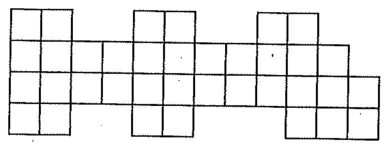
ACE

Assignment Guide for Problem 1.1

Applications: 1–24 | Connections: 37–54

Answers to Problem 1.1

A. **1.** Possible answer:



2. Possible answers: The floor plan could be a 9-by-4 rectangle, or a 5-by-7 rectangle with an additional square somewhere. (See Figure 1.)

B. **1.** Design A: area = 12 m^2, perimeter = 26 m
Design B: area = 18 m^2, perimeter = 38 m
Design C: area = 12 m^2, perimeter = 18 m
Design D: area = 16 m^2, perimeter = 18 m

2. Designs A and C can be designed with the same number of floor tiles because they have the same area. They will have different numbers of rail sections because their perimeters are different. In Design A, the tiles are more spread out, while in Design C, the tiles are more compact and have fewer sides exposed.

3. Design A costs the most ($1,010), and Design C costs the least ($810).

4. Yes, you can make many rectangles with an area of 18 m^2, including 1×18, 2×9, 3×6, 6×3, 9×2, and 18×1. The perimeters of these rectangles area:
1×18 and 18×1: perimeter = 38 m
2×9 and 9×2: perimeter = 22 m
3×6 and 6×3: perimeter = 18 m

C. **1.** Design I: 36 m^2
Design II: 40 m^2
Design III: 55 m^2
Strategies for finding area may vary. Any reasonable strategy, including counting the number of squares, is acceptable.

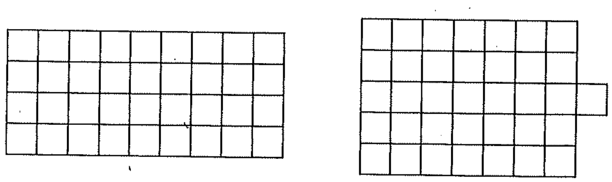
2. Design I: 24 m
Design II: 26 m
Design III: 32 m
Strategies for finding area may vary. Strategies for finding perimeter may vary. Any reasonable strategy, including counting the units of the design's outline, is acceptable.

3. **a.** Possible answers:
The perimeter is length plus width plus length plus width, or the perimeter is twice the sum of the length and width, or the perimeter is twice the length added to twice the width.
$P = \ell + w + \ell + w$, or $P = (\ell + w) \times 2$, or $P = 2 \times \ell + 2 \times w$

b. Possible answers:
The area is the length times the width.
$A = \ell \times w$, or $A = w \times \ell$

c. $P = 2(6 + 15) = 2(21) = 42$ cm or
$P = 2(6) + 2(15) = 12 + 30 = 42$ cm
$A = 6 \times 15 = 90$ square centimeters

Figure 1



ACE

Assignment Guide for Problem 1.2

Applications: 25–30 | Connections: 55–57

Answers to Problem 1.2

A.

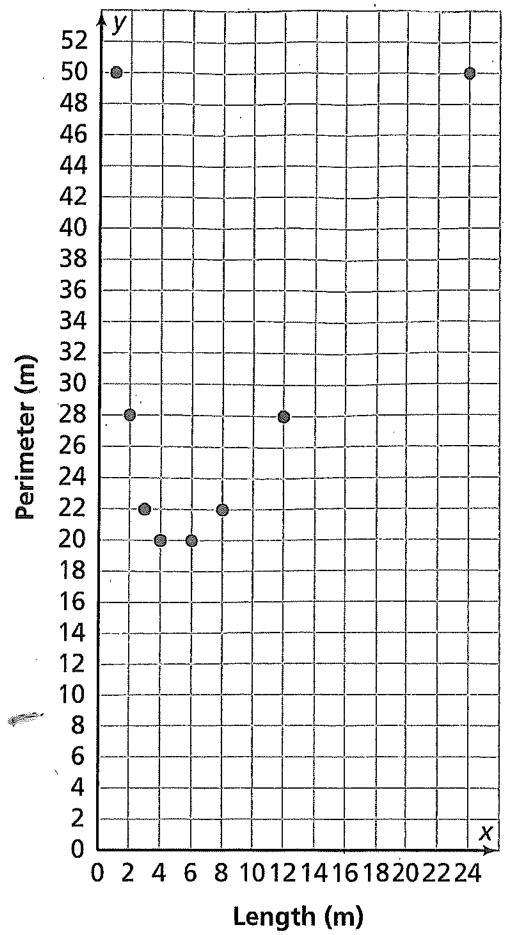
Length (m)	Width (m)	Perimeter (m)	Area (m^2)
1	24	50	24
2	12	28	24
3	8	22	24
4	6	20	24
6	4	20	24
8	3	22	24
12	2	28	24
24	1	50	24

B. **1.** Perimeter; possible explanation: Perimeter is the distance or length around the outside of a shape. The walls fit around the outside, so the number of panels depends on how long the distance around the outside is.

2. The 1 m × 24 m (or 24 m × 1 m) shelter requires the most panels. The floor plan is long and skinny, with the least open space and the most wall sections.

3. The 4 m × 6 m (or 6 m × 4 m) shelter requires the fewest panels. The floor plan is the most square-like of the possibilities. The floor plan would have the most open space and the fewest panels.

C. **1.**



2. The graph is curved. Moving from left to right, the points get lower and lower, then begin to rise again. This is because if one side of the storm shelter is very short, the other must be very long, so the perimeter will be large. As the two dimensions become close to each other, the perimeter becomes smaller. Past a certain point, the perimeter becomes large again.

D. **1.** The 6 m × 6 m floor plan has the least perimeter (24 m) because it is the most square-like floor plan. The 36 m × 1 m floor plan has the greatest perimeter (74 m) because it is the skinniest floor plan.

2. A long, skinny rectangle has the largest perimeter for a fixed area, while the rectangle that is most square-like has the smallest perimeter for a fixed area.

The more square-like a shape is, the more of the square units there are on the inside of the shape and less that touch the edge. When fewer units touch the edge, the perimeter is smaller. Prompt students to use the data in the table and the diagrams of the rectangles to support their reasoning when answering Question C.

ACE

Assignment Guide for Problem 1.3

Applications: 31–36 | Connections: 58–69
Extensions: 70–78

Answers to Problem 1.3

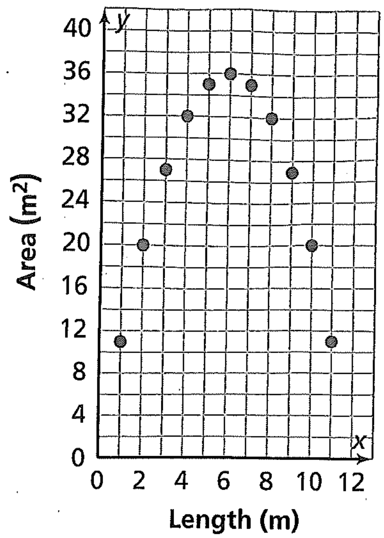
A. 1.

Length (m)	Width (m)	Perimeter (m)	Area (m^2)
1	11	24	11
2	10	24	20
3	9	24	27
4	8	24	32
5	7	24	35
6	6	24	36
7	5	24	35
8	4	24	32
9	3	24	27
10	2	24	20
11	1	24	11

2. For rectangles with whole-number dimensions and a perimeter of 24 m, a 1 m × 11 m (or 11 m × 1 m) rectangle will have the least area and a 6 m × 6 m rectangle (a square) will have the greatest area.

3. Possible answer: A 6 m × 6 m dog pen would be the best design to choose because it has the most area for a dog to roam around.

B. 1.



2. The graph is curved. As you read it from left to right, the points rise to a certain place, then go down. In the table, as the length increases in the table, the width decreases. The area increases until we have a square. Here, the area is maximized and so increasing the length further begins to decrease the area.

3. This graph is curved like the other one was, but the curve is different. The other graph was decreasing to a minimum, then increasing. This curve increases to a maximum, then decreases. This graph is symmetrical, the other was not. The two graphs show different relationships: the possible areas for a fixed perimeter and the possible perimeters for a fixed area.

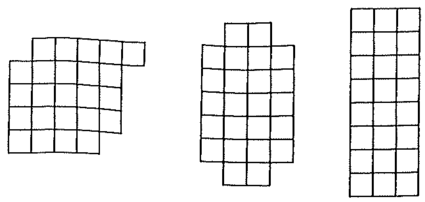
C. With 36 m of fencing, the rectangle with whole-number side lengths that has the least area will be 17 m × 1 m (or 1 m × 17 m). The greatest area is formed with a 9 m × 9 m rectangle (square).

D. The longest and thinnest shape has the least area for a fixed perimeter. A square shape has the most area for a fixed perimeter.

ACE Answers Investigation 1

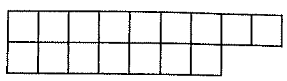
Applications

1. a. Possible answers:

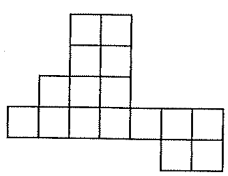


b. The bumper-car ride has an area of 24 m^2, which is the total number of square meters used to cover the floor plan of the bumper-car ride. The perimeter of 22 m is the total number of rail sections that are needed to surround the bumper-car ride.

2. Answers will vary. Maximum perimeter for whole-number dimensions is 34 units, minimum is 16 units. Possible answers:

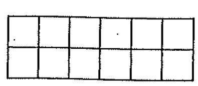


Perimeter: 22 units

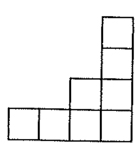


Perimeter: 26 units

3. Answers will vary. Maximum area for whole-number dimensions is 16 square units, minimum is 7 square units. Possible answers:

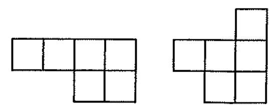


Area: 12 square units

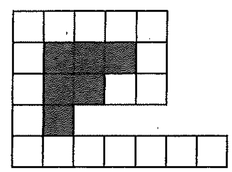


Area: 8 square units

4. Answers will vary. Possible answers:



5. Possible answers: The perimeters are not the same, because I counted the number of units around the edge of each figure and found that their perimeters were different.

6. 

Adding these six tiles reduced the perimeter of the figure. Only two of the new tiles have exposed edges, while together they cover ten previously exposed edges in the original figure.

