Applications

1. 1; Possible explanation: $\frac{9}{9}$ is 1, so $\frac{10}{9}$ is more than 1.
2. $\frac{1}{2}$; Possible explanation: Since $\frac{8}{16}=\frac{1}{2}$, $\frac{9}{16}$ is close to $\frac{1}{2}$.
3. 1; Possible explanation: $\frac{5}{6}$ is only $\frac{1}{6}$ less than 1 whole.
4. $\frac{1}{2}$; Possible explanation: $\frac{1}{2}$ of 100 is 50, so $\frac{48}{100}$ is closer to $\frac{1}{2}$.
5. $\frac{1}{2}$; Possible explanation: $\frac{1}{2}$ is 0.50 and $\frac{3}{4}$ is 0.75, so 0.67 is close to $\frac{1}{2}$.
6. 0; Possible explanation: 0.0009999 is a very small amount. It does not have any tenths in it, and $\frac{1}{2}$ is equivalent to 5 tenths.
7. 1; Possible explanation: $\frac{7}{8}$ is a little less than 1 and $\frac{4}{9}$ is a little less than $\frac{1}{2}$. Together, a little less than 1 and a little less than $\frac{1}{2}$ is a little less than $1\frac{1}{2}$ or closer to 1 than to 2.
8. 2; Possible explanation: $1\frac{3}{4}$ is closest to 2, and $\frac{1}{8}$ is a small number that will make the sum closer to, but not greater than, 2.
9. 3; Possible explanation: $1\frac{1}{3}$ is a little more than 1.3. 1.3 and 1.3 is 2.6, which is greater than $2\frac{1}{2}$, and closest to 3.
10. 0; Possible explanation: It would take two $\frac{1}{4}$'s to equal exactly $\frac{1}{2}$. Since $\frac{1}{8}$ is less than $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$ is less than $\frac{1}{2}$ and closer to 0.
11. 2; three tenths and eight tenths have a sum greater than 1, so the total sum here is greater than 2.
12. 2; $1\frac{4}{10}$ is equivalent to 1.4 which, when added to 0.375, is 1.775, which is close to 2.
13. 0.5; Possible explanation: $\frac{3}{5}$ is a bit more than half, and $\frac{1}{10}$ is a small fraction, not big enough to push $\frac{3}{5}$ close to 1.
14. 0.5; Possible explanation: $\frac{1}{4}$ is exactly in the middle of 0 and $\frac{1}{2}$. $\frac{1}{10}$ is a small fraction so the sum is closer to $\frac{1}{2}$, but not greater than $\frac{1}{2}$.
15. 0; Possible explanation: $\frac{1}{4}=\frac{2}{8}$. Also, $\frac{1}{4}$ is exactly halfway between 0 and $\frac{1}{2}$. $\frac{1}{9}<\frac{1}{8}$, so $\frac{1}{9}+\frac{1}{8}<\frac{1}{4}$. Therefore the sum is closer to 0 than to $\frac{1}{2}$.
16. a. This is incorrect. $\frac{1}{8}<\frac{1}{4}$, so the sum here is less than $\frac{3}{4}$.
    b. Correct. Each fraction is equal to $\frac{1}{2}$, so the sum is 1.
    c. Correct. $\frac{5}{6}>\frac{3}{4}$, so the sum of $\frac{5}{12}$ and $\frac{5}{6}$ is also greater.
    d. Correct. $\frac{5}{10}=\frac{1}{2}$ and $\frac{3}{8}>\frac{1}{4}$, so the sum is greater than $\frac{3}{4}$.
17. Possible answer: $\frac{1}{4}$ and $\frac{1}{7}$
18. Possible answer: $\frac{3}{8}$ and $\frac{4}{9}$
19. Possible answer: $\frac{5}{8}$ and $\frac{1}{2}$
20. Possible answer: $\frac{5}{8}$ and $\frac{7}{8}$
21. a. The greatest possible sum is $1.05+\frac{9}{10}$.
    b. The least possible sum is $\frac{1}{4}+\frac{3}{5}$.
22. a. No; In the price list, the whole numbers add up to 9: $2+1+1+1+3+1=9$. The cheese is \$1.95, which brings the total to \$9.95. There's clearly enough in the cost of other items to put the total over \$10. (The exact cost of the groceries is \$12.42.)
    b. Possible answers: Milk, cheese, avocado (\$4.92); eggs, cheese, honey, bread (\$4.91)
    c. cereal, honey, and avocado (\$4.94)
23. $\frac{5}{8}$ is closest to $\frac{1}{2}$. Possible explanation: $\frac{4}{8}=\frac{1}{2}$. The other $\frac{1}{8}$ makes $\frac{5}{8}$ just a little more than $\frac{1}{2}$, but not close to $\frac{8}{8}$, or 1 whole.

24. Soo has enough molding. $\frac{7}{8}$ is $\frac{1}{8}$ less than 1 whole and $\frac{8}{7}$ is $\frac{7}{7}$ more than 1 whole or $1\frac{1}{7}$. Since $\frac{1}{7} > \frac{1}{8}$, when the $\frac{1}{7}$ is added to the $\frac{7}{8}$ it will be greater than 1 yard, and the total will be greater than 2 yards.

25. He had about 8 quarts.

26. No; If you add $\frac{3}{4}$ to $1\frac{3}{4}$, you will get $2\frac{1}{2}$. But $\frac{5}{8}$ is a little less than $\frac{3}{4}$, so there is not enough.

