

Unit 6, Lesson 7: Revisit Percentages

Lesson Goals

- If $A\%$ of B is C , find A or B by writing equations of the form $px = q$.

Required Materials

7.1: Number Talk: Percentages (5 minutes)

Setup: Display one problem at a time. Allow 30 seconds of quiet think time per problem, followed by a whole-class discussion.

Student task statement

Solve each problem mentally.

1. Bottle A contains 4 ounces of water, which is 25% of the amount of water in Bottle B. How much water is there in Bottle B?
2. Bottle C contains 150% of the water in Bottle B. How much water is there in Bottle C?
3. Bottle D contains 12 ounces of water. What percentage of the amount of water in Bottle B is this?

Possible responses

1. 16 ounces
2. 24 ounces
3. 75%

7.2: Representing a Percentage Problem with an Equation (20 minutes)

Setup:

Students in groups of 2. 5–10 minutes of quiet work time and time to share responses with a partner, followed by a whole-class discussion.

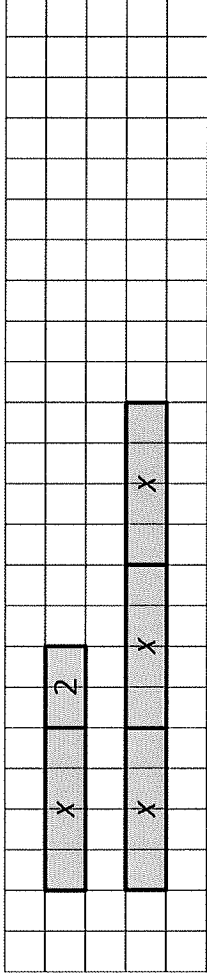
8.2: Using Diagrams to Show That Expressions are Equivalent (20 minutes)

Setup:

Access to graph paper. Ask students to draw diagrams and discuss how the diagrams help us distinguish expressions that are equal and those that are not. Students in groups of 2. Partners answer each question independently and then check with each other after each. 15 minutes of work time, followed by a whole-class discussion.

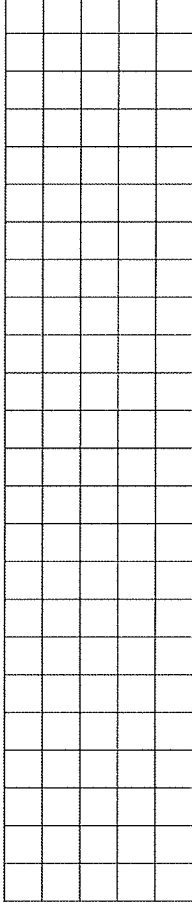
Student task statement

Here is a diagram of $x + 2$ and $3x$ when x is 4. Notice that the two diagrams are lined up on their left sides.

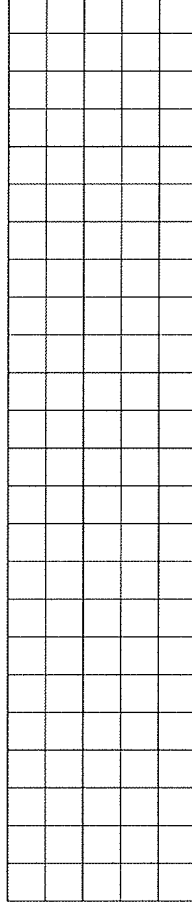


In each of your drawings below, line up the diagrams on one side.

1. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 3.



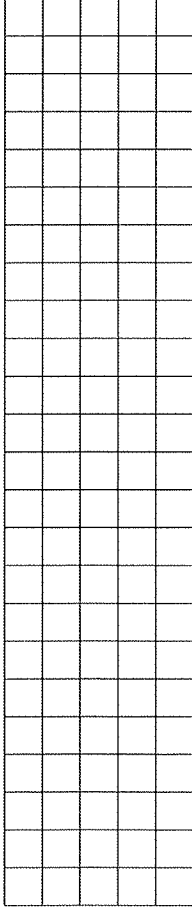
2. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 2.



