2. $C = 2.5l$
 $C = 2.5(12) = 25$ centimeters

3. 10 centimeters \approx 4 inches,
30 centimeters \approx 12 inches, and
100 centimeters \approx 40 inches.

4. For part (1), you can use the graph by finding 5 (or 12 or 7.5) on the x-axis, finding the corresponding point on the line, and then looking at the y-axis for the length in centimeters.

For part (3), you can use the graph by finding 10 (or 30 or 100) on the y-axis, finding the corresponding point on the line, and then looking at the x-axis for the length in inches.

Students might construct a rule for unit conversion in the opposite direction:

$l = \frac{C}{2.5}$. In that case, their graphs will have axis units swapped, and the steepness of the line will be different.

(See Figure 5.)

Figure 4

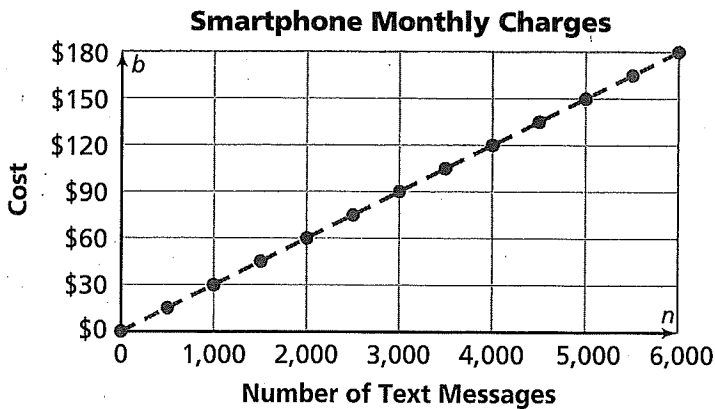
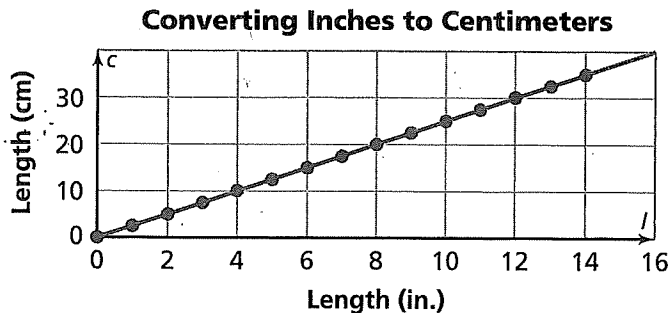
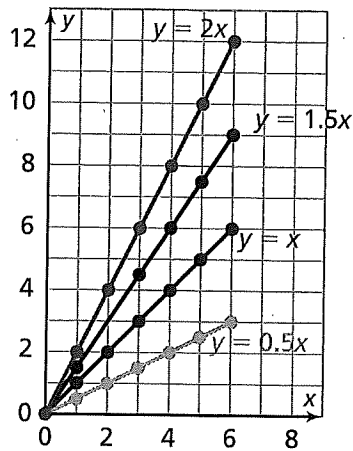


Figure 5



D. 1.

x	0	1	2	3	4	5	6
$y \cdot 2x$	0	2	4	6	8	10	12
$y \cdot 0.5x$	0	0.5	1	1.5	2	2.5	3
$y \cdot 1.5x$	0	1.5	3	4.5	6	7.5	9
$y \cdot x$	0	1	2	3	4	5	6



- The pattern of change in each table is that each increase of 1 in the value of x leads to an increase of m in the value of y . The graphs are all linear patterns (and it does make sense to connect the points) with slope indicated by the size of the coefficient m . Greater values of m lead to steeper slopes.
- a-b.** Table: You can look in the column for $x = 2$ to find the value for y for each equation.
 $y = 4, 1, 3,$ and 2 for equations $y = 2x,$
 $y = 0.5x,$ $y = 1.5x,$ and $y = x,$ respectively.
 Graph: You can trace the y -value for $x = 2$
 for each graph.
 Equation: You can substitute 2 for x in the
 equation.

Table: Similarly, you can use the table to find the value of x in each equation by seeing where $y = 6$. You will have to extend the table for the equation $y = 0.5x$.