7. $P = 4 \times 12 \text{ ft} = 48 \text{ ft}$, $A = 12 \text{ ft} \times 12 \text{ ft} = 144 \text{ ft}^2$

8. $P = 22 \times 12 \text{ ft} = 264 \text{ ft}$, $A = 144 \text{ ft}^2 \times 21 = 3{,}024 \text{ ft}^2$

9. $P = 30 \times 12 \text{ ft} = 360 \text{ ft}$, $A = 144 \text{ ft}^2 \times 26 = 3{,}744 \text{ ft}^2$

10. $P = 26 \times 12 \text{ ft} = 312 \text{ ft}$, $A = 144 \text{ ft}^2 \times 20 = 2{,}880 \text{ ft}^2$

11. $P = 16$ units, $A = 7 \text{ units}^2$

12. $P = 16$ units, $A = 16 \text{ units}^2$

13. $P \approx 11$ units, $A \approx 5.5 \text{ units}^2$

14. $P = 40$ in., $A = 100 \text{ in.}^2$

15. $P = 40$ m, $A = 75 \text{ m}^2$

16. $P = 2\ell + 2w$, $A = \ell \cdot w$

17. Check students' sketches. (See Figure 1.)

18. $A = 65\ \text{cm}^2$, $P = 38$ cm

19. $A = 36\ \text{cm}^2$, $P = 36$ cm

20. a. The next thing that she did was she stretched out the string and measured it. She was finding the perimeter of the figure.

b. She got the same answer that she got by counting, 18 cm.

c. No, she can't because the string method measures length, not area. Instead, she must count all the squares.

21. a. $6\ \text{ft} \times 8\frac{1}{2}\ \text{ft} = 51\ \text{ft}^2$

b. 29 ft of molding

c. Two walls have an area of $6\ \text{ft} \times 6\ \text{ft} = 36\ \text{ft}^2$, and two walls would have an area of $6\ \text{ft} \times 8.5\ \text{ft} = 51\ \text{ft}^2$. The total surface area would be $36 + 36 + 51 + 51 = 174\ \text{ft}^2$. You would need 4 pints of paint because $174\ \text{ft}^2 \div 50\ \text{ft}^2 = 3.48$ and you round up to 4 so that you will have enough paint.

d. Check students' work. Answers will vary, but students should use processes similar to those they used in parts (a)–(c). Students also need to make sure that they round the number of pints of paint up to the nearest whole number to make sure they have enough paint to cover the walls.

22. a. Since $40 \times 120 = 4{,}800$, the cost of this model is $4{,}800 \times \$95 = \$456{,}000$

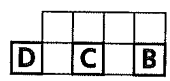
b. $4{,}800 \div 100 = 48$ cars

23. Designs will vary and costs are dependent on the number of tiles and rail sections used. Two possible answers:
7 m by 5 m: area = 35 m^2, perimeter = 24 m, cost = \$1,650.00

6 m by 6 m: area = 36 m^2, perimeter = 24 m, cost = \$1,680.00

Students will have to make a guess to get started and then alter the guess to increase or decrease three inter-related variables. Look for ways that students proceeded from their first guesses.

24. A



25. A 4 ft-by-4 ft square requires the least amount of material for the sides: 16 ft of board.

Figure 1

Rectangle	Length (in.)	Width (in.)	Area (square in.)	Perimeter (in.)
A	5	6	30	22
B	4	13	52	34
C	$6\frac{1}{2}$	8	52	29

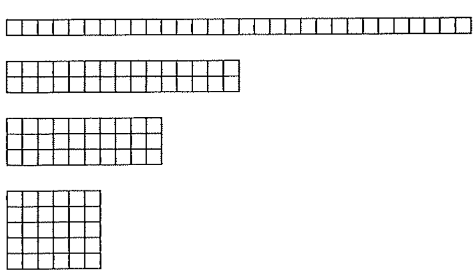
26. a.

Length (ft)	Width (ft)
1	240
2	120
3	80
4	60
5	48
6	40
8	30
10	24
12	20
15	16

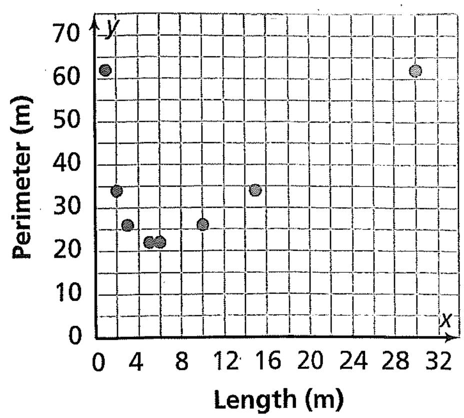
b. Possible answer: A car needs at least 8 ft for the length, so the 8 ft-by-30 ft design would probably be too snug. The 10 ft-by-24 ft, 12 ft-by-20 ft, and 15 ft-by-16 ft designs would all be appropriate as garages.

27. a.

Length (m)	Width (m)	Area (m^2)	Perimeter (m)
1	30	30	62
2	15	30	34
3	10	30	26
5	6	30	22
6	5	30	22
10	3	30	26
15	2	30	34
30	1	30	62



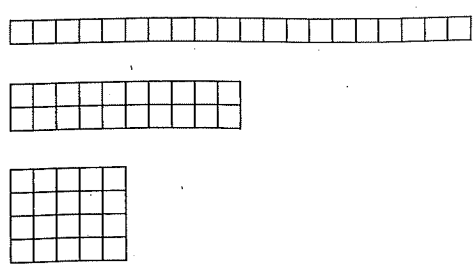
b.



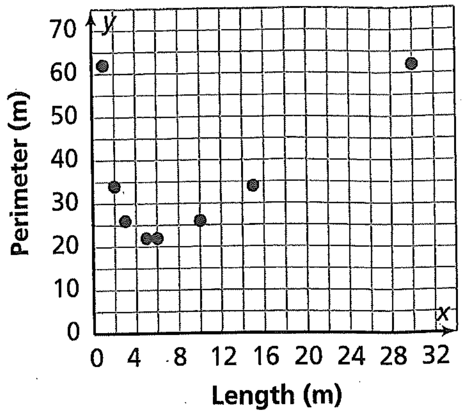
c. On the table, look for the greatest (least) number in the perimeter column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read left to the perimeter axis to get the perimeter. The greatest perimeter is 62 meters (a 1 m × 30 m rectangle). The least perimeter is 22 meters (a 5 m × 6 m rectangle).

28. a.

Length (m)	Width (m)	Area (m^2)	Perimeter (m)
1	20	20	42
2	10	20	24
4	5	20	18
5	4	20	18
10	2	20	24
20	1	20	42



b.



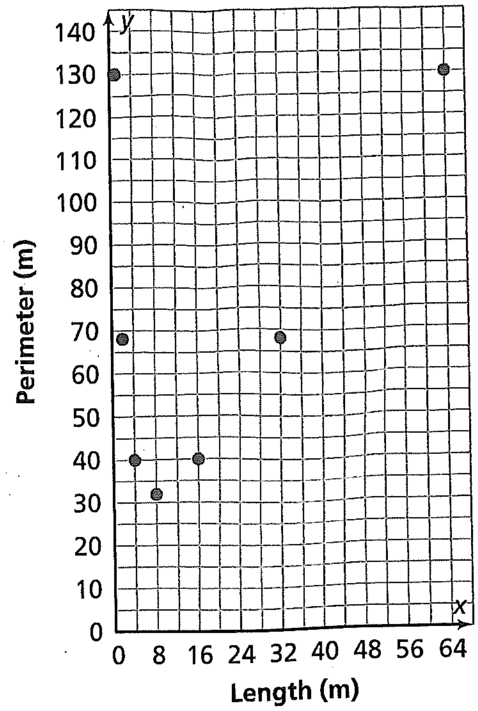
c. On the table, look for the greatest (least) number in the perimeter column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read left to the perimeter axis to get the perimeter. The greatest perimeter is 42 meters (a 1 m × 20 m rectangle). The least perimeter is 18 meters (a 4 m × 5 m rectangle).

29. a.

Length (m)	Width (m)	Area (m^2)	Perimeter (m)
1	64	64	130
2	32	64	68
4	16	64	40
8	8	64	32
16	4	64	40
32	2	64	68
64	1	64	130

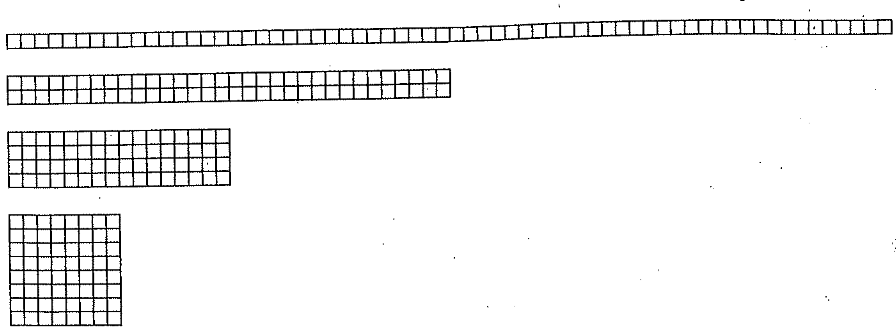
(See Figure 2.)

b.



c. On the table, look for the greatest (least) number in the perimeter column. The dimensions will be next to this entry in the length and width columns. On the graph, look for the highest (lowest) point. Then read left to the perimeter axis to get the perimeter. The greatest perimeter is 130 meters (a 1 m × 64 m rectangle). The least perimeter is 32 meters (a 8 m × 8 m rectangle).

Figure 2



34. There are no such rectangles. This is because we need to double the sum of the length and width. If length and width are both whole numbers, their sum is a whole number. Doubling any whole number gives an even number. Five is odd.

35. No; there are always many different possible perimeters for rectangles with given areas.

36. a. One possible answer: A 4 unit-by-12 unit rectangle whose perimeter is 32 units.

b. One possible answer: A 4 unit-by-10 unit rectangle whose area is 40 square units.

Connections

37. C

38. Possible answers: A tile on the classroom floor is about 1 ft^2. A coffee table is about 1 yd^2.

39. 1 yd^2 is greater. It is 9 ft^2.

40. They are the same length. 5×12 in. $=$ 60 in.

41. 12 m is greater because 120 cm is 1.2 m.

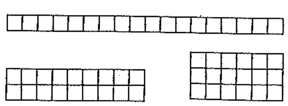
42. 120 ft is greater because 12 yd is 36 ft.

43. They are the same length. 50 cm $=$ 500 mm.

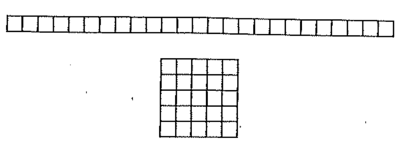
44. Possible answer: One square meter is greater because a meter is greater than a yard.

45. The area is the same because she just shifted one part of the rectangle to another part of it. The perimeter is longer because the distance around the new shape is longer than the original rectangle.