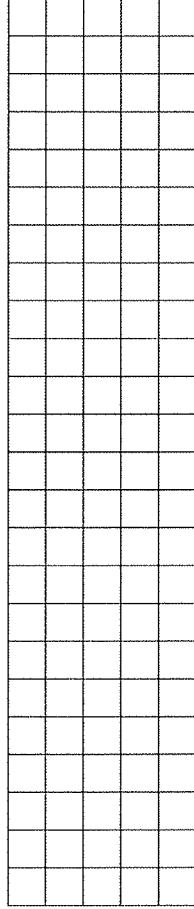
3. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 1.

Possible responses

1. Diagram shows a length of 5 for $x + 2$ and 9 for $3x$.
2. Diagram shows a length of 4 for $x + 2$ and 6 for $3x$.
3. Diagram shows a length of 3 for $x + 2$ and 3 for $3x$.
4. Diagram shows a length of 2 for $x + 2$ and 0 for $3x$.
5. They are equal when $x = 1$, not equal for other values.
6. Answers vary. Diagrams should be the same length regardless of choice of x .
7. They are always equal. Explanations vary.



4. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 0.



5. When are $x + 2$ and $3x$ equal? When are they not equal? Use your diagrams to explain.

6. Draw a diagram of $x + 3$, and a separate diagram of $3 + x$.

7. When are $x + 3$ and $3 + x$ equal? When are they not equal? Use your diagrams to explain.

8.3: Identifying Equivalent Expressions (10 minutes)

Setup: 5 minutes of quiet work time, followed by a whole-class discussion.

Student task statement

Here is a list of expressions. Find any pairs of expressions that are equivalent. If you get stuck, try reasoning with diagrams.

$a + 3$

$a \div \frac{1}{3}$

$\frac{a}{3}$

a

$a + a + a$

$a \cdot 3$

$3a$

$3 + a$

Possible responses

- $a + 3$ and $3 + a$
- $a \div \frac{1}{3}$ and $a \cdot 3$
- $a + a + a$ and $3a$ (also $a \div \frac{1}{3}$ and $a \cdot 3$)
- $\frac{1}{3}a$ and $\frac{a}{3}$
- $1a$ and a

Are you ready for more?

Below are four questions about equivalent expressions. For each one:

- Decide whether you think the expressions are equivalent.
- Test your guess by choosing numbers for x (and y , if needed).

1. Are $\frac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$ equivalent expressions?

2. Are $\frac{x + x + x + x}{x}$ and $x + x + x$ equivalent expressions?

3. Are $2(x + y)$ and $2x + 2y$ equivalent expressions?

4. Are $2xy$ and $2x \cdot 2y$ equivalent expressions?

Possible Responses

1. Yes
2. No
3. Yes
4. No

Lesson Synthesis (5 minutes)

Ensure students understand what is meant by equivalent expressions (equal for any value of their variables) and how they are different from expressions that are just equal for a given value of their variables. We can use length diagrams to show this distinction, but we can also use our knowledge of operations and their properties.

8.4: Decisions About Equivalence (Cool-down, 5 minutes)

Setup: None.

Student task statement

Decide if the expressions in each pair are equivalent. Explain how you know.

1. $x + x + x + x$ and $4x$

2. $5x$ and $x + 5$

Possible responses

1. Equivalent, because the diagrams that represent these expressions would have the same length for any value of x .
2. Not equivalent. For example, if $x = 1$, $5x = 5$ and $x + 5 = 6$, so they do not have the same value.

Unit 6, Lesson 8: Equal and Equivalent

Lesson Goals

- Understand that two expressions that are equal for every value of their variable are equivalent expressions.
- Use diagrams based on lengths of rectangles to show that some expressions are equal only for a value of their variable while others are equal for all values of their variable.
- Use the meanings and properties of operations to determine if expressions with variables are equivalent.