27. **a.** Marigolds: $\frac{3}{20}$; Lantana: $\frac{1}{20}$; Impatiens: $\frac{3}{10}$; Petunias: $\frac{1}{10}$; Lilies: $\frac{1}{5}$; Begonias: $\frac{1}{20}$; Tulips: $\frac{1}{20}$; Daisies: $\frac{1}{20}$; Irises: $\frac{1}{20}$

b. $\frac{4}{20} - \frac{1}{20} = \frac{3}{20}$

c. $\frac{4}{20} + \frac{1}{20} + \frac{1}{20} = \frac{6}{20}$, or $\frac{3}{10}$

d. Incorrect. Possible explanation: The number sentence for the situation is $\frac{3}{20} - \frac{1}{20} = \frac{1}{10} + \frac{1}{20}$. If you work out the subtraction problem on the left of the equal sign and the addition problem on the right, the answers are not the same.

e. Possible combinations that total $\frac{3}{10}$, the fraction planted with impatiens:

Marigolds + Petunias + Lantana:
$\frac{3}{20} + \frac{1}{10} + \frac{1}{20} = \frac{3}{20} + \frac{2}{20} + \frac{1}{20} = \frac{6}{20} = \frac{3}{10}$

Lilies + Petunias: $\frac{4}{20} + \frac{2}{20} = \frac{6}{20}$, or $\frac{3}{10}$

Marigolds + Begonias + Tulips + Daisies: $\frac{3}{20} + \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{6}{20}$, or $\frac{3}{10}$

28. **a.** $\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$ of the page is used for ads.

b. $1 - \frac{3}{16} = \frac{13}{16}$ of the page remains.

29. $\frac{3}{4}$ (three $\frac{1}{4}$-page ads, or $3 \times \frac{1}{4}$) plus $\frac{4}{8}$ (four $\frac{1}{8}$-page ads, or $4 \times \frac{1}{8}$) plus $\frac{10}{16}$ (ten $\frac{1}{16}$-page ads, or $10 \times \frac{1}{16}$) $= 1\frac{7}{8}$ pages.

30. $2\frac{3}{4} - 1\frac{5}{8} = 1\frac{5}{8}$ pages

31. $\frac{1}{9} + \frac{1}{18} = \frac{3}{18} = \frac{1}{6}$ of the lasagna is eaten, leaving $\frac{5}{6}$ of the lasagna uneaten.

32. $\frac{3}{4} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \frac{7}{8}$ of a small bag of chips

33. $2\frac{11}{15}$

34. $7\frac{3}{8}$

35. $8\frac{5}{6}$

36. $6\frac{2}{12}$ or $6\frac{1}{6}$

37. $\frac{3}{4} + \frac{4}{5}$ is greater: $\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1\frac{1}{2} = 1\frac{10}{20}$; $\frac{3}{4} + \frac{4}{5} = \frac{15}{20} + \frac{16}{20} = 1\frac{11}{20}$

38. $\frac{7}{6} - \frac{2}{3}$ is greater: $\frac{7}{6} - \frac{2}{3} = \frac{7}{6} - \frac{4}{6} = \frac{3}{6} = \frac{1}{2} = \frac{5}{10}$; $\frac{3}{5} - \frac{5}{10} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

39. $2\frac{5}{6} + 1\frac{1}{3} = 4\frac{1}{6}$

40. $15\frac{5}{8} + 10\frac{5}{6} = 26\frac{11}{4}$

41. $4\frac{4}{9} + 2\frac{1}{5} = 6\frac{29}{45}$

42. $6\frac{1}{4} - 2\frac{5}{6} = 3\frac{5}{12}$

43. $3\frac{1}{2} - 1\frac{4}{5} = 1\frac{7}{10}$

44. $8\frac{2}{3} - 6\frac{5}{7} = 1\frac{20}{21}$

45. $\frac{3}{4}$

46. $\frac{3}{6}$ or $\frac{1}{2}$

47. $\frac{3}{8}$

48. $\frac{3}{10}$

49. $\frac{3}{12}$

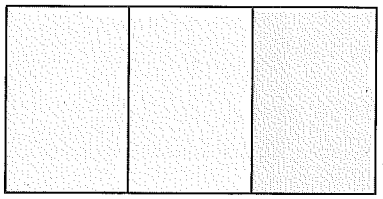
50. $\frac{3}{14}$

In all of the problems, you add unit fraction where one fraction is half the size of the other. The fraction in each part with the lesser denominator is twice the value of the unit fraction with the greater denominator. You can think of the unit fraction with the lesser denominator as two unit fractions with the greater denominator. This gives a sum with a 3 in the numerator over the greater denominator.

51. No. If $\frac{14}{16}$ of all the pizza were eaten, this would be less than one whole pizza. If there are eight sections in each pizza, then people are eating eighths. And all together they ate $\frac{14}{8}$ or $1\frac{6}{8}$ pizzas.

Connections

52. Divide the rectangle into thirds. Each third of the rectangle represents 50% of the original whole. So, 100% is represented by the blue area.



53. 15 beans. There are 9 beans shown in the picture representing three fifths. So each fifth must contain 3 beans. Since the whole is five fifths, there must be 15 beans on the counter.

54. **a.** $\frac{1}{2}$ is 0.5 and 50%

$\frac{1}{3}$ is about 0.33 and 33%

$\frac{1}{4}$ is 0.25 and 25%

$\frac{2}{3}$ is about 0.66 or 66% (Some students will argue for 0.67 and 67%, if the convention of rounding up is used.)