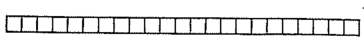
46. a.



b.



c.



d. The factors of a number and the dimensions of the rectangles that can be made from that number of tiles are the same. For example, the factors of 25 are 1, 5, and 25, so each number can be one dimension of a rectangle with 25 square units of area.

47. 31.45

48. 49

49. $\frac{765}{32}$ or $23\frac{29}{32}$ or 23.906

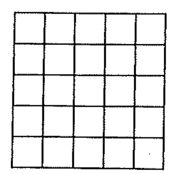
50. $\frac{105}{72}$ or $\frac{35}{24}$ or $1\frac{11}{24}$ or 1.4583

51. a. 8

b. 16

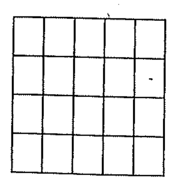
c. 6

52. a. Possible answer:



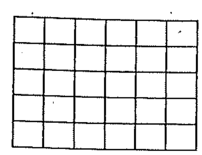
Each brownie is 2 in. by 2 in.

b. Possible answer:



Each brownie is 2.5 in. by 2 in.

c. Possible answer:



Each brownie is 2 in. by $1\frac{2}{3}$ in.

d. If they make 20 brownies, then each could be 2 in. by $2\frac{1}{2}$ in. The area of the bottom of the brownie is 5 in.^{2}.

e. If they make 30 brownies, then each could be 2 in. by $1\frac{2}{3}$ in. The area of the bottom of the brownie is $3\frac{1}{3}$ in.^{2}.

53. **a.** $A = 86{,}250$ ft^2, $P = 1{,}210$ ft

b. $A = 86{,}250 \div 9 = 9{,}583\frac{1}{3}$ yd^2, $P = 403\frac{1}{3}$ yd

c. 15 ft $\times$ 25 ft $=$ 375 ft^2, $86{,}250 \div 375 =$ 230 classrooms

Note: The shape of the classroom is not necessarily maintained.

54. **a.** 38.25 square feet

b. Both students are correct. The area of any rectangle can be found by multiplying the length and the width, regardless of whether the values are whole numbers or fractions. Nathan is using the partial products method, finding the area of the four regions then finding the sum. He is using the Distributive Property that was developed in *Prime Time.*

55. The area of a square is side $\times$ side and all sides of a square are equal. Therefore, the side length is $\sqrt{169} = 13$ and the perimeter is $4 \times 13 = 52$.

56. No. There are many rectangles that have an area of 120 cm^2, such as 5×24, 2×60, 40×3, etc.

57. F

58. The 36 card tables should be arranged in a straight line, seating 74 people.

59. **a.** 1 by 60, 2 by 30, 3 by 20, 4 by 15, 5 by 12, and 6 by 10.

b. 1 by 61

c. 1 by 62 and 2 by 31

d. The factors of a number and the dimensions of the rectangles that can be made from that number of tiles are the same. For example, the factors of 62 are 1, 2, 31, and 62.

60. This is always true, because $E + E + E + E = E$, $O + O + O + O = E$, and $E + E + O + O = E$, where E stands for an even number, O for an odd number.

Alternatively, the formula $2(\ell + w)$ shows that 2 is a factor of any perimeter with whole-number length and width.

61. All of them are correct. A rectangle has four sides: two lengths and two widths.

62. $2 \times (5 + 7.5) = 25$

63. By using Stella's formula, $2 \times (50 + w) = 196$, the width is 48 cm.

64. Matt is correct because a square has four sides and all the sides (two lengths and two widths) are equal.

65. $A = 121$ in.^{2}, $P = 44$ in.

66. $A = 156.25$ in.^{2}, $P = 50$ in.

67. $\sqrt{144} = 12$ cm

68. **a.** Possible answer:

Largest rectangle: $\frac{5.6 \text{ cm}}{3.5 \text{ cm}} = 1.6$

Second-largest rectangle: $\frac{3.5 \text{ cm}}{2.15 \text{ cm}} \approx 1.63$

Third-largest rectangle: $\frac{2.15 \text{ cm}}{1.3 \text{ cm}} \approx 1.65$

b. Possible answer: The Nautilus shell is so popular because the dimensions of its spiral shape are close to the golden ratio, which makes it visually appealing.

69. **a.** Possible answer:

Largest rectangle: $\frac{7.6 \text{ cm}}{4.7 \text{ cm}} \approx 1.617$

Second-largest rectangle: $\frac{2.5 \text{ cm}}{1.4 \text{ cm}} \approx 1.786$

Third-largest rectangle: $\frac{1.1 \text{ cm}}{0.7 \text{ cm}} \approx 1.571$

Some of the ratios are less than the golden ratio, and others are greater. Overall, the ratios are all close in value to each other and to the golden ratio.

c. About 104 ft; you can approximate its width by using the golden ratio, 1 : 1.62.

Extensions

70. You may want to ask students to write their answers as fractions because the patterns are more obvious.

a. $\frac{1}{4}$ m

b. Rectangle: side lengths are $\frac{1}{4}$ m, $\frac{1}{8}$ m, $\frac{1}{4}$ m, $\frac{1}{8}$ m; perimeter is $\frac{3}{4}$ m.

c. Rectangle: side lengths are $\frac{1}{4}$ m, $\frac{3}{16}$ m, $\frac{1}{4}$ m, $\frac{3}{16}$ m; perimeter is $\frac{7}{8}$ m.

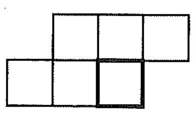
d. Rectangle: side lengths are $\frac{1}{4}$ m, $\frac{7}{32}$ m, $\frac{1}{4}$ m, $\frac{7}{32}$ m; perimeter is $\frac{15}{16}$ m.

e. Perimeter $= \frac{31}{32}$ m

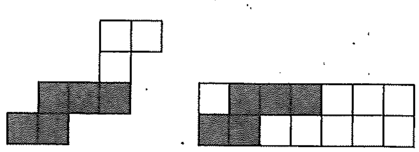
71. 3 cm × 6 cm rectangle

72. No; there are many different combinations of lengths and widths that could add up to a certain perimeter. A square, on the other hand, has four equal sides, so you can find the length of one of the sides by dividing the perimeter by 4.

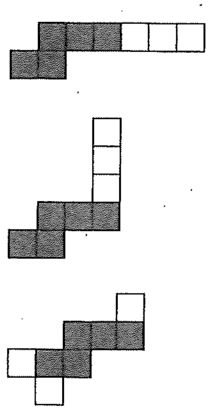
73. a. Yes. See diagram below.



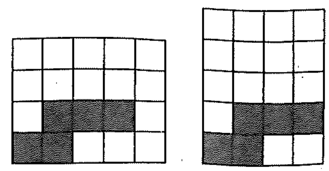
b. Answers will vary. Possible answers:



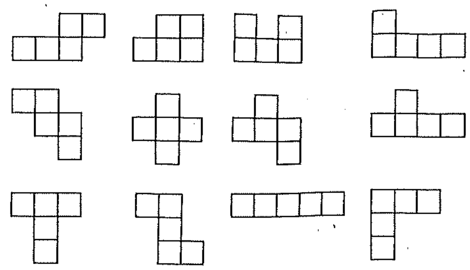
c. 3; each of the three tiles must touch only one edge as they are added. Possible answers:



d. 15 tiles can be added. Each figure must enclose the pentomino in a 4-by-5 rectangle. Possible answers:

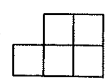


74. a.



b. Possible answer: I conducted a systematic search.

c. This pentomino has the least perimeter, 10 units, because four of the tiles have two edges joined. All of the other pentominos have a perimeter of 12 units.



75. a. The area of Loon Lake is 38–42 square units (380,000–420,000 square meters). The area of Ghost Lake is 34–37 square units (340,000–370,000 square meters).

b. Answers will vary, but one possibility is to use a grid such as a 0.5 with smaller units.

76. a. You could wrap a string around the lake on the grid and then measure the string.

b. The perimeter of Loon Lake is 25–26 units (2,500–2,600 m). The area of Ghost Lake is 45–50 units (4,500–5,000 m).

77. **a.** Ghost Lake. Ghost Lake would also make a better Nature Preserve since it has more shoreline for bird nests, a variety of vegetation, etc.

b. Loon Lake has more room to cruise.

c. Loon Lake is better for swimming, boating, and fishing.

d. Ghost Lake has more shoreline for campsites.

78. **a.** approximately 31 cm^2

b. approximately 40 cm

c. The amount of rubber in the sole is related to the area of the foot. The amount of thread required to stitch the sole to the rest of the shoe is related to the perimeter (although you would have to ignore the part of the perimeter between the toes!).

Summarize

Display the answers to Question A and any different answers you noticed during the Explore.

- *How did you find these perimeter measurements?*
- *How did you find these area measurements?*

Students will likely have found the area by counting. Ask whether they tried any other strategies. Make sure that students can understand the methods described.

- *Which of the triangles are right triangles?*
- *How do you think right triangles are related to rectangles?*
- *How could you get a right triangle from a rectangle?*

For Question B display some of the student answers in a table.

- *How are the areas of the triangle and smallest enclosing rectangle related?*
- *Will the perimeter of the smallest rectangle be twice the triangle's perimeter?*

When discussing Question C, ask:

- *How can you write a rule to find the area of a triangle?*
- *How are the length and width of the rectangle related to the triangle?*

Explain briefly that the labels *length* and *width* do not apply to triangles.

- *In a triangle these parts are called base and height. How are the base and height of a rectangle related to the length and width of a rectangle?*

ACE

Assignment Guide for Problem 2.1

Applications: 1–6 | Connections: 27–32

Answers to Problem 2.1

A. 1. Triangle A: ≈ 18.8 cm; Triangle B: ≈ 29.2 cm; Triangle C: ≈ 19.5 cm; Triangle D: ≈ 24.2 cm; Triangle E: ≈ 21.2 cm; Triangle F: ≈ 23 cm

Strategies will vary but should include that the length of a slanted side was measured with a centimeter ruler.

2. Triangle A: 15 cm^2; Triangle B: 35 cm^2; Triangle C: 12 cm^2; Triangle D: 27 cm^2; Triangle E: 21 cm^2; Triangle F: 24 cm^2.

Strategies will vary but may include cutting and rearranging the triangle into a rectangle, or surrounding a triangle by a rectangle, then finding the area of the rectangle and dividing it by 2.

B. 1.

Design	Area of Rectangle (cm^2)	Area of Triangle (cm^2)
A	30	15
B	70	35
C	24	12
D	54	27
E	42	21
F	48	24

2. The area of a rectangle is twice the area of a triangle or the area of the triangle is half the area of the rectangle.