Required Materials

- graph paper

8.1: Algebra Talk: Solving Equations by Seeing Structure (5 minutes)

Setup: Display one problem at a time. Allow 30 seconds of quiet think time per problem, followed by a whole-class discussion.

Student task statement

Find a solution to each equation mentally.

- $3 + x = 8$

- $10 = 12 - x$

- $x^2 = 49$

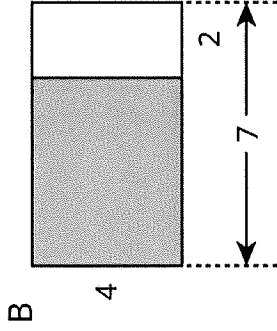
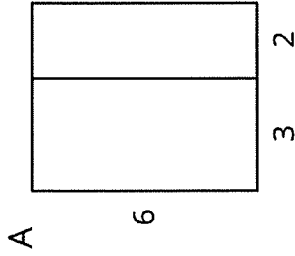
- $\frac{1}{3}x = 6$

Possible responses

- 5
- 2
- 7
- 18

Student task statement

Possible responses



Explanations vary.

1. $6 \cdot 3 + 6 \cdot 2$

$6 \cdot 5$

$6(3 + 2)$

2. $4 \cdot 5$

$4 \cdot 7 - 4 \cdot 2$

$4(7 - 2)$

1. Select **all** the expressions that represent the area of the large, outer rectangle in figure A. Explain your reasoning.

- $6 + 3 + 2$
- $6 \cdot 3 + 6 \cdot 2$
- $6 \cdot 3 + 2$
- $6 \cdot 5$
- $6(3 + 2)$
- $6 \cdot 3 \cdot 2$

2. Select **all** the expressions that represent the area of the shaded rectangle on the left side of figure B. Explain your reasoning.

- $4 \cdot 7 + 4 \cdot 2$
- $4 \cdot 7 \cdot 2$
- $4 \cdot 5$
- $4 \cdot 7 - 4 \cdot 2$
- $4(7 - 2)$
- $4(7 + 2)$
- $4 \cdot 2 - 4 \cdot 7$

9.3: Distributive Practice (15 minutes)

Setup: 10 minutes of quiet work time, followed by a whole-class discussion.

Student task statement

Complete the table. If you get stuck, skip an entry and come back to it, or consider drawing a diagram of two rectangles that share a side.

column 1	column 2	column 3	column 4	value
$5 \cdot 98$	$5(100 - 2)$	$5 \cdot 100 - 5 \cdot 2$	$500 - 10$	490
$33 \cdot 12$	$33(10 + 2)$			
		$3 \cdot 10 - 3 \cdot 4$	$30 - 12$	
	$100(0.04 + 0.06)$			
		$8 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4}$		
			$9 + 12$	
			$24 - 16$	

Possible responses

See lesson plan.

Anticipated misconceptions

Students might understand how to expand an expression with parentheses but struggle with how to approach a sum. Encourage students to think about the rectangle diagrams they have seen and draw a diagram of a partitioned rectangle. Ask students what the sum represents and help them to see that it can represent the sum of the areas of the two smaller rectangles. Remind students that the rectangles have the same width, and ask what that width might have been to produce the two areas, what factor the two areas have in common. Then have them consider the other factors (the lengths) that would produce those products for the areas.

Are you ready for more?

1. Use the distributive property to write two expressions that equal 360. (There are many correct ways to do this.)
2. Is it possible to write an expression like $a(b + c)$ that equals 360 where a is a fraction? Either write such an expression, or explain why it is impossible.
3. Is it possible to write an expression like $a(b - c)$ that equals 360? Either write such an expression, or explain why it is impossible.
4. How many ways do you think there are to make 360 using the distributive property?