$\frac{3}{4}$ is 0.75 and 75%

$\frac{1}{6}$ is about 0.16 and 16% (Some students will argue for 0.17 and 17%, if the convention of rounding up is used.)

$\frac{1}{5}$ is 0.2 or 20%

$\frac{1}{8}$ is 0.125 or 12.5%

b. (See Figure 1.)

55. D

56. a and d

a. This set of fractions can be renamed as hundredths because each denominator 2, 4, and 5, are factors of 100.

b. This set of fractions cannot be renamed as hundredths. The denominator 10 is a factor of 100, but 11 and 12 are not.

c. This set of fractions cannot be renamed as hundredths. The denominators 6 and 8 are not factors of 100.

d. This set of fractions can be renamed as hundredths because all the denominators, 5, 10, and 20, are factor of 100.

57. **a.** The least possible sum is $1\frac{1}{4}$. This is the sum of the least numbers in each of these intervals.

b. The greatest possible sum is 2. That is the sum of the greatest number in each of these intervals.

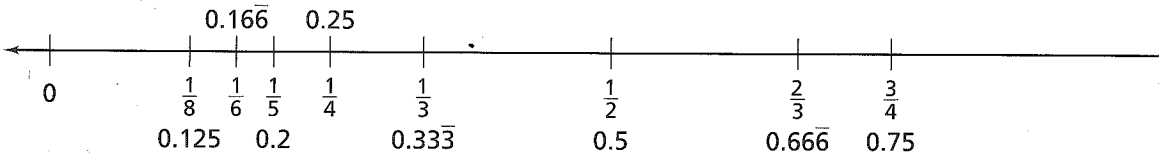
58. $N = 2$

59. $N = 3$

60. $N = 8$

61. $N = 24$

Figure 1



62. $\frac{2}{6} = \frac{4}{12}$

63. $\frac{8}{12} = \frac{2}{3} = \frac{4}{6}$

64. $\frac{3}{9} = \frac{2}{6} = \frac{6}{18}$

65. $18.156 < 18.17$

66. $4.0074 > 4.0008$

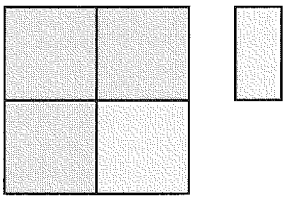
67. G

68. Answers will vary. Possible answers include: Foley, Burg, Walker, Theule, and Lapp.
$\frac{5}{16} + \frac{3}{16} + \frac{5}{16} + \frac{3}{16} + \frac{1}{4} = 1\frac{1}{4}$

69. **a.** $1\frac{1}{3}$ of a whole; 3 squares $= \frac{1}{13}$ of a whole; 9 squares = 1 whole; so 12 squares $= 1\frac{1}{3}$ of a whole.

b. $\frac{7}{9}$ of a whole; 3 squares $= \frac{1}{3}$ or $\frac{3}{9}$ of a whole; 9 squares $= \frac{3}{3}$ or $\frac{9}{9}$ of a whole; so 7 squares $= \frac{7}{9}$ of a whole.

70. **a.** **Note:** The purple shaded region represents 1. The green shaded region represents the $\frac{1}{3}$ portion of $1\frac{1}{3}$. The blue shaded region represents $\frac{1}{6}$.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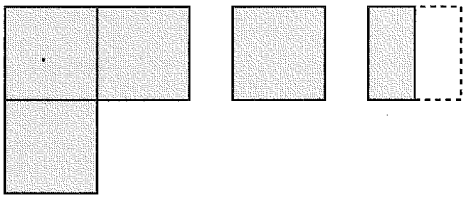
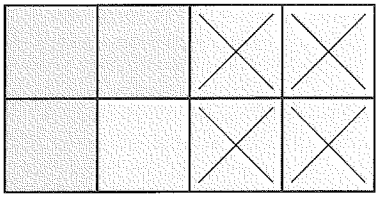


b. **Note:** The Xs indicate the regions subtracted. The purple and green shaded regions each represent 1. The blue shaded region represents $\frac{2}{3}$. The remaining portion is $\frac{4}{3}$.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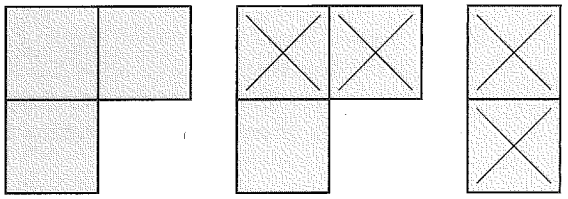


71. **a.** Since $\frac{70}{150} = \frac{7}{15}$ and $\frac{30}{150} = \frac{2}{10}$, the sum of these two numbers is the same no matter which forms we use.

b. Answers will vary. Possible answers include: $\frac{14}{30} + \frac{4}{20}$ and $\frac{14}{30} + \frac{6}{30}$.

c. Any problem with common denominators should be easy to solve. This includes Maribel's problem as well as this one: $\frac{14}{30} + \frac{6}{30}$.

Extensions

72. Answers will vary. Possible answers include: $\frac{2}{5}$

73. Answers will vary. Possible answers include: $\frac{2}{7}$

74. It is always possible to find a fraction between any two fractions on the number line. One way to know this is that we can rewrite each given fraction as an equivalent fraction, using common denominators, and then choose a new fraction with a numerator between the two new numerators. For example, given $\frac{1}{3}$ and $\frac{1}{2}$ we can rewrite as $\frac{4}{12}$ and $\frac{6}{12}$, and then we know $\frac{5}{12}$ is between these.