C. 1. Possible answer: $(b \times h) \div 2$ (**Note:** $(\ell \times w) \div 2$ is acceptable at this time.)

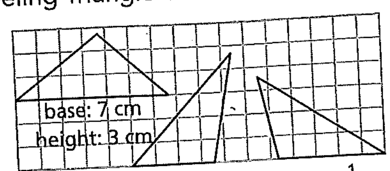
2. $A = \frac{1}{2}\left(8 \times 3\frac{1}{2}\right) = 14$ in.^{2}

Assignment Guide for Problem 2.2

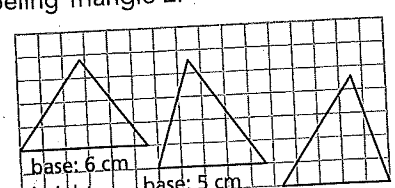
Applications: 7–12 | Connections: 33–35

Answers to Problem 2.2

A. 1. Possible choices for positioning and labeling Triangle 1:



Possible choices for positioning and labeling Triangle 2:



2. The following are based on approximate measurements. This is why different measurements give different areas for each triangle. Some students may count the squares. Some may imbed each triangle in a rectangle and subtract off excess area. Some may use the height-base relationship.

Triangle *D*

Base (cm)	Height (cm)	Area (cm²)
7 (longest side)	3	$10\frac{1}{2}$
4 (shortest side)	≈$5\frac{1}{4}$	≈$10\frac{1}{2}$
≈$5\frac{1}{4}$ (other side)	4	≈$10\frac{1}{2}$

Triangle *E*

Base (cm)	Height (cm)	Area (cm²)
6 (longest side)	4	12
5 (shortest side)	≈$4\frac{4}{5}$	≈12
5 (other side)	≈$4\frac{4}{5}$	≈12

B. 1. Theoretically, the answer is no. Changing the side of the base should not change the area of the triangle. However, as students are measuring some error will occur, so their calculations may be slightly different.

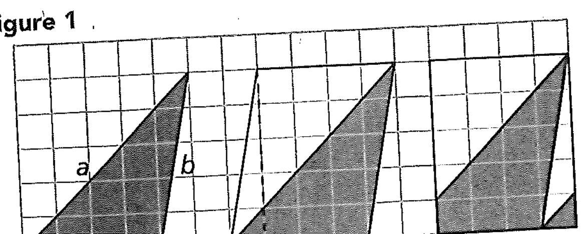
2. In some cases, the base that is chosen may result in a more accurate measure for the base, height, or both.

C. The formula does work, but it is difficult to draw a rectangle in this case to show that the area of the triangle is half the area of rectangle. The diagram in the answer might help. At this level students can use their empirical data to confirm the conjecture that the area of triangle is the same no matter which side and corresponding height is used as a base and height. (See Figure 1.)

It is easier to find the area when the height falls inside the rectangle.

It is easier to find the position that gives the nicest measurements, such as whole-number measurements.

Figure 1



- *What conditions give a longer perimeter? A shorter perimeter?*
- *What are some other locations that could be used as the third vertex of a triangle that belongs in this family?*
- *How many different triangles do you think you can make that belong to this family?*

If students have trouble seeing this, ask them to visualize sliding a point (representing the triangle's vertex for height) on a line 4 cm above and parallel to the base from above one end of the base to the other. The areas of the triangles will be the same, but the perimeters will be different.

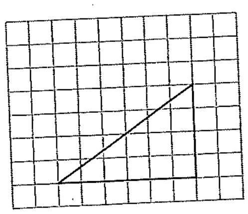
If you have time, ask a few students to share the triangle families they created on grid paper. As they share their triangle families, also ask them to describe their triangles by giving the base, height, and area.

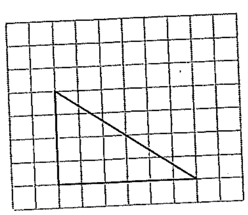
ACE Assignment Guide for Problem 2.3

Applications: 22–23 | Connections: 36–39

Answers to Problem 2.3

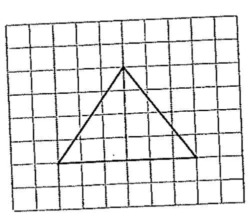
A. 1. Possible triangles:



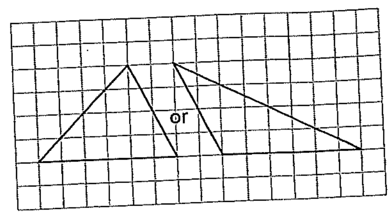


2. For possible triangles, see Question A, part (1).

3. There is one possibility:



4. Possible triangles:

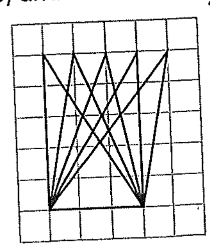


5. 12 cm^2

B. 1. The four triangles have the same base, height, and area.

2. Because they have the same base, height, and area.

C. 1. Answers will vary. Here is one possibility of a triangle family that has a base of 3, a height of 5, and an area of $7\frac{1}{2}$ cm^2.



2. The triangles have the same area, but not necessarily the same shape. Note that this question is similar to the focus question.

Summarize

Start by asking students what they have found when making these designs.

- *What kinds of constraints make drawing a triangle easy? What kinds of constraints make drawing a triangle difficult?*

If a student identifies a certain type of condition as difficult, ask the other students whether they came up with strategies for drawing triangles under that kind of constraint.

- *Were there any questions for which you could only make one triangle to fit the constraints?*

When discussing Question A, you want students to notice that fixing the base and height of a triangle does not limit the shape of the triangle but does fix the area.

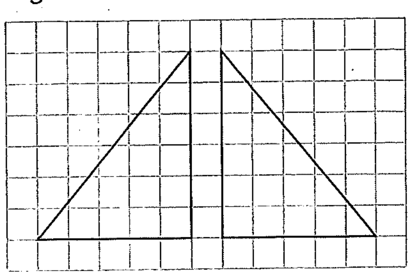
You could end the Summarize by posing the last question in the Explore.

Assignment Guide for Problem 2.4

Applications: 24–26 | Extensions: 40–42

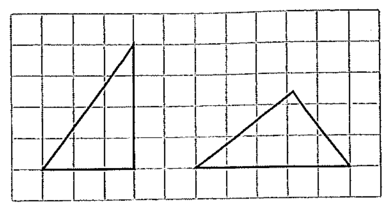
Answers to Problem 2.4

A. Drawings will vary. Students may recognize that this question is making reference to the idea of "triangle families" in Problem 2.3. You can draw many triangles (actually, an infinite number) with the same base and height. The areas of the triangles will be the same, but the perimeters will be different. Possible drawings:



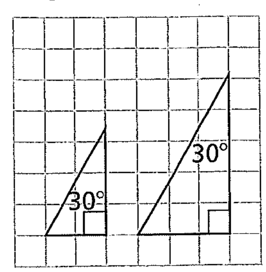
B. Drawings will vary. Correct examples include a triangle with base 6 units and height 5 units, a triangle with base 10 units and height 3 units, and a triangle with base 15 units and height 2 units. The perimeters will all be different. Note that any triangle from Question A will satisfy this constraint.

C. Only one triangle is possible. Of course, this triangle may be oriented in different ways.



Note: In the Grade 7 Unit *Shapes and Designs*, students learn that at most one triangle is possible from three given side lengths.

D. Drawings will vary. You can draw an infinite number of right triangles with a 30° angle. The triangles will have different areas and different perimeters, but they will all have the same angle measures and the same shape. Possible drawings:



Note: The ideas of similarity are covered in the Grade 7 Unit *Stretching and Shrinking*.

ACE Answers Investigation 2

Applications

1. $A = (5 \times 5) \div 2 = 12.5$ square units
   $P \approx 5 + 5 + 7 = 17$ units
   Possible explanation: To find area, count the number of square units covering the figure, use the rule, or find half of the area of the smallest rectangle that surrounds the triangle. To find perimeter, measure around the edges of the triangle with a string and compare the length marked off on the string to the units on the grid or measure the length of each edge of the triangle and add the measurements.

2. $A = (7 \times 6) \div 2 = 21$ square units
   $P \approx 7 + 7.2 + 6.8 = 21$ units

3. $A = (3 \times 7) \div 2 = 10.5$ square units
   $P \approx 3 + 7.25 + 7.25 = 17.5$ units

4. $A = (4 \times 7) \div 2 = 14$ square units
   $P \approx 7.3 + 7.3 + 4 = 18.6$ units
   For possible explanation, see Exercise 1.

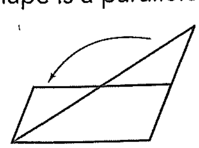
5. $A = (8 \times 7) \div 2 = 28$ square units
   $P \approx 7 + 9.5 + 8.2 = 24.7$ units
   For possible explanation, see Exercise 1.

6. $A = (2 \times 8) \div 2 = 8$ square units
   $P \approx 2 + 8 + 8.2 = 18.2$ units

7. **a.** $(h \times \frac{1}{2}) \times b =$ the area of the right triangle

   b. 6 cm, $(10 \times \frac{1}{2}) \times b = 30$, $b = 30 \div 5 = 6$

   c. Yes, this will work for any triangle. In the case of a right triangle, the new shape is a rectangle, but in general the new shape is a parallelogram.



8. **a.** 39 cm^2

   b. 7.5 cm^2

   c. 40 m^2

   d. 35 ft^2

9. Vashon is correct because no matter which side of the triangle he chooses for the base, as long as he chooses the corresponding height, the area will be the same.

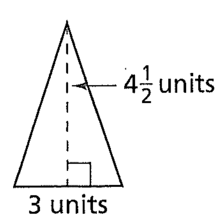
10. **a.** Dimensions for the simplest orientation of each triangle:

	Base (cm)	Height (cm)	Area (cm^2)
A	5	6	15
B	10	7	35
C	3	8	12
D	9	6	27
E	6	7	21
F	6	8	24

   b. The areas should be the same. Any differences should be small and related to different approximations in measuring.

11. Talisa is correct, because it does not matter which side of the triangle you use as the base, as long as you choose the appropriate corresponding height. For example, a right triangle with a base of 4 units and a height of 5 units will have an area of 10 square units; the area would be the same if the base is 5 and the height is 4 units.

12. **a.**



   b. $6\frac{3}{4}$ square units

13. The height is 3.2 m because $0.5 \times 2.5 \times 3.2 = 4$; or $h = \frac{4}{0.5 \times 2.5} = 3.2$.

14. $A = 28$ cm^2, $P = 22$ cm

15. $A = 120$ cm^2, $P = 60$ cm

16. $A = 60$ cm^2, $P = 36$ cm

17. $A = 7.03$ in.^{2}, $P = 14.25$ in.