Graph: You can trace the x -value for $y = 6$ for each graph.

Equation: You can substitute 6 for y in the equation and use fact families or guess-and-check to solve for x .

- A problem context for $y = 0.5x$ might be something like this: "If the operator of a juice bottle machine makes 50 cents profit on each sale, the profit in dollars on sale of x bottles will be given by $y = 0.5x$ "

A problem context for $y = 1.5x$ might be something like this: "If it takes 1.5 yards of material to make a decorative flag, the equation $y = 1.5x$ gives the amount of material required to make x flags."

A problem context for $y = 2x$ might be something like this: "A National Hockey League team earns 2 points for every game it wins, so the team's point total (from wins) will be given by $y = 2x$ (though teams can earn 1 point for losses that occur in overtime)."

A problem context for $y = x$ might be something like this: "If Mike runs 1 meter per second, how far y does Mike run in x seconds?"

- All relationships can be given by equations in the general form $y = mx$. The pattern of growth in the dependent variable is an increase of m for each increase of 1 in the value of the independent variable. All graphs have a linear pattern of points that are steeper for greater values of m .

During the discussion of Question C, you might ask:

- How does this equation compare to the equations you wrote for Questions A and B?
- How does this graph compare to the graphs you made in Questions A and B?
- If the value of one variable is known, how can you find the value of the other variable?
- What are the advantages and disadvantages of each representation?



Assignment Guide for Problem 3.3

Applications: 6–9 | Connections: 31–35

Answers to Problem 3.3

A. 1. a. (See Figures 1 and 2.)

b. The table and graph both show a steady rate of increase. The pattern of change in the table is that each increase of 4 in the number of people in a group leads to a constant increase in the group cost. The graph shows a linear pattern with a start value of 50.

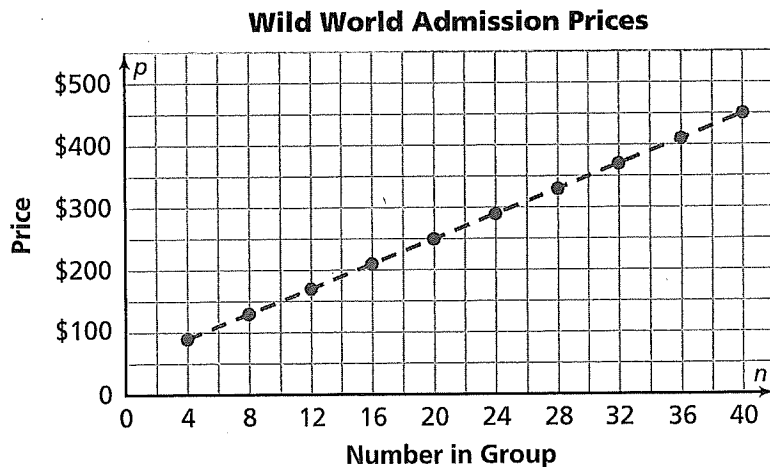
- a. You can calculate the admission price for a group with any number of people by multiplying the number of people by 10 and adding 50 to that result.
- b. The equation $p = 10n + 50$ relates admission price p to group size n .
- c. Both Problems show a linear relationship (There is a constant rate of change.). Both graphs are straight lines, indicating that the change is constant. In Problem 3.2, you can express the relationship between the dependent and independent variables with one operation. In this Problem, you need two operations to express the relationship.

Figure 1

Wild World Admission Prices

Number in Group	4	8	12	16	20	24	28	32	36	40
Price	\$90	\$130	\$170	\$210	\$250	\$290	\$330	\$370	\$410	\$450

Figure 2



3. a. Table: 18 is halfway from 16 to 20. The group cost for 16 is \$210, and for 20, it is \$250. So the group cost for 18 people will be $\$210 + \$20 = \$230$.

Graph: You can trace the y-value for $x = 18$.

Equation: You can substitute 18 for the n-value in the equation and solve for p.

$$p = 10 \times 18 + 50 = 230$$

- b. Table: 350 is halfway from 330 to 370. 28 people cost \$330, and 32 people cost \$370. So 30 people will cost \$350.