Another method would be to take the number that is halfway between the two fractions, which is the average of the two fractions. For example, to find the fraction between $\frac{3}{4}$ and $\frac{4}{5}$ we can take the average.

$\frac{1}{2}\left(\frac{3}{4}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{15}{20}+\frac{16}{20}\right)=\frac{1}{2}\times\frac{31}{20}=\frac{31}{40}$

Now $\frac{3}{4}=\frac{30}{40}<\frac{31}{40}<\frac{32}{40}=\frac{4}{5}$,

so $\frac{31}{40}$ is between $\frac{3}{4}$ and $\frac{4}{5}$.

75. **a.** $\frac{1}{32}$ page costs \$5, $\frac{1}{16}$ page costs \$10, $\frac{1}{8}$ page costs \$20, $\frac{1}{4}$ page costs \$40, $\frac{1}{2}$ page costs \$80, 1 full page costs \$160.

b. $40\times3+20\times4+10\times1=\$210$

c. Yes. $\frac{2}{8}$ page costs \$40 and $\frac{4}{16}$ page costs \$40, for a total of \$80.

d. Answers will vary. Possible answers include: Any combination adding up to $\frac{1}{2}$ page. This includes two $\frac{1}{4}$-page ads, or one $\frac{1}{4}$-page ad and two $\frac{1}{8}$-page ads.

76. **a.** 2 acres; 16 people can clear twice as much in the same amount of time.

b. $\frac{1}{4}$ acre; one fourth of the people can clear one fourth as much in the same amount of time.

c. 24 people; three times as many people are needed to clear three times the area in the same amount of time.

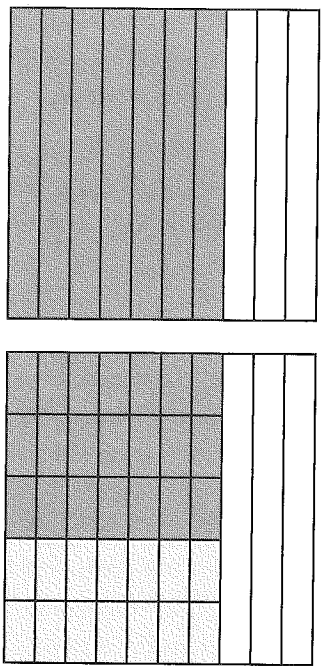
d. 48 people; twice as many people are needed to complete part (c) in half of the time.

77. **a.** Answers will vary. Possible answers include: $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$.

b. Answers will vary. Possible answers include: $\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$.

Applications

1. a.



b. $\frac{14}{50}$ or $\frac{7}{25}$

2. a. $\frac{2}{3}$ of $\frac{3}{4}$ (Figure 1)
   $\frac{3}{4}$ of $\frac{2}{3}$ (Figure 2)

   b. They are equal.

   c. These expressions are equal. Another way to say this is that multiplication is commutative.

Figure 1

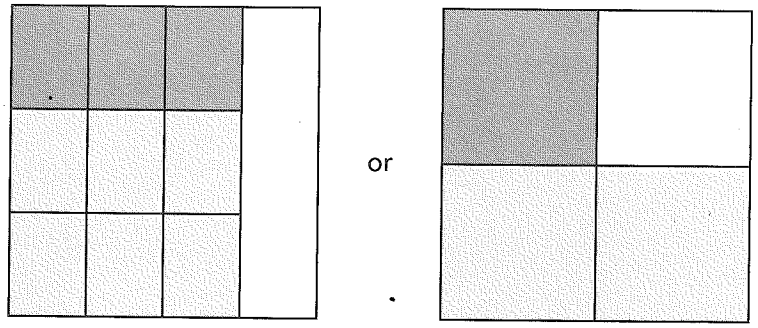
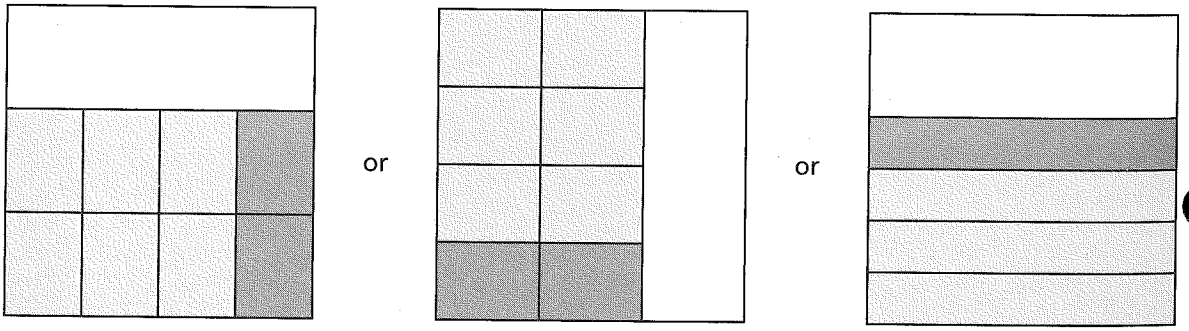


Figure 2



3. **a.** (Figure 3)

b. $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ or $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$

4. **a.** $\frac{1}{6}$

b. $\frac{1}{8}$

c. $\frac{1}{3}$

d. $\frac{3}{8}$

5. **a.** Less. You are finding a fraction of 1.

b. Less. You are finding part of a fraction.

c. Less. You are finding part of $\frac{2}{3}$.

d. Less. You can think of this as $\frac{2}{3} \times \frac{3}{4}$. In this case, you are finding part of $\frac{3}{4}$.