18. Keisha is incorrect because these triangles are the same size and shape (congruent) and therefore have the same area. Also they both have an area of $(3 \text{ cm} \times 4 \text{ cm}) \div 2 = 6$ cm^2.

19. $A = 24$ cm^2

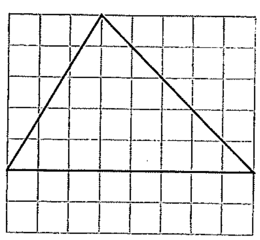
20. $A = 24$ cm^2

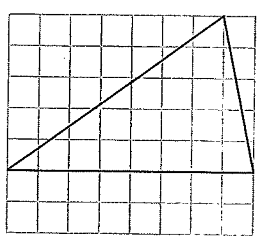
21. $A = 24$ cm^2

22. Marlika is correct. The base and height of a triangle determine its area, not the size of its angles.

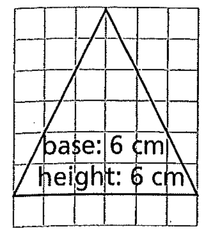
23. D

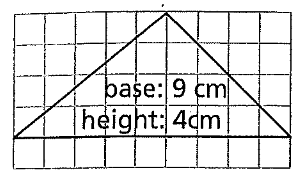
24. The triangles have the same area. Some possible triangles:



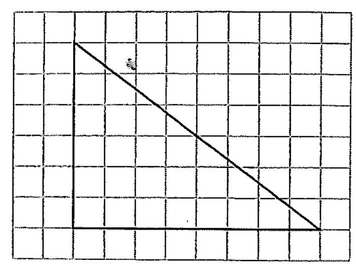


25. These triangles have the same area but not the same perimeter. Some possible triangles:





26. The only possible triangle (although it may be oriented multiple ways):



Note: With these side lengths, the only possible triangle is a right triangle. Students may not know this yet, as the Pythagorean Theorem will justify this for students in the Grade 8 Unit *Looking for Pythagoras*. Therefore, if their drawings are slightly inaccurate, they may think there is more than one. If this occurs, tell them that there is only one triangle. You could give them straws of lengths 6, 8, and 10 centimeters to demonstrate the uniqueness.

Connections

27. $A = 28\ \text{cm}^2$; $P \approx 22.6$ cm

28. $A = 28\ \text{cm}^2$; $P \approx 23$ cm
Possible explanation: To find area, divide shape into a 5-by-4 rectangle with two triangles on either side. The two triangles can come together to make a 2-by-4 rectngle. So, $20\ \text{cm}^2 + 8\ \text{cm}^2 = 28\ \text{cm}^2$. To find perimeter, measure around the edges of the shape with a string and compare the length marked off on the string to the units on the grid.

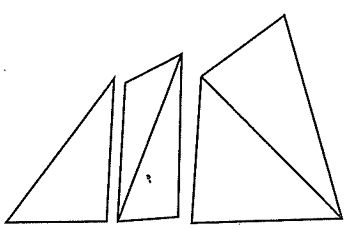
29. $A = 6\ \text{cm}^2$; $P \approx 11.4$ cm

30. $A = 27\ \text{cm}^2$; $P \approx 24.4$ cm

31. $A = 31.5\ \text{cm}^2$; $P \approx 25.8$ cm
Possible explanation: Base = 9 cm, height = 7 cm; so $A = \frac{1}{2}(9)(7) = 31.5\ \text{cm}^2$. To find perimeter, measure around the edges of the shape with a string and compare the length marked off on the string to the units on the grid.

32. $A = 15\ \text{cm}^2$; $P \approx 18.9$ cm
Possible explanation: Base = 5 cm, height = 6 cm; so $A = \frac{1}{2}(5)(6) = 15\ \text{cm}^2$. To find perimeter, measure around the edges of the shape with a string and compare the length marked off on the string to the units on the grid.

33. a.

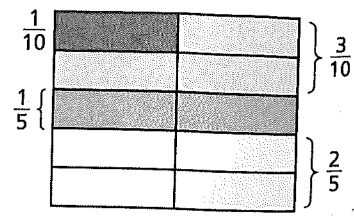


b. You would need to measure the base and height of each triangle in the sail to find its area and then add the areas of the triangles to find the area of the cloth.

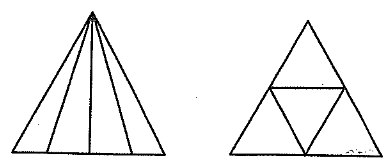
34. G

35. The four isosceles triangles all have areas of $(42\text{ cm} \times 28\text{ cm}) \div 2 = 588\ \text{cm}^2$, and the square base has an area of $42\text{ cm} \times 42\text{ cm} = 1{,}764\ \text{cm}^2$. Therefore, you would need $588 \times 4 + 1{,}764$ or $4{,}116\ \text{cm}^2$ of glass.

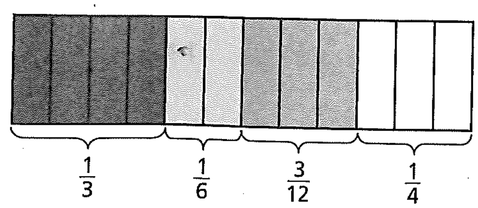
36. Answers will vary. Students may divide squares into tenths, and then mark off $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, and $\frac{4}{10}$.



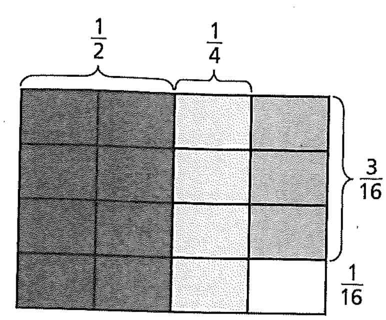
37. Answers will vary. Two possibilities:



38. Answers will vary. Students may divide the rectangle into twelfths, and then mark off $\frac{4}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, and $\frac{3}{12}$.



39. Answers will vary. Students may divide the rectangle into sixteenths, and then mark off $\frac{1}{16}$, $\frac{8}{16}$, $\frac{3}{16}$, and $\frac{4}{16}$.



Extensions

40. Possible answer: The hexagon can be subdivided into a rectangle and two triangles. The area of each of the figures can be found easily, and then the results can be added together. For the perimeter you could measure one side and multiply it by six, because this is a regular hexagon.

41. C

42. **a.** There is enough information to find the area of the polygon in Figures 1 and 3.

b. The areas of the other patterns are

1. 32 cm^2

2. Not enough information.

(**Note:** Once students have studied the Pythagorean Theorem in the *Looking for Pythagoras* Unit in Grade 8, they will be able to determine that the height of each isosceles triangle is 1.5 cm. The area of each triangle is 3 cm^2. So the total area is 28 cm^2.)

3. 48 cm^2

4. Not enough information.

(**Note:** The height of each triangle is $2\sqrt{3} \approx 3.5$ cm. The area of each triangle is $4\sqrt{3} \approx 7$ cm. So the total area is $48 + 8\sqrt{3} \approx 62$ cm^2. Again, students will be able to find the area of this polygon after the Grade 8 Unit *Looking for Pythagoras*.)

Make a table of the measures they found for base, height, and area of each parallelogram.

- *How is the area of a parallelogram related to the area of a triangle? How are the bases and heights of the triangle and parallelogram related?*
- *If the bases and heights are the same, why is the area of the parallelogram twice the area of the triangle?*
- *How did you use b and h to write a formula for the area of a parallelogram?*

ACE

Assignment Guide for Problem 3.1

Applications: 1–9 | Connections: 39

Answers to Problem 3.1

A. 1. Figure A: $\approx 12\frac{1}{2}$ cm; Figure B: $\approx 12\frac{1}{5}$ cm; Figure C: ≈ 21 cm; Figure D: ≈ 24 cm; Figure E: ≈ 16 cm; Figure F: $\approx 21\frac{4}{5}$ cm

2. Possible answers: add the lengths of the sides; add the lengths of the two sides that form an angle and double; or double one side length, double the other side length and total.

B. 1. Figure A: 8 cm^2; Figure B: 8 cm^2; Figure C: 24 cm^2; Figure D: 35 cm^2; Figure E: $15\frac{3}{4}$ cm^2; Figure F: 18 cm^2

2. Possible answers: count the number of whole square centimeters and estimate how many partial square centimeters there are; cut off part of the parallelogram by cutting perpendicular to the base, rearranging to make a rectangle, and then finding the area of the rectangle.

C. 1–2.

Parallelograms

Figure	Area (cm^2)	Height (cm)	Base (cm)
A	8	2	4
B	8	4	2
C	24	4	6
D	35	7	5
E	$15\frac{3}{4}$	$3\frac{1}{2}$	$4\frac{1}{2}$
F	18	6	3

Triangles

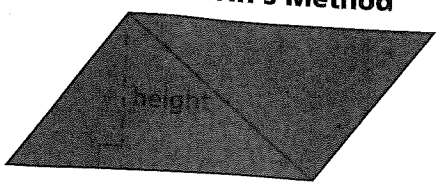
Figure	Area (cm^2)	Height (cm)	Base (cm)
A	4	2	4
B	4	4	2
C	12	4	6
D	$17\frac{1}{2}$	7	5
E	$7\frac{7}{8}$	$3\frac{1}{2}$	$4\frac{1}{2}$
F	9	6	3

Sample answer for Question C, part 1: If you multiply the base and the height, you get the area.

3. The areas of all the triangles are the same. The areas of the parallelograms are twice the area of one of the triangles. The triangles have the same bases and heights as the parallelograms. Therefore, the area of the parallelogram is 2 × area of a triangle whose base is one side of the parallelogram, and its height is the same as the height of the triangle.

D. 1. No; they will both get the same formula for finding the area of a parallelogram. Any parallelogram can be decomposed into two identical triangles each with area $\frac{1}{2} \times b \times h$. Students may see the area formula of a parallelogram as $2 \times \left(\frac{1}{2} \times b \times h\right) = b \times h$, which shows that both formulas are equivalent.

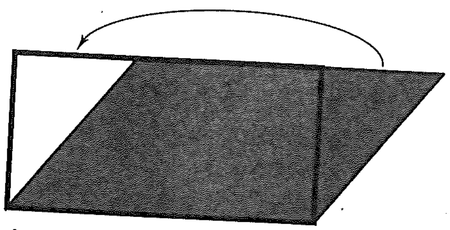
Torrin's Method



area of each triangle $= \frac{1}{2}bh$

area of parallelogram $= 2(\frac{1}{2}bh) = bh$

Maya's Method



Area of Parallelogram = Area of Rectangle
= base × height

2. a. The area of a parallelogram $= b \times h$. Students may say that this formula works because it is equivalent to the area of the two triangles that make up a parallelogram, or they may say that this formula works because it is the equivalent of the formula for the area of the rectangle that results from cutting off a triangle from one side of a parallelogram and attaching it to the other side. In both cases (decomposing into two triangles, rearranging into a rectangle), the area remains constant; nothing is lost or gained.