Possible Responses

1. Answers vary. Possible expressions: $36(7 + 3)$, $10(20 + 16)$
2. Yes. For example, $\frac{1}{2}(700 + 20)$.
3. Yes. For example, $12(50 - 20)$.
4. There are infinite such expressions if you allow fractions or decimals, and quite a large number indeed even if you don't.

Lesson Synthesis (5 minutes)

Products and sums can represent areas of rectangles and be written as equivalent expressions using the distributive property. The distributive property can work in two directions, either removing or inserting parentheses, and can make mental calculations simpler.

9.4: Complete the Equation (Cool-down, 5 minutes)

Setup: None.

Student task statement

Write a number or expression in each empty box to create true equations.

1. $7(3 + 5) = \square + \square$

2. $15 - 10 = \square(3 - 2)$

Possible responses

1. 21, 35

2. 5

Unit 6, Lesson 9: The Distributive Property, Part 1

Lesson Goals

Required Materials

- Apply the distributive property with addition and subtraction to generate equivalent expressions
- Represent the distributive property with side lengths and areas of rectangles and use the diagrams to write equivalent expressions

9.1: Number Talk: Ways to Multiply (5 minutes)

Setup: Display one problem at a time. Allow 30 seconds of quiet think time per problem, followed by a whole-class discussion.

Student task statement

Find each product mentally.

$$5 \cdot 102$$

$$5 \cdot 98$$

$$5 \cdot 999$$

Possible responses

- 510
- 490
- 4,995

9.2: Ways to Represent Area of a Rectangle (15 minutes)

Setup: 10 minutes of quiet work time, followed by a whole-class discussion.

Unit 6, Lesson 10: The Distributive Property, Part 2

Lesson Goals

Required Materials

- Represent the distributive property with known and unknown side lengths and areas of rectangles, and use the diagrams to write equivalent expressions with variables.
- Apply the distributive property to generate equivalent expressions with variables.

10.1: Possible Areas (5 minutes)

Setup: 2–3 minutes of quiet work time, followed by a whole-class discussion.

Student task statement

1. A rectangle has a width of 4 units and a length of m units. Write an expression for the area of this rectangle.
2. What is the area of the rectangle if m is 3 units?
2.2 units? $\frac{1}{3}$ unit?
3. Could the area of this rectangle be 11 square units? Why or why not?

Possible responses

1. $4m$ (or equivalent)
2. 12 square units, 8.8 square units, $\frac{4}{3}$ square units
3. Yes, the area could be 11 square units. m would have to be $\frac{11}{4}$ units, since $4 \cdot \frac{11}{4} = 11$.

Anticipated misconceptions

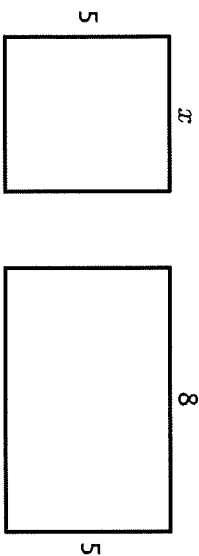
If students are struggling but they haven't drawn a diagram of a rectangle, suggest that they do so.

10.2: Partitioned Rectangles When Lengths are Unknown (10 minutes)

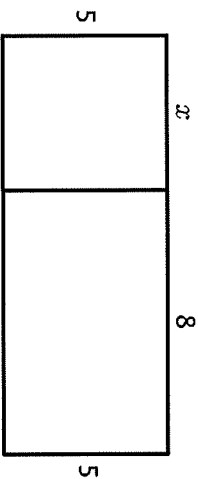
Setup: Students in groups of 2-3, 5 minutes to work with their groups, followed by a whole-class discussion.

Student task statement

- Here are two rectangles. The length and width of one rectangle are 8 and 5. The width of the other rectangle is 5, but its length is unknown so we labeled it x . Write an expression for the sum of the areas of the two rectangles.



- The two rectangles can be composed into one larger rectangle as shown. What are the width and length of the new, large rectangle?