6. Possible answer: $\frac{1}{2} \times 2 = 1$

7. Possible answer: $\frac{1}{3} \times 2 = \frac{2}{3}$

8. Possible answer: $\frac{1}{4} \times 10 = 2\frac{1}{2}$

9. Possible answer: $\frac{1}{3} \times 2 = \frac{2}{3}$

10. **a.** 9 cups of pretzels, 8 cups of popcorn, 4 cups of peanuts, and 3 cups of chocolate chips

b. $11\frac{1}{4}$ cups of pretzels, 10 cups of popcorn, 5 cups of peanuts, and $3\frac{3}{4}$ cups of chocolate chips

11. **a.** $\frac{1}{2}$

b. $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{2}$ or $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

12. $15\frac{3}{4}$ pounds

13. **a.** Answers will vary. Possible answer: Approximately 6, because $8\frac{5}{6}$ is near 9. The exact answer is $5\frac{8}{9}$.

b. Answers will vary. Possible answer: Approximately 10 since $14\frac{1}{2}$ is near 15. The exact answer is $9\frac{2}{3}$.

c. Answers will vary. Possible answer: Approximately $1\frac{1}{2}$ because $2 \times \frac{2}{3}$ is less than $1\frac{1}{2}$, and adding half of $\frac{2}{3}$ (i.e, $\frac{1}{3}$) should give a result a bit bigger than $1\frac{1}{2}$. The exact answer is $1\frac{2}{3}$.

14. **a.** 18 caramel squares

b. $2\frac{3}{4} \times \frac{3}{4} = \frac{6}{4} = 1\frac{1}{2}$

15. $12 \times \frac{11}{3} = 136$

16. $53\frac{1}{2}$

17. **a.** 6

b. 12

c. 30

d. 30

e. Possible answers: Each product is a multiple of 6; as the numerator increases, so does the product.

18. $4\frac{3}{8}$ cups

19. $\frac{5}{9}$

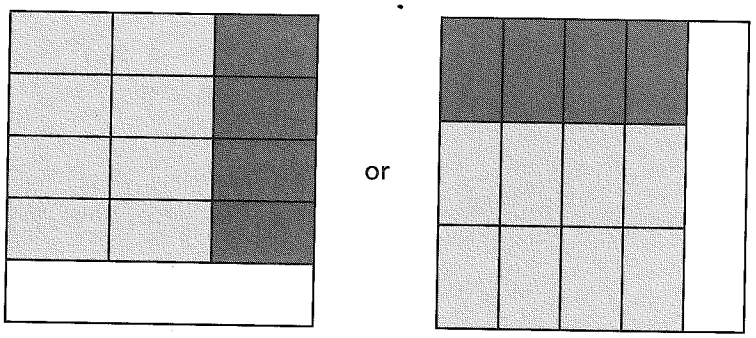
20. $\frac{1}{4}$

21. $7\frac{14}{27}$

22. $2\frac{14}{25}$

23. $28\frac{2}{3}$

Figure 3



24. $\frac{9}{14}$

25. $1\frac{13}{20}$

26. $2\frac{5}{8}$

27. $1\frac{9}{11}$

Connections

28. $\frac{11}{60}$, or about $\frac{1}{6}$

29. $\frac{17}{30}$, or about $\frac{1}{2}$

30. Yes. If Roshaun started with more than twice as much money as Lea, then $\frac{1}{4}$ of Roshaun's money would be more than $\frac{1}{2}$ of Lea's money.

31. $\frac{7}{16}$ of a yard

32. Aran gets $\frac{1}{2}$ of the pretzels. Jon gets $\frac{1}{3}$ of the pretzels. Kiona gets $\frac{1}{6}$ of the pretzels.

33. $\frac{2}{5}$ of the class wants to go to Navy Pier.

34. a. $\frac{3}{5}$ of the class consists of girls with brown hair.

b. Answers will vary. Possible answers: 20 students. The number of students needs to be divisible by both 4 and 5 (so that $\frac{3}{4}$ and $\frac{3}{5}$ both represent whole numbers of people). Any multiple of 20 is possible.

35. a. $30 \times \frac{1}{4} + 6 \times \frac{3}{8} + \frac{7}{16} = 10\frac{3}{16}$ inches

b. No. We can know this by finding the exact sum of $8 \times \frac{3}{8} + 8 \times \frac{7}{16} = 6\frac{1}{2}$, or by estimating. One estimation strategy: There are 16 beads altogether; each bead is shorter than an inch, so the whole collection is less than 16 inches.