390 is halfway from 370 to 410.

32 people cost \$370, and 36 people cost \$410. So 34 people will cost \$390.

Graph: You can trace the x-value for $y = 350$.

You can trace the x-value for $y = 390$.

- B. 1. (See Figure 3.)

2. To find your bonus card balance after any number of rides, you multiply the number of rides by 6 and subtract that result from 100.

3. $P = 100 - 6r$, where P is the number of points left and r is the number of rides taken.

4. Cost per ride appears as subtraction because there is a decrease in the value of the bonus card balance for each ride. 100 bonus points at the start appears as addition in the equation.

5. (See Figure 4.)

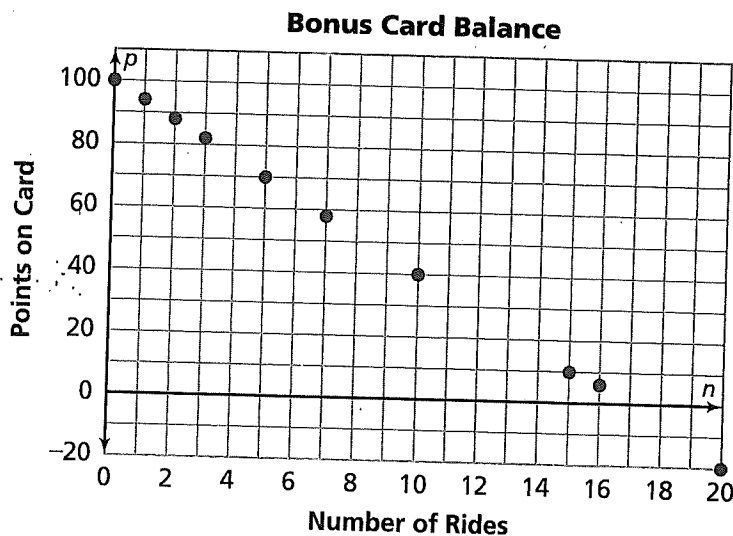
The values of the dependent variable decrease at a constant rate as the values of the independent variable increase.

Figure 3

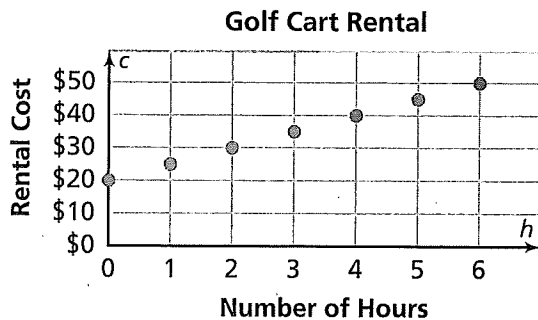
Bonus Card Balance

Number of Rides	0	1	2	3	5	7	10	15
Points on Card	100	94	88	82	70	58	40	10

Figure 4



- C. 1. The variable h represents hours of rental time and the variable c represents rental cost. The 20 represents the minimum rental charge of \$20 and the 5 says that each hour of time used adds \$5 to the cost of the rental.
2. (See Figure 5.)



In this case, it is not clear whether (or how) one should connect the points on the graph. Rentals like this often charge a full hour for any part of an hour used, so the proper graph would be a step function that is level between whole hours.

3. The cost per hour shows up in the table and graph as a steady rate of increase of 5 dollars for every 1 hour. It shows up in the equation as the coefficient 5.
4. The 20 is the entry in the table when the number of hours is 0. The 20 is also where the graph intersects the y -axis.
5. If you substitute 0 for x , $y = 20 + 5(0) = 20$. Therefore, neither $(0, 4)$ nor $(0, 20)$ satisfy the relationship represented by the equation.

If you substitute 7 for x , $y = 20 + 5(7) = 55$. Therefore, $(7, 55)$ does satisfy the relationship represented by the equation.

You could also use a graph or a table to check for these points.

Figure 5

Golf Cart Rental

Number of Hours	0	1	2	3	4	5	6
Rental Cost	\$20	\$25	\$30	\$35	\$40	\$45	\$50