- Write an expression for the total area of the large rectangle as the product of its width and its length.

Possible responses

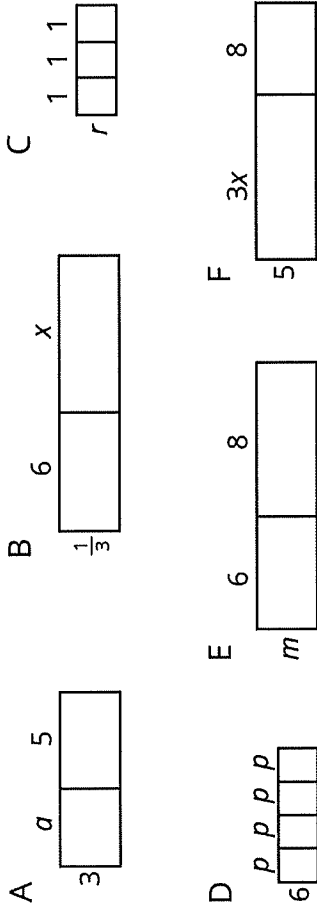
- $5x + 40$ or $5x + 5 \cdot 8$
- The length is $x + 8$ and width is 5
(or vice versa)
- $5(x + 8)$ or $(x + 8) \cdot 5$

10.3: Areas of Partitioned Rectangles (20 minutes)

Setup: Students in the same groups. 10 minutes to work with their groups, followed by a whole-class discussion.

Student task statement

For each rectangle, write expressions for the length and width and two expressions for the total area. Record them in the table. Check your expressions in each row with your group and discuss any disagreements.



width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles

Possible responses

See lesson plan.

Are you ready for more?

Here is an area diagram of a rectangle.

	y	z
w	A	24
x	18	72

1. Find the lengths w , x , y , and z , and the area A . All values are whole numbers.
2. Can you find another set of lengths that will work? How many possibilities are there?

Possible Responses

There are four solutions to this problem. The value of A is always 6.

- $w = 1, x = 3, y = 6, z = 24, A = 6$
- $w = 2, x = 6, y = 3, z = 12, A = 6$
- $w = 3, x = 9, y = 2, z = 8, A = 6$
- $w = 6, x = 18, y = 1, z = 4, A = 6$

Lesson Synthesis (5 minutes)

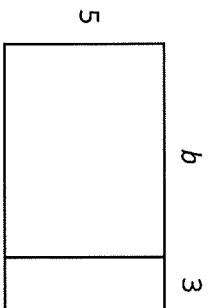
Ensure that students see that their work with expressions that contain variables is an extension of the work they did in the last lesson with numbers. Rectangular area diagrams can illustrate the distributive property in situations where one quantity is represented by a variable. We can write the area of a partitioned rectangle in two different ways, which emphasizes the idea of equivalent expressions as being two different ways of writing the same quantity.

10.4: Which Expressions Represent Area? (Cool-down, 5 minutes)

Setup: None.

Student task statement

Select all the expressions that represent the large rectangle's total area.



- 1. $3(5 + b)$
- 2. $5(b + 3)$
- 3. $5b + 15$
- 4. $15 + 5b$
- 5. $3 \cdot 5 + 3b$

Possible responses

- $5(b + 3)$
- $5b + 15$
- $15 + 5b$

Unit 6, Lesson 11: The Distributive Property, Part 3

Lesson Goals

- Apply the distributive property with addition and subtraction to generate equivalent expressions with variables.

Required Materials

11.1: The Shaded Region (5 minutes)

Setup: 2 minutes of quiet work time, followed by a whole-class discussion.

Student task statement

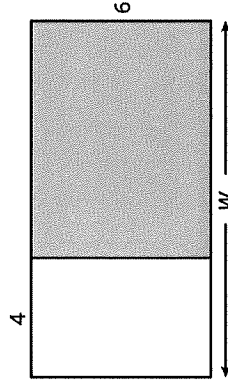
A rectangle with dimensions 6 cm and w cm is partitioned into two smaller rectangles.