36. C

37. F

38. B

39. $\frac{2}{9}$

40. $2\frac{1}{2}$

41. $\frac{5}{9}$

42. $\frac{1}{4}$

43. a. 60 minutes

b. 30 minutes

c. 30 minutes

d. 6 minutes

e. 75 minutes

f. $3\frac{1}{10}$ or 3.1 hours

44. $1\frac{1}{8}$ cups of flour
$\frac{1}{2}$ teaspoon of baking soda
$\frac{1}{2}$ teaspoon of salt
$\frac{1}{2}$ cup of butter
$\frac{3}{8}$ cup of sugar
$\frac{1}{2}$ teaspoon of vanilla extract
1 large egg
$1\frac{1}{6}$ cups of chocolate morsels
$\frac{1}{3}$ cup of chopped nuts

45. $\frac{3}{4}$ cup of firmly packed brown sugar
$\frac{1}{3}$ cup of shortening
$\frac{1}{2}$ tablespoon of water
$\frac{1}{2}$ teaspoon of vanilla extract
1 large egg
$\frac{3}{4}$ cup of flour
$\frac{1}{6}$ cup of cocoa powder
$\frac{1}{4}$ teaspoon of salt
$\frac{1}{8}$ teaspoon of baking soda
1 cup of chocolate chips

46. Answers will vary on all parts. Possible answers are given for each part.

a. The product will be greater than, but closer to, 1. The exact answer is $1\frac{9}{20}$.

b. The product will be greater than, but closer to, 4. The exact answer is $4\frac{7}{20}$.

c. The product will be greater than, but closer to, 1. The exact answer is $1\frac{3}{7}$.

d. The product will be between 9 and 10. Estimating whether this product is closer to 9 or to 10 is challenging. The exact product is $9\frac{23}{48}$, which is only slightly closer to 9 than it is to 10.

e. See solutions to a—d.

47. $6\frac{1}{2}$

48. $7\frac{1}{10}$

49. $6\frac{19}{30}$

50. $2\frac{23}{24}$

51. $2\frac{9}{10}$

52. $4\frac{1}{15}$

53. B

54. **a.** $1\frac{1}{5}$; Fala probably assembled five fifths to make one whole and had one fifth left over.

b. Jorell probably multiplied numerators and denominators to get $\frac{6}{5}$.

c. Hiroshi probably got the decimal answer 1.2.

55. F

Extensions

56. $\frac{1}{4}$

57. $\frac{5}{24}$

Applications

1. 1. a. 2 lattes with $\frac{1}{9}$ of a cup of milk left over. This $\frac{1}{9}$ cup of milk is enough to make $\frac{1}{3}$ of a latte.

   b. 2 lattes with $\frac{1}{6}$ of a cup of milk left over. This $\frac{1}{6}$ cup of milk is enough to make $\frac{1}{2}$ of a latte.

   c. exactly 11 lattes

2. Answers will vary. You have $1\frac{3}{4}$ pounds of cheese. One batch of macaroni and cheese requires $\frac{1}{2}$ pound of cheese. How many batches can you make?

   Finding *how many* $\frac{1}{2}$'s are in $1\frac{3}{4}$ is a division problem. You can make $3\frac{1}{2}$ batches.

3. a. 80

   b. 40

   c. $26\frac{2}{3}$

   d. 200

   e. 100

   f. $28\frac{4}{7}$

   g. 140

   h. 70

   i. $23\frac{1}{3}$

   j. $20 \div \frac{2}{7}$ is half as large as $20 \div \frac{1}{7}$. This makes sense because the groups of $\frac{2}{7}$ are twice as large, so there are half as many of them in the total of 20. Similarly, $20 \div \frac{6}{7}$ is $\frac{1}{6}$ the size of $20 \div \frac{1}{7}$.

4. 10

5. $22\frac{1}{2}$

6. 12

7. $6\frac{2}{5}$

8. 20

9. 40

10. 80

11. No, he does not have quite enough flour. $16 \div \frac{3}{4} = 21\frac{1}{3} < 22$

12. She can make five frames: $108 \div 18\frac{3}{8} = 5\frac{129}{147}$. An easier computation for many people may be to estimate and then check the solution. $108 \div 18 = 6$, so 5 is a reasonable guess for $108 \div 18\frac{3}{8}$. Checking by multiplication, $5 \times 18\frac{3}{8} = 91\frac{7}{8}$. This is less than 108, but the remainder is not enough to make an additional frame.

13. Each rabbit gets $\frac{21}{48}$ of an ounce of parsley (or a tiny bit less than half an ounce). $5\frac{1}{4} \div 12 = \frac{21}{48}$

14. a. $\frac{1}{16}$ pound

b. $\frac{1}{16}$ pound

c. $\frac{3}{12}$ pound, or $\frac{1}{4}$ pound

d. $\frac{2}{25}$ pound

e. $\frac{3}{4}$ pound

15. a. $1\frac{1}{3}$ gallons

b. $28 \times \frac{4}{3} = \frac{112}{3} = 37\frac{1}{3}$ miles

16. D

17. $\frac{4}{15}$; possible diagram: this diagram is made by renaming $\frac{4}{5}$ as $\frac{12}{15}$. Since there are 12 fifteenths, you can divide by 3, or group the fifteenths into 3 groups. Each of the three groups will have 4 fifteenths in it. (Figure 1)

18. $\frac{1}{3}$; this diagram shows 5 thirds. Since you are dividing by 5, you can make 5 groups. Each group will have $\frac{1}{3}$ in it. (Figure 2)

19. $\frac{1}{3}$; this problem is equivalent to Exercise 18, so the diagram is the same as Figure 2.

20. C

21. F

22. Greater than 1. $\frac{7}{9}$ is greater than $\frac{1}{9}$, so there are multiple groups of $\frac{1}{9}$ in $\frac{7}{9}$.

23. Greater than 1. $\frac{2}{3}$ is greater than $\frac{1}{9}$, so there are multiple groups of $\frac{1}{9}$ in $\frac{2}{3}$.