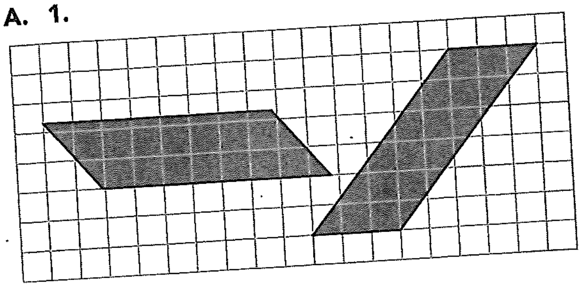
b. $A = 7\frac{2}{3} \times 12 = 92$ cm

Assignment Guide for Problem 3.2

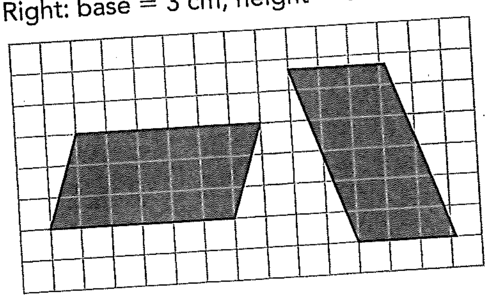
Applications: 10–21 | Connections: 40–42
Extensions: 80–87

Answers to Problem 3.2

A. 1.



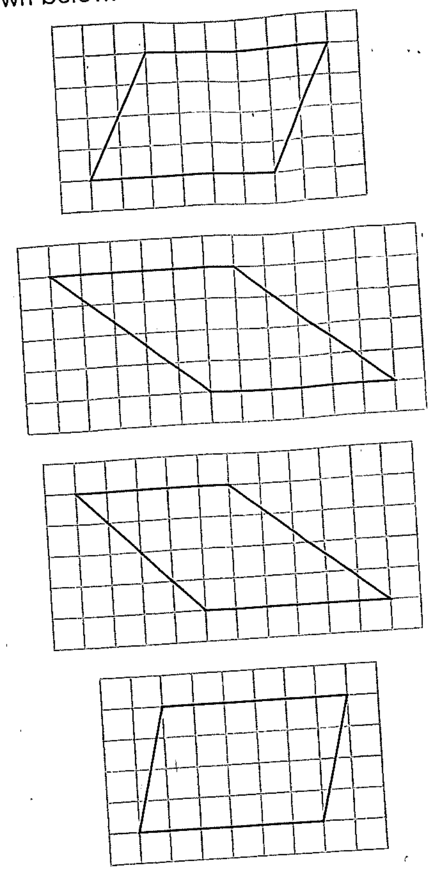
For parallelogram F:
Left: base ≈ 7.8 cm, height ≈ 2.3 cm
Right: base = 3 cm, height = 6 cm



For parallelogram G:
Left: base = 6 cm, height = 3 cm
Right: base ≈ 3.2 cm, height ≈ 5.6 cm

2. In each case, the base times the height is (approximately) equal to the area of the parallelogram: 18 cm^2.

B. 1. Solutions will vary. Four examples are shown below.



2. The area of each parallelogram is 24 cm^2. As with a family of triangles, each member of this family of parallelograms has the same base, height, and area.

3. All parallelograms with the same base and height will have the same area.

Summarize

You can start the summary by asking students to think about the Problem in general.

- *What kinds of constraints make drawing a figure easy? What kinds of constraints make drawing a figure difficult?*
- *Were there any questions for which you could make only one figure that fit the constraints?*
- *For which descriptions of a parallelogram was it possible to make more than one shape?*
- *Which measures are necessary for determining area and perimeter of parallelograms? Of triangles?*

Question C is the case in which base and height are held constant. There are infinitely many parallelograms that can be drawn with the same base and height. These parallelograms form a "family" with the same base, height, and area.

- *Do these parallelograms have the same area? Why?*
- *Do the parallelograms have the same perimeter? Why?*

Question D presents a situation in which side lengths are fixed.

- *What is the same and what is different with this set of parallelograms?*
- *Why does the area change?*

ACE

Assignment Guide for Problem 3.3

Applications: 22–33

Answers to Problem 3.3

A. Drawings will vary. The most common drawings will be 1 unit-by-18 unit, 2 unit-by-9 unit, and 3 unit-by-6 unit rectangles. The rectangles do not have the same perimeter.

B. It is not possible to draw two different rectangles. Some students may draw the same rectangle twice with different orientations.

C. Drawings will vary. All the parallelograms will have an area of 28 cm^2. A geoboard is an excellent tool for demonstrating that you can keep the same base and height but move the side parallel to the base to get different parallelograms with the same area.

D. Drawings will vary. The areas of the parallelograms can vary from almost 0 to 36 cm^2.

E. Drawings will vary. The base times the height of all the parallelograms will be 30 cm^2. The perimeters of the parallelograms will vary.

Assignment Guide for Problem 3.4

Applications: 34–38 | Connections: 43
Extensions: 44–48

Answers to Problem 3.4

A. **1.** Figure 1: parallelogram
Figure 2: trapezoid
Figure 3: right triangle
Figure 4: isosceles triangle

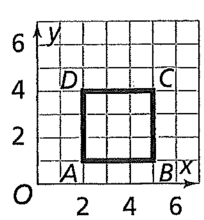
2. Figure 1: (2, 8) (3, 11) (7, 11) (6, 8)
Figure 2: (7, 7) (8, 9) (10, 9) (11, 7)
Figure 3: (8, 1) (11, 6) (11, 1)
Figure 4: (2, 2) (2, 6) (7, 4

3. It is possible to use the coordinates to find the lengths of sides when the sides are horizontal or vertical. Figure 1: Horizontal sides (top and bottom) are 4 units long each; Figure 2: Top is 2 units, bottom is 4 units; Figure 3: Bottom is 3 units, right side is 5 units; Figure 4: Left side is 4 units.

4. Figure 1: base = 4 units, height = 3 units, area = 12 units^2; Figure 2: base$_1$ = 4 units, base$_2$ = 2 units, height = 2 units, area = 6 units^2; Figure 3: base = 3 units, height = 5 units, area = 7.5 units^2; Figure 4: base = 4 units, height = 5 units, area = 10 units^2

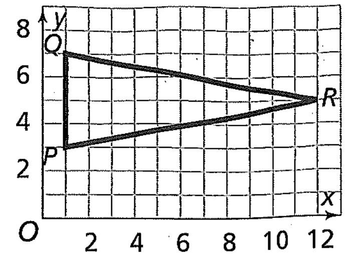
5. The coordinates of the points will change, because the locations of the points are different. The area, perimeter, side lengths, base, and height will remain the same because the shapes themselves are not stretching or shrinking, only their locations are changing.

B. **1.** Point C: (5, 4); Point D: (2, 4); area = 9 units^2

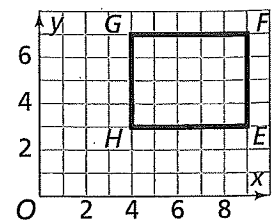


Note: There is a second possible answer that involves coordinates in the fourth quadrant. Point C (5, −2); Point D: (2, −2).

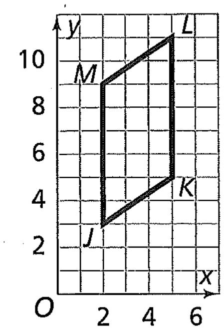
2. Point R: (12, 5); area = 22 units^2



3. Point H: (4, 3); area = 20 units^2



4. Point M: (3, 9); area = 21 units^2



C. **1.** Rectangle, top left: $x = 4$
Right triangle: $x = 10$
Parallelogram: $x = 14$
Rectangle, bottom right: $x = 12$

2. No. The top left rectangle and the parallelogram have the same area (24 square units), but the right triangle has half the area of each of the quadrilaterals (12 square units). The bottom right rectangle has an area of 72 square units, which is three times the area of the top left rectangle.

3. No. The parallelogram has a greater perimeter than the top left rectangle because the slanted sides of the parallelogram are longer than the vertical sides of the rectangle. The top left rectangle has a greater perimeter than the right triangle because the base and height of the rectangle are longer (combined) than the hypotenuse of the right triangle. The bottom right rectangle has the greatest perimeter. The base of the bottom right rectangle has the same length as the base of the parallelogram, so students just have to compare the nonbase sides of the other shapes. The bottom right rectangle's vertical sides are longer than the parallelogram's slanted sides.

Note: Students will not be able to definitively calculate the slanted side of the parallelogram, but an estimated answer such as the one provided should suffice.

D. Some students may count the squares. They should be encouraged to look for another method.

1. The area of each polygon is 16 cm^2. Some students may decompose the polygons into triangles and rectangles as shown below. They can then find the area of each triangle and rectangle and add the areas together to find the area of the entire polygon.

For the first polygon, the area of upper and lower triangles is $2 \text{ cm} \times 2 \text{ cm} \times \frac{1}{2} = \text{cm}^2$ and the area of the rectangle is $2 \times 6 = 12 \text{ cm}^2$. So the area of the first polygon is 16 cm^2 (i.e., 2 cm^2 (upper triangle) + 2 cm^2 (lower triangle) + 12 cm^2 (rectangle) = 16 cm^2).

For the second polygon, the area of the upper and lower triangles is $4 \text{ cm} \times 1 \text{ cm} \times \frac{1}{2} = 2 \text{ cm}^2$ and the area of the rectangle is $3 \times 4 = 12 \text{ cm}^2$. So the area of the second polygon is 16 cm^2 (i.e., 2 cm^2 (upper triangle) + 2 cm^2 (lower triangle) + 12 cm^2 (rectangle) = 16 cm^2).

For the third polygon, the area of upper triangle is $4 \text{ cm} \times 2 \text{ cm} \times \frac{1}{2} = 4 \text{ cm}^2$ and the area of the rectangle is $3 \times 4 = 12 \text{ cm}^2$. So the area of the third polygon is 16 cm^2 (i.e., 4 cm^2 (upper triangle) + 12 cm^2 (rectangle) = 16 cm^2). (See Figure 1.)

Some students may enclose each polygon in a rectangle, find the area of the rectangle, and then subtract off the areas of the regions that are in the rectangle but not in the polygon. This may require finding areas of triangles or counting.

2. Angie is correct. There are two ways to explain why she is correct. Explanation 1: If you count the number of unit squares, there are 16 in each polygon. Thus, all the polygons have the same area. Explanation 2: For each polygon, if you measure the area of the shaded rectangle and the area of the triangle(s) and add them, you get the area of the entire polygon. This comes out to 16 cm^2 for each polygon, so all the polygons have the same area.