Explain why each of these expressions represents the area, in cm^2 , of the shaded portion.

- $6w - 24$
- $6(w - 4)$

Possible responses

Answers vary.



11.2: Matching to Practice Distributive Property (Optional, 15 minutes)

Setup: 10 minutes of quiet work time, followed by a whole-class discussion.

Student task statement

Match each expression in column 1 to an equivalent expression in column 2. If you get stuck, consider drawing a diagram.

Column 1

A. $a(1 + 2 + 3)$

B. $2(12 - 4)$

C. $12a + 3b$

D. $\frac{2}{3}(15a - 18)$

E. $6a + 10b$

F. $0.4(5 - 2.5a)$

G. $2a + 3a$

Column 2

1. $3(4a + b)$

2. $12 \cdot 2 - 4 \cdot 2$

3. $2(3a + 5b)$

4. $(2 + 3)a$

5. $a + 2a + 3a$

6. $10a - 12$

7. $2 - a$

Possible responses

1. $a(1 + 2 + 3)$ and $a + 2a + 3a$

2. $2(12 - 4)$ and $12 \cdot 2 - 4 \cdot 2$

3. $12a + 3b$ and $3(4a + b)$

4. $\frac{2}{3}(15a - 18)$ and $10a - 12$

5. $6a + 10b$ and $2(3a + 5b)$

6. $0.4(5 - 2.5a)$ and $2 - a$

7. $2a + 3a$ and $(2 + 3)a$

Student task statement

The distributive property can be used to write equivalent expressions. In each row, use the distributive property to write an equivalent expression. If you get stuck, draw a diagram.

product	sum or difference
$3(3 + x)$	
	$4x - 20$
$(9 - 5)x$	
	$4x + 7x$
$3(2x + 1)$	
	$10x - 5$
	$x + 2x + 3x$
$\frac{1}{2}(x - 6)$	
$y(3x + 4z)$	
	$2xyz - 3yz + 4xz$

Possible responses

See lesson plan.

11.3: Writing Equivalent Expressions Using the Distributive Property (Optional, 15 minutes)

Setup: 10 minutes of quiet work time, followed by a whole-class discussion.

Lesson Synthesis (5 minutes)

Consider having students work on a creative visual display that shows they understand that the distributive property can be used to both write a sum as a product and to write a product as a sum.

11.4: Writing Equivalent Expressions (Cool-down, 5 minutes)

Setup: None.

Student task statement

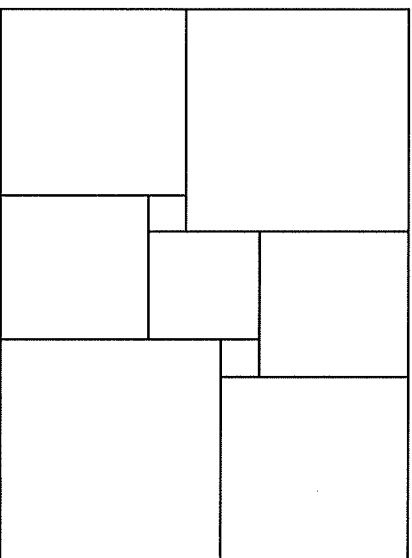
1. Use the distributive property to write an expression that is equivalent to $12 + 4x$.
2. Draw a diagram that shows the two expressions are equivalent.

Possible responses

1. Answers vary. Sample response:
 $4(3 + x)$
2. Answers vary. See lesson plan for a sample diagram.

Are you ready for more?

This rectangle has been cut up into squares of varying sizes. Both small squares have side length 1 unit. The square in the middle has side length x units.



1. Suppose that x is 3. Find the area of each square in the diagram. Then find the area of the large rectangle.
2. Find the side lengths of the large rectangle assuming that x is 3. Find the area of the large rectangle by multiplying the length times the width. Check that this is the same area you found before.
3. Now suppose that we do not know the value of x . Write an expression for the side lengths of the large rectangle that involves x .