24. Less than 1. $\frac{1}{18}$ is less than $\frac{1}{9}$, so there is not even one whole group of $\frac{1}{9}$ in $\frac{1}{18}$.

25. Greater than 1. 1 is greater than $\frac{1}{9}$, so there are multiple groups of $\frac{1}{9}$ in 1.

26. $2\frac{1}{2}$

27. 6

28. 4

29. 15

30. $6\frac{2}{3}$

31. $\frac{3}{14}$

32. $\frac{3}{20}$

33. $1\frac{1}{5}$

34. $1\frac{7}{8}$

35. Answers will vary.

Exercise 29: You have 10 feet of string. You are making bracelets that require $\frac{2}{3}$ of a foot per bracelet. How many bracelets can you make?

Exercise 31: You have $\frac{6}{7}$ of a pound of powdered sugar that you need to split equally among four large pancakes. How much powdered sugar goes on each pancake?

Figure 1

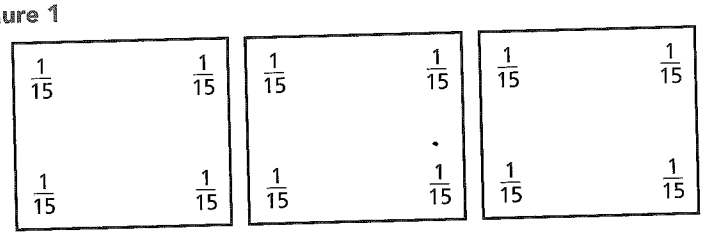
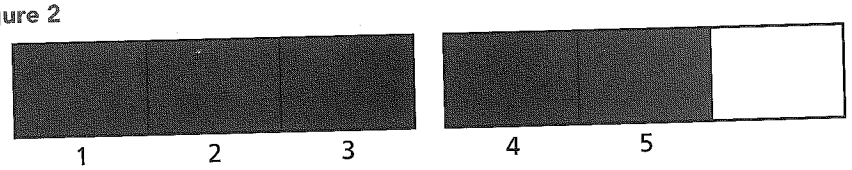


Figure 2



Connections

Answers will vary for 36–39.

36. $\frac{2}{3}$ and $\frac{8}{12}$

37. $\frac{5}{6}$ and $\frac{20}{24}$

38. $\frac{4}{3}$ and $\frac{24}{18}$

39. $\frac{4}{3}$ and $\frac{16}{12}$

40. $1\frac{1}{4}$ hours

41. $1\frac{1}{10}$

42. $1\frac{17}{24}$

43. 2

44. $4\frac{7}{12}$

45. $\frac{2}{21}$

46. $\frac{21}{32}$

47. $\frac{1}{2}$

48. $12\frac{5}{6}$

49. $\frac{9}{10}$ of a mile

50. a. (Figure 3)

b. (Figure 4)

c. (Figure 5)

d. (Figure 6)

e. Answers will vary. Possible answer: To find the difference between two consecutive marks, take the difference between the two end points. Then divide by 4 because there are four equal divisions between the two endpoints. Then add that amount to the left endpoint to find the value for the first mark. Add that same value to that answer, and then add that amount once more to the next answer.

51. 3

52. 1

53. $\frac{1}{2}$

Extensions

54. Both are correct. Some students may think of this as a division situation, while others may think of it as a multiplication situation, so student thinking may match one or the other of these more closely. But both are correct.