3. Answers will vary. Possible answers are shown below.

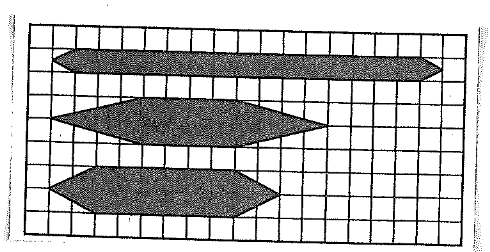
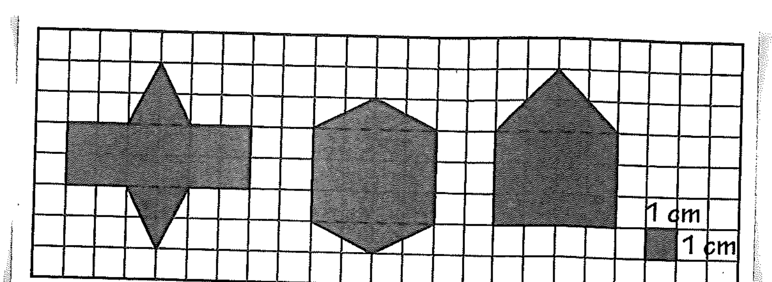


Figure 1



ACE Answers Investigation 3

Applications

1. Area = 20 cm^2 (base = 5 cm, height = 4 cm), perimeter = 20 cm (Each side is 5 cm.)

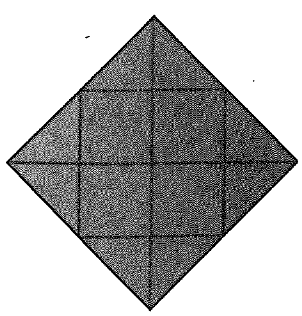
2. Area = 9 cm^2 (base = 3 cm, height = 3 cm), perimeter $\approx$ 12.4 cm (The side lengths are 3 cm and ~ 3.2 cm.) Explanations will vary.

3. Area = 30 cm^2 (base = 6 cm, height = 5 cm), perimeter $\approx$ 25 cm (The side lengths are 6 cm and ~ 6.5 cm.)

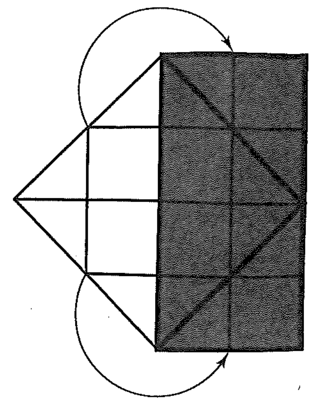
4. Area = 8 cm^2 (base = 2 cm, height = 4 cm), perimeter $\approx$ 15.4 cm (The side lengths are 2 cm and ~ 5.7 cm.)

5. Area = 6 cm^2 (base = 6 cm, height = 1 cm), perimeter $\approx$ 20.2 cm (The side lengths are 6 cm and 4.1 cm.)

6. Area = 8 cm^2, perimeter $\approx$ 11.2 cm (Each side length is about 2.8 cm.)
Note: The area of this square can be found easily. The length of the sides, from the Pythagorean Theorem, are $\sqrt{8} \approx 2.8284271$. We don't expect students will know this, but some families who help their children may offer this as an answer. Some students may say that the length of the sides is 3 cm, and thus the perimeter is 12 cm. Although this is close, you will want to discuss why it is not possible for the length to be 3 cm. If it were, the area would be 9 cm^2, and they can easily see from the drawing that this is not the case. The figure can be split into 4 pieces.



These pieces can be rearranged to form a rectangle with an area 8 cm^2.



7. Area = 20 cm^2 (base = 5 cm, height = 4 cm), perimeter ~ 19 cm (The side lengths are 5 cm and about 4.5 cm.) Explanations will vary.

8. a. The base is 4 units, the height is 5 units, and the area of each parallelogram is 20 square units.
   b. The bases, heights, and areas are the same.
   c. The parallelograms are a family because the bases, heights, and areas are the same.

9. a. $109\frac{1}{4}$ cm^2; $9\frac{1}{2} \times 11\frac{1}{2} = 109\frac{1}{4}$
   b. $109\frac{1}{4}$ cm^2; $9\frac{1}{2} \times 11\frac{1}{2} = 109\frac{1}{4}$
   c. Usually, the perimeter of a parallelogram will be greater than the perimeter of a rectangle when both have the same base and height. The only times the perimeters will be equal is when the parallelogram is itself a rectangle.

10. $A = 24$ cm^2, $P = 22$ cm

11. $A = 30$ cm^2, $P = 30$ cm

12. $A = 29\frac{3}{4}$ cm^2, $P = 25\frac{7}{10}$ cm

13. $A = 72$ in.^{2}, $P =$ 35 in.

14–19.

Exercise	Approximate Area (cm^2)	Approximate Perimeter (cm)
14	$8 (b = 2, h = 4)$	12
15	$4.5 (b = 3, h = 3)$	$10\frac{1}{4}$
16	$15 (b = 5, h = 3)$	$16\frac{4}{5}$
17	$18 (b = 4.5, h = 4)$	$17\frac{1}{5}$
18	$3 (b = 3, h = 2)$	$12\frac{3}{10}$
19	$18 (15 + 3)$	18

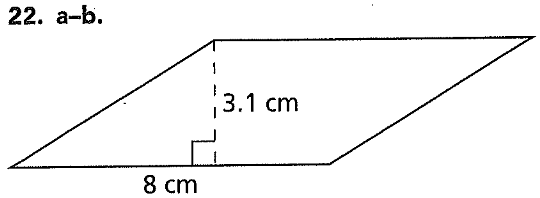
20. a. The area of Tennessee is approximately 41,800 mi^2.

b. The estimate is slightly smaller than the actual area because Tennessee is not quite a parallelogram. There are chunks of land attached to the north and west sides of the state beyond the shape of a parallelogram. There is also a chunk of land in the southeast that is not part of Tennessee but was included in the parallelogram estimate. The area of this land that was included in the estimate is greater than the chunks to the north and west that were left out of the estimate.

21. These parallelograms all have the same area because the first two have a base of 4 and a height of 3, and the last parallelogram has a base of 3 and a height of 4. Since area is base times height, the areas are all the same.

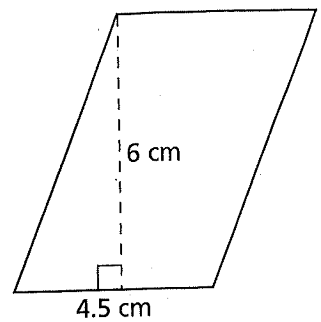
For Exercises 22–27, answers will vary. Possible answers are listed.

22. a–b.



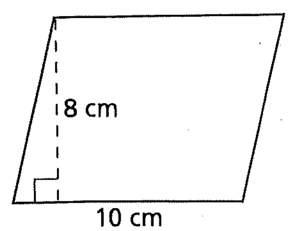
c. Yes, you can draw more than one parallelogram with base 8 and perimeter 28. Rotating the side segments around the two vertices on the base will keep the same side lengths and base. Keeping the side lengths the same while changing the measure of the interior angles will result in a parallelogram with the same perimeter but a different shape and a different area.

23. a–b.



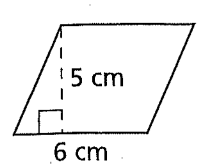
c. Yes, you can draw more than one parallelogram with base $4\frac{1}{2}$ and area 27. You just slide the top vertices over, keeping the same base and height. The area remains the same, but the perimeter changes.

24. a–b.



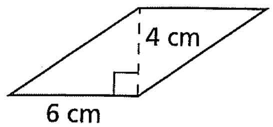
c. Yes, you can draw more than one parallelogram with base 10 and height 8. You just slide the top vertices over, keeping the same base and height. The area remains the same, but the perimeter changes.

25. a–b.



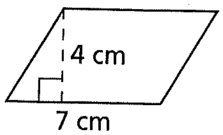
c. Yes, you can draw more than one parallelogram with base 6 and area of 30. Use a height of 5, and slide the top vertices over.

26. **a–b.**



c. Yes, you could have some of these combinations: $b = 1$, $h = 24$; $b = 2$, $h = 12$; $b = 3$, $h = 8$; $b = 4$, $h = 6$. They would all have an area of 24 cm^2.

27. **a–b.**



c. Yes, you could have some of these side lengths: $b = 1$, $s = 11$; $b = 2$, $s = 10$; $b = 3$, $s = 9$; $b = 4$, $s = 8$; $b = 5$, $s = 7$; $b = 6$, $s = 6$, where b = base length, and s = nonbase side length. All the parallelograms would have a perimeter of 24 cm^2.

28. **a.** There are three parallelograms. Each consists of two small triangles.

b. Yes

c. 8 square units

29. **a.** 4 ft^2

b. 24 ft^2

30. Mr. Lee will need 72 tiles. One possible method: $24 \div 3 = 8$ tiles along the length, $18 \div 2 = 9$ tiles along the width, $8 \times 9 = 72$ tiles. Another method: $18 \div 3 = 6$ tiles along the length, $24 \div 2 = 12$ tiles along the width, $6 \times 12 = 72$ tiles. Another method: $24 \times 18 = 432$ ft^2 is the area for the whole ceiling divided by the area of 1 tile, which is 6 ft^2, to find the total number of tiles, 72.

31. The area of the lot minus the area of the house is the area left for grass. $20{,}000 - 2{,}250 = 17{,}750$ ft^2

32. **a.** $\frac{4}{9}$ of the park will be used for skateboarding.
$(\frac{2}{3}\ell \times \frac{2}{3}w = \frac{4}{9}(\ell \times w) = \frac{4}{9}A)$

b. The dimensions of the playground area are 20 ft by 70 ft, giving an area of 1,400 ft^2 and a perimeter of 180 ft.