Possible Responses

1. Answers are given in a sequence in which they can be derived:
 Small squares: 1 square unit each
 Center square: 9 square units
 Top center: 16 square units
 Top right: 25 square units
 Bottom right: 36 square units
 Bottom center: 16 square units
 Bottom left: 25 square units
 Top left: 36 square units
 The area of the large rectangle is the sum of these numbers: 165 square units.
2. 11 units by 15 units. $11 \cdot 15 = 165$.
3. Answers vary, depending on which side lengths students choose to work with and how much they simplify their expressions. Sample response: the length is $2x + (2x - 1)$, or $4x - 1$ units, and the width is $2x + (1 + x) + (2 + x)$, or $4x + 3$ units.

Unit 6, Lesson 12: Meaning of Exponents

Lesson Goals Required Materials

- Understand the meaning of positive integer exponents.
- Express numbers in terms of repeated multiplication.
- Use informal language to describe quantities that can be expressed using repeated multiplication.

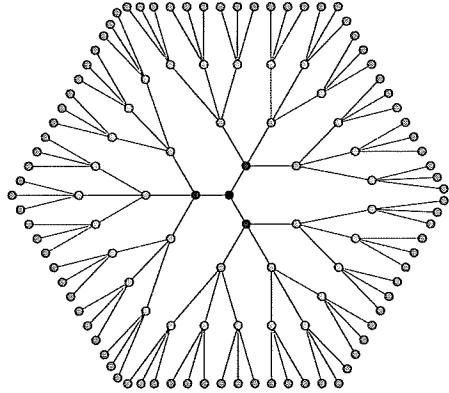
12.1: Notice and Wonder: Dots and Lines (5 minutes)

Setup:

Students in groups of 2. Display the problem for all to see. 1 minute of quiet think time, followed by partner and whole-group discussions.

Student task statement

What do you notice? What do you wonder?



Possible responses

Answers vary.

Anticipated misconceptions

Some students may try to count the dots in the two outer levels. To encourage students to use the patterns in the image, ask them if there is an easier way they could use their count from the level before to determine the next one.

12.2: The Genie's Offer (20 minutes)

Setup:

5 minutes of quiet work time for the first two questions. Show exponential function on calculators. 5 minutes of quiet work time on the last two questions, followed by a whole-class discussion.

Student task statement

You find a brass bottle that looks really old. When you rub some dirt off of the bottle, a genie appears! The genie offers you a reward. You must choose one:

- \$50,000, or
- A magical \$1 coin. The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

1. The number of coins on the third day will be $2 \cdot 2 \cdot 2$. Write an equivalent expression using exponents.
2. What do 2^5 and 2^6 represent in this situation? Evaluate 2^5 and 2^6 without a calculator. Pause for discussion.
3. How many days would it take for the number of magical coins to exceed \$50,000?
4. Will the value of the magical coins exceed a million dollars within the 28 days? Explain or show your reasoning.

Possible responses

1. 2^3
2. The number of coins on the 5th day, the number of coins on the 6th day. $2^5 = 32$, $2^6 = 64$
3. 16 days
4. Answers vary.

Anticipated misconceptions

Students might evaluate 2^5 as $2 \cdot 5$. If this happens, have them make a table showing the number of coins accumulated each day. It will soon be apparent that many more than 10 coins will be accumulated after 5 days.

Are you ready for more?

A scientist is growing a colony of bacteria in a petri dish. She knows that the bacteria are growing and that the number of bacteria doubles every hour.

When she leaves the lab at 5 p.m., there are 100 bacteria in the dish. When she comes back the next morning at 9 a.m., the dish is completely full of bacteria. At what time was the dish half full?

Possible Responses

8 a.m., because a half-full dish will take one hour to become a full dish.