55. 8

56. 24

57. $\frac{1}{8}$

58. $\frac{1}{32}$

59. Any of these will work. Possible answer: I would use 1 pint and fill it 8 times.

Figure 3

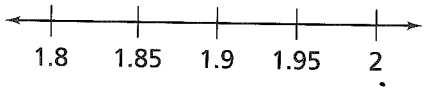


Figure 4

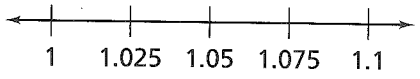


Figure 5

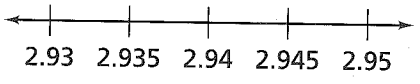


Figure 6

Applications

1. $\frac{1}{16}+\frac{1}{12}=N$
   $\frac{1}{12}+\frac{1}{16}=N$
   $N-\frac{1}{16}=\frac{1}{12}$
   $N-\frac{1}{12}=\frac{1}{16}$
2. $\frac{5}{4}-\frac{4}{5}=N$
   $\frac{5}{4}-N=\frac{4}{5}$
   $N+\frac{4}{5}=\frac{5}{4}$
   $\frac{4}{5}+N=\frac{5}{4}$
3. $N-1\frac{1}{3}=2\frac{2}{3}$
   $N-2\frac{2}{3}=1\frac{1}{3}$
   $1\frac{1}{3}+2\frac{2}{3}=N$
   $2\frac{2}{3}+1\frac{1}{3}=N$
4. $N+\frac{4}{3}=\frac{1}{3}$
   $\frac{4}{3}+N=\frac{1}{3}$
   $\frac{1}{3}-\frac{4}{3}=N$
   $\frac{1}{3}-N=\frac{4}{3}$
5. $1\frac{5}{12}$
6. $\frac{1}{20}$
7. $\frac{17}{20}$
8. $\frac{1}{2}$
9. $\frac{1}{8}$
10. $-\frac{1}{8}$
11. $m=\frac{6}{10}$
12. Answers will vary: $m=\frac{3}{10}$, $n=\frac{3}{10}$, or any choices of m and n with $m+n=\frac{3}{5}$, will solve the problem.
13. $m=\frac{6}{10}$
14. $\frac{2}{3}\times\frac{5}{7}=\frac{10}{21}$
    $\frac{5}{7}\times\frac{2}{3}=\frac{10}{21}$
    $\frac{10}{21}\div\frac{5}{7}=\frac{2}{3}$
    $\frac{10}{21}\div\frac{2}{3}=\frac{5}{7}$
15. $\frac{3}{4}\div1\frac{1}{2}=\frac{1}{2}$
    $\frac{3}{4}\div\frac{1}{2}=1\frac{1}{2}$
    $\frac{1}{2}\times1\frac{1}{2}=\frac{3}{4}$
    $1\frac{1}{2}\times\frac{1}{2}=\frac{3}{4}$
16. $N=\frac{2}{3}$
17. $N=\frac{2}{15}$
18. $N=\frac{2}{3}$
19. $N=\frac{3}{5}$
20. $N=7$
21. $N=\frac{1}{3}$
22. a. $m=\frac{5}{8}$
    b. $m=\frac{5}{8}$
    c. $m=\frac{5}{8}$
23. $32\frac{1}{2}$
24. 15
25. a. $24-\frac{1}{2}-1-1=21\frac{1}{2}$ buns
    b. 64 servings, with $\frac{1}{6}$ of a bun left over (which is $\frac{1}{2}$ of a serving)
26. $\frac{4}{9}$
27. $18\frac{2}{3}$
28. $1\frac{7}{44}$ hours (which is about 1 hour and 10 minutes) for one way and $2\frac{7}{22}$ hours for the round trip.

Connections

29.a. $N = \frac{6}{7}$

b. $N = \frac{3}{4}$

c. The original expressions are not equivalent. In part (a), you need to add $\frac{1}{4}$ and $\frac{1}{3}$ before multiplying by N. In part (b), you need to multiply $\frac{1}{3}$ by N before adding $\frac{1}{4}$.

30. $\frac{1}{2}$

31. 2

32. $\frac{1}{3}$

33. 3

34. $\frac{3}{2}$

35. $\frac{4}{3}$

36. $\frac{2}{5}$

37. $\frac{4}{5}$

38. $\frac{12}{7}$

39. $\frac{1}{3}$ and 3. These are reciprocals.

40. $\frac{1}{4}$ and 4. These are reciprocals.

41. $\frac{1}{2}$ and 2. These are reciprocals.

42. $\frac{1}{8} + \frac{5}{6}$ is larger. There are many ways to know this without computing. One way is to reason that you can add 1 small thing and 5 large things or 1 large thing and 5 small things. 5 large things will be larger (assuming the large things are the same size in both instances, and that the small things are also). The two sums are $\frac{23}{24}$ and $\frac{19}{24}$, respectively.

Another way to tell that $\frac{1}{8} + \frac{5}{6}$ is larger is to note that $\frac{1}{6} > \frac{1}{8}$. Thus,
$\frac{5}{6} + \frac{1}{8} = \left(\frac{1}{6} + \frac{4}{6}\right) + \frac{1}{8} > \left(\frac{1}{6} + \frac{4}{8}\right) + \frac{1}{8} = \frac{1}{6} + \frac{5}{8}$.
The two sums are $\frac{23}{24}$ and $\frac{19}{24}$, respectively.

43. $\frac{5}{6} - \frac{1}{8}$ is larger. There are many ways to know this without computing. One way is to observe that for a large difference, you want the numbers to be far apart. Because $\frac{5}{6} > \frac{5}{8}$ and $\frac{1}{8} < \frac{1}{6}$, the first difference will be greater than the second. The two differences are $\frac{17}{24}$ and $\frac{11}{24}$ respectively.
Another way to tell that $\frac{5}{6} - \frac{1}{8}$ is larger is to note that $\frac{5}{6} > \frac{5}{8}$ and $\frac{1}{6} > \frac{1}{8}$. Thus, $\frac{5}{6} - \frac{1}{8} > \frac{5}{8} - \frac{1}{8} > \frac{5}{8} - \frac{1}{6}$. (Here you use the fact that subtracting a larger number from a given number results in a smaller number.) The two differences are $\frac{17}{24}$ and $\frac{11}{24}$ respectively.

44. $N = \frac{1}{3}$

45. $N = 1$

46. In a simpler form this sentence is $1\frac{13}{24} + m + n = 3$. Using fact families to rewrite it, you have $m + n = 1\frac{11}{24}$. So now you can choose any number for m (less than $1\frac{11}{24}$ if you are working with positive numbers) and calculate n, since $n = 1\frac{11}{24} - m$. Possible solutions are $m = 1$ and $n = \frac{11}{24}$, or $m = \frac{5}{24}$ and $n = 1\frac{6}{24}$, or $m = \frac{15}{24}$ and $n = \frac{20}{24}$, and so forth.

Extensions

47. 0

48. 0

49. 1

50. 1

51. Answers will vary. Identity means the number that leaves the starting value unchanged.

52. $N = -\frac{1}{2}$

53. $N = -\frac{2}{3}$

54. $N = 2$

55. $N = \frac{3}{2}$

56. **a.** Yes; the additive inverse of a is $-a$. The additive inverse is also called the *opposite* of a number.

b. Nearly all numbers do, but 0 has no multiplicative inverse. The multiplicative inverse is also called the *reciprocal* of a number.