33. **a.** 6 $in.^2$ of fabric for each nonsquare parallelogram and 4 $in.^2$ of fabric for each square

b. 20 $in.^2$ of fabric for all the squares together

c. 56 $in.^2$ of gray fabric will be visible.
$100 - (40 + 16) = 100 - 56 = 44$

34. **a.** 1: (2, 8), (2.5, 11), (4.5, 8), (5, 11)
2: (7, 8), (7, 11), (12, 11)
3: (8, 2), (8, 6), (11, 1), (11, 7)
4: (2, 2), (4.5, 2), (2, 6), (4.5, 6)

b. 1: horizontal = 2.5 units, 2.5 units
2: horizontal = 5 units, vertical = 3 units
3: vertical = 4 units, 6 units
4: horizontal = 2.5 units, 2.5 units, vertical = 4 units, 4 units

c. 1: parallelogram
2: right triangle
3: trapezoid
4: rectangle

35. **a.** 1: (1, 7), (5, 11), (5,7)

2: (7, 8), (8, 10), (10, 10), (11, 8)

3: (6, 1), (6, 4), (10, 0), (10, 3)

4: (1, 2), (1, 5), (3, 2), (3, 5)

b. 1: horizontal = 4 units, vertical = 5 units
2: horizontal = 2 units, 4 units
3: vertical = 3 units, 3 units
4: horizontal = 2 units, 2 units, vertical = 3 units, 3 units

c. 1: right triangle
2: trapezoid
3: parallelogram
4: rectangle

36. **a.** $x = 1$, $m = 5$, $n = 2$
b. $y = 3$
c. $x = 3$, $y = 4$
d. $x = 3$, $y = 1$

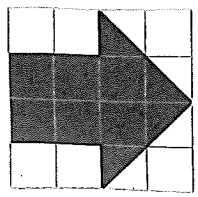
37. **a.** 1: horizontal = 4, vertical = 4
2: horizontal = 4, vertical = 4
3: horizontal = 4, vertical = 5
4: vertical = 10

b. 1: $A = 4 \times 4 = 16$
2: $A = 4 \times 4 \times \frac{1}{2} = 8$
3: $A = 4 \times 5 = 20$
4: $A = 10 \times 2 = 20$

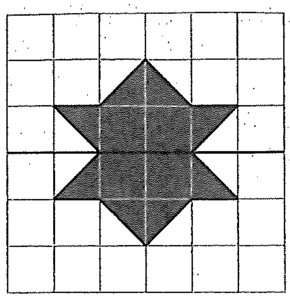
38. **a.** All polygons and the fox shape have same area: 8 cm^2. There are three ways to measure the area of these shapes:

Count inside units

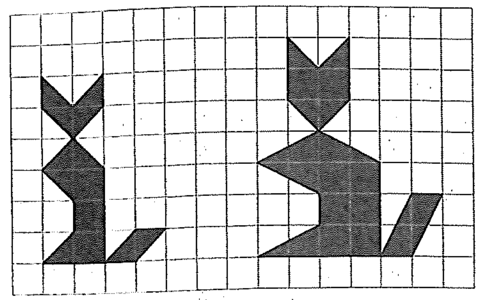
Draw a rectangle around the shape and subtract the pieces not used



Count half the inside squares and double (except the fox shape)



b. Answers will vary.
Some students may double the area but not maintain the fox shape.
Possible answer:



Connections

39. D

40. J

41. **a.** The angles will change, area will decrease, and perimeter will stay the same.

b. Opposite sides have equal length and are parallel. Opposite angles have equal measure.

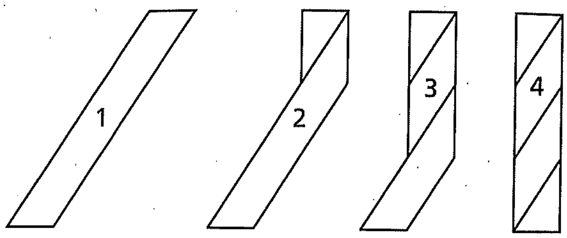
42. Answers will vary. Possible examples: two sheets of notebook paper or two speed limit signs

43. **a.** 240 rectangular floor pieces are needed. One possible method:
$120 \div 5 = 24$ pieces along the length,
$40 \div 4 = 10$ pieces along the width,
$24 \times 10 = 240$ pieces.

b. 240 tiles $\times$ \$20 per tile = \$4,800 for the floor. Answers for number of bumper cars will vary. Sample:
30 bumper cars $\times$ \$10 per car = \$300.
The total cost is \$5,100.

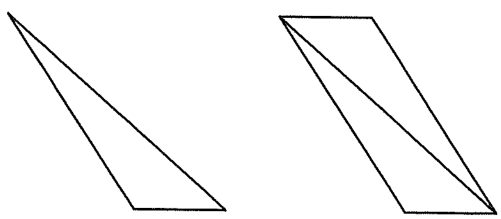
Extensions

44. **a.** Possible answer:



It may take several cuttings, but every parallelogram can be rearranged to make a rectangle with the same base and height as the original.

b. Possible answer:



Every triangle can be put together with a copy of itself to make a parallelogram. The area of the parallelogram is the base times the height, so the area of each triangle is half of that.

45. **Note:** An easier way to find the area of the parallelogram was developed in class. You can draw a diagonal and then find the area of each triangle.

a. Yes, both methods are correct. By decomposing the parallelogram into smaller shapes and then recomposing the shapes into a rectangle with the same base and height as the parallelogram, the area of the original parallelogram is equivalent to the area of the new rectangle.

b. Answers will vary. Possible solutions: The strategies used in this Exercise and the strategies used in class have many things in common. Students rearranged pieces of the parallelograms and demonstrated an understanding of the fact that area remains constant even after cutting and rearranging pieces of a shape.

The students in this Exercise rearranged the area of the parallelogram into one shape: a rectangle with the same area as the parallelogram. In class, students decomposed the parallelogram into two shapes: congruent triangles with half the area of the parallelogram. In both situations, the resulting area was the same as the area of the parallelogram.

c. Yes, these strategies work for all parallelograms. In the case of a parallelogram whose height falls outside of the parallelogram, the shape may need to be rotated to use the two methods given. Anastasia's method also works well for triangles and trapezoids.

46. **a.**

Trapezoid	Approximate Area (cm^2)	Approximate Perimeter (cm)
I	5	$9\frac{1}{4}$
II	7	$12\frac{1}{10}$
III	$10\frac{1}{2}$	$13\frac{4}{5}$
IV	6	$10\frac{1}{2}$
V	$3\frac{1}{2}$	$10\frac{13}{20}$
VI	9	$14\frac{1}{10}$

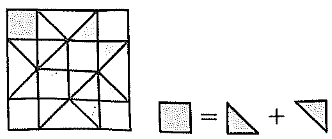
b. Most students will find the area of the trapezoids by partitioning them into a rectangle and one or two triangles, finding the area of these, and adding the areas. You may want to challenge some of your top students to modify the method used for parallelograms. They may come up with the formula: Area $= \frac{1}{2} \times (\text{base} + \text{top}) \times \text{height}$.

c. See part (a) above.

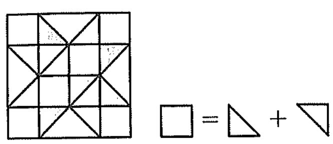
d. The perimeter was found by measuring the length of each side in centimeters and adding them.

47. $A = 42$ in.^{2}, $P = 26\frac{3}{5}$ in.

48. **a.** For each block, the area of the small (unit) square is 25 cm^2 (5 cm by 5 cm) and eleven of these squares will be yellow. Thus, Amy needs $11 \times 25 =$ 275 cm^2 of yellow fabric.



b. For each block, the area of the small (unit) square is 25 cm^2 (5 cm by 5 cm) and five of these squares will be green. Thus, Amy needs $5 \times 25 = 125$ cm^2 of green fabric.



Another way to measure the area of the green fabric for one pattern block is by taking the area of the whole pattern block minus the area of the yellow fabric from part (a). That is, 400 cm^2 − 275 cm^2 = 125 cm^2.

c. The total area of the quilt is 4,800 cm^2. There are two ways to measure it: First, for one block, the area of one block pattern is 400 cm^2 (i.e., $20 \times 20 = 400$). The total area of snowflake quilt is 12 times the area of one block, or $12 \times 400 = 4{,}800$ cm^2. Second, the width and length of this snowflake quilt are 80 cm and 60 cm, respectively. The total area of this quilt is $80 \times 60 =$ 4,800 cm^2.

d. For each pattern block, the areas of yellow fabric and green fabric are 275 cm^2 and 125 cm^2, respectively. The total area of yellow and green fabric is 12 times the area of each color. That is, 3,300 cm^2 and 1,500 cm^2 (i.e., $12 \times 275 = 3{,}300$ cm^2 and $12 \times 125 =$ 1,500 cm^2), respectively.

Begin to discuss Question B by asking the following questions.

- *Explain how you decided where to fold each net.*
- *How can you find the dimensions from the nets? From the box?*

Encourage students to share strategies for finding surface area.

- *What features of the box do you see that might make it easier to find the surface area?*
- *What is the surface area of each box? How did you find it?*
- *Suppose the sides were not drawn on grid paper. How might you find the surface area?*

Have the class save the boxes from Question B to use in the next Problem.

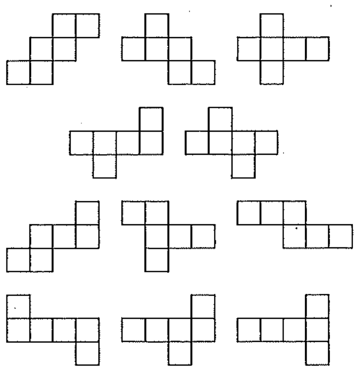
ACE

Assignment Guide for Problem 4.1

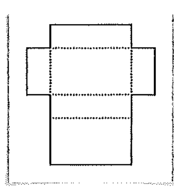
Applications: 1–14 | Connections: 47–51

Answers to Problem 4.1

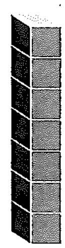
A. 1. There are 35 different nets that can be made with six squares (these are called hexominos), but only the 11 shown below will fold into a cube. The total area of each net is 6 square units. A unit cube has 6 faces, each of which has an area of 1 square unit.



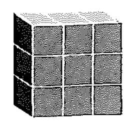
2. Possible net:



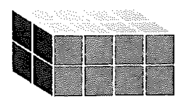
B. **Box P**



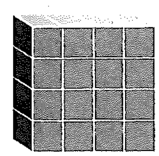
Box Q



Box R



Box S



1. Box P: 1 cm × 1 cm × 6 cm
   Box Q: 1 cm × 3 cm × 3 cm
   Box R: 2 cm × 2 cm × 4 cm
   Box S: 1 cm × 4 cm × 4 cm
2. Each combination of two dimensions results in the dimensions of a pair of congruent faces.
3. Box P: 26 cm^2; Box Q: 30 cm^2; Box R: 40 cm^2; Box S: 48 cm^2
4. Box P: 6 unit cubes; Box Q; 9 unit cubes; Box R: 16 unit cubes; Box S: 16 unit cubes
5. Answers will vary. The box should hold 6 unit cubes and have dimensions 1 cm × 2 cm × 3 cm.

- *If the length of a side of a cube is 15 centimeters, use each of the three formulas, Kurt's, Natasha's, and Dushane's, to compute the volