ACE

### Assignment Guide for Problem 1.1

Applications: 1–7 | Connections: 34–35
Extensions: 41

## Answers to Problem 1.1

**A.** **1.** You test the number by dividing it into the original number. If there is a whole-number answer, then the number is a factor of the original number.

**2.** You divide the known factor into the original number. The quotient is another factor of the number.

**3.** The factors of 18 are 1, 2, 3, 6, 9, and 18. The divisors of 18 are 1, 2, 3, 6, 9, and 18. They are the same. In each case, you can find a whole number that you can multiply by the factor or divisor and get the product 18.

**B.** Answers may vary. Possible answer: 24 has 1, 2, 3, 4, 6, 8, 12, and 24 as factors, while 7 only has 1 and 7 as factors.

**C.** You know that you have found all of the factors of a number when there are no numbers left that are less than the original number and that divide evenly into the original number.

**Note:** Some students may notice that it is enough to test only the numbers that are less than half the original number.


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## ACE Assignment Guide for Problem 1.2

Applications: 8–13 | Connections: 37–38
Extensions: 42

## Answers to Problem 1.2

**A. 1.**

| First Move | Proper Factors | My Score | Opponent's Score |
|---|---|---|---|
| 1 | None | 0 | 0 |
| 2 | 1 | 2 | 1 |
| 3 | 1 | 3 | 1 |
| 4 | 1, 2 | 4 | 3 |
| 5 | 1 | 5 | 1 |
| 6 | 1, 2, 3 | 6 | 6 |
| 7 | 1 | 7 | 1 |
| 8 | 1, 2, 4 | 8 | 7 |
| 9 | 1, 3 | 9 | 4 |
| 10 | 1, 2, 5 | 10 | 8 |
| 11 | 1 | 11 | 1 |
| 12 | 1, 2, 3, 4, 6 | 12 | 16 |
| 13 | 1 | 13 | 1 |
| 14 | 1, 2, 7 | 14 | 10 |
| 15 | 1, 3, 5 | 15 | 9 |
| 16 | 1, 2, 4, 8 | 16 | 15 |
| 17 | 1 | 17 | 1 |
| 18 | 1, 2, 3, 6, 9 | 18 | 21 |
| 19 | 1 | 19 | 1 |
| 20 | 1, 2, 4, 5, 10 | 20 | 22 |
| 21 | 1, 3, 7 | 21 | 11 |
| 22 | 1, 2, 11 | 22 | 14 |
| 23 | 1 | 23 | 1 |
| 24 | 1, 2, 3, 4, 6, 8, 12 | 24 | 36 |
| 25 | 1, 5 | 25 | 6 |
| 26 | 1, 2, 13 | 26 | 16 |
| 27 | 1, 3, 9 | 27 | 13 |
| 28 | 1, 2, 4, 7, 14 | 28 | 28 |
| 29 | 1 | 29 | 1 |
| 30 | 1, 2, 3, 5, 6, 10, 15 | 30 | 42 |

**2.** Sample answers:
1 is a factor of all numbers. Some numbers have only two factors, 1 and that number itself. All of the numbers with two factors are odd except 2. The numbers 4, 9, 16, and 25 have an odd number of factors; they alternate even/odd/even/odd, so the next number with an odd number of factors will be even.

**B. 1.** 29; it gives you the greatest point advantage, 28 points, over your opponent.

**2.** 1; if you choose 1, you lose your turn because 1 has no proper factors for your opponent to choose. Other than 1, 24 and 30 are the worst moves. Your opponent gets 12 points more than you get.

**C. 1.** 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29; prime numbers are good first moves. A prime number has only two factors: the number itself and 1. Since the only proper factor of a prime number is 1, your opponent scores only 1 point. Large prime numbers are the best first moves. For example, 29 gives you 29 points and your opponent only 1 point.

**2.** 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, and 30; a composite number with many proper factors would be a bad first move, because your opponent would get many points. Composite numbers with proper-factor sums that are greater than the number are the worst first moves. For example, if you choose 30 or 24 as a first move, your opponent will score 12 points more than you will. Not all composite numbers are bad first moves. For example, if you choose 25, you will get 25 points, and your opponent will only get 6.

**D. 1.** two factors: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29

**2.** three factors: 4, 9, and 25

**3.** four factors: 6, 8, 10, 14, 15, 21, 22, 26, and 27

**4.** six factors: 12, 20, and 28

**5.** It is better to choose numbers with only 2 or 3 factors as a move because your opponent always scores fewer points than you.


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**E.** 28; $1 + 2 + 4 + 7 + 14 = 28$.

**F.** **1.** Anna is wrong. She forgot 3, 8, and 12, which are also proper factors of 24.

**2.** To find all the proper factors of a number $n$, you can check all of the whole numbers 1 to $\frac{n}{2}$. Since factors of a number occur in pairs, you can find the factor pairs starting with (1, $n$) and stop when the factor pairs begin to repeat in reverse order. (Factor pairs occur in the next problem.)

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## ACE Assignment Guide for Problem 1.3

Applications: 14–21 | Connections: 36, 39, 40
Extensions: 43, 44
**Note:** Do not assign #44 without #43.

## Answers to Problem 1.3

**A.** **1.** Yes; you can see this by checking each number in the grid.

**2.** No; every possible product appears in the grid. You can see this by listing every possible product and checking to see that they are all on the game board or by systematically checking the multiples of each number in the factor list.

**3.** $1 \times 1 = 1$; $2 \times 2 = 4$; $3 \times 3 = 9$; $4 \times 4 = 16$; $5 \times 5 = 25$; $6 \times 6 = 36$; $7 \times 7 = 49$; $8 \times 8 = 64$; and $9 \times 9 = 81$

**4.** 1, 25, 49, 64, and 81; these numbers have only one option for a factor pair restricted to the factor list, and both paper clips must be on the same number.

**B.** **1.** $5 \times 1 = 5$; $5 \times 2 = 10$; $5 \times 3 = 15$; $5 \times 4 = 20$; $5 \times 5 = 25$; $5 \times 6 = 30$; $5 \times 7 = 35$; $5 \times 8 = 40$; and $5 \times 9 = 45$

**2.** Answers may vary. Other multiples include 50; 60; 6,000,000; 5,555; and 85.

**C.** **1.** $4 \times 9 = 36$; $6 \times 6 = 36$; $9 \times 4 = 36$. The other factor pairs of 36 do not appear on the board.

**2.** No; 1, 2, 3, 4, 6, 9, 12, 18, and 36 all of the factors of 36. If one of the factors in the factor pair is greater than 9, then you cannot use that factor pair to make 36 in the Product Game. So $1 \times 36$, $2 \times 18$, and $3 \times 12$ are other ways to make 36 that cannot be done using the factor list.

**D.** Some of the things that should surface are: In the Factor Game, you start with a number and find all of its factors. So this number is the multiple of each of the numbers that are factors. This number is also the product of two of the factors.

In the Product Game, you start with factors and find the product of two factors. The product is also a multiple of each of the factors. Some numbers have more than one factor pair.


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## ACE Assignment Guide for Problem 1.4

Applications: 22–23 | Extensions: 45–49

## Answers to Problem 1.4

**A.** Answers will vary. All sheets will have two rectangles with one as a factor. Square numbers 1, 4, 9, 16, and 25 will have an odd number of rectangles.

**B.** **1.** 24 and 30; possible answer: composite and abundant.

**Composite numbers** have more than two factors. The number 1 is neither prime nor composite.

**Prime** numbers have exactly two factors, themselves and 1.

**Abundant** means "more than enough," which is appropriate since the sum of an abundant number's proper factors is more than the number.

**Deficient** means "not enough," which is appropriate since the sum of a deficient number's proper factors is less than the number.

**Perfect** means "exactly right," which is appropriate because the sum of a perfect number's proper factors is equal to the number.

**2.** 1 has only one rectangle, and 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29 each have two.

Possible answer: Except for 1, all these numbers are prime.

**3.** 1, 4, 9, 16, and 25; square numbers have an odd number of factors.

**4.** The dimensions of the rectangles are the factors of the number. For example, with 12 tiles, you can make rectangles with dimensions $1 \times 12$, $2 \times 6$, $3 \times 4$, $4 \times 3$, $6 \times 2$, and $12 \times 1$. The factors of 12 are the dimensions of the rectangles: 1, 2, 3, 4, 6, and 12.

**C.** Answers may vary. The dimensions of the rectangles are the factor pairs of 12. They are $1 \times 12$, $2 \times 6$, $3 \times 4$, $4 \times 3$, $6 \times 2$, or $12 \times 1$. Which rectangle the class should choose depends on the size and number of crafts they want to exhibit. A $3 \times 4$ or $4 \times 3$ space will be closest to a square shape; but a $1 \times 12$ or $12 \times 1$ table will be long and narrow.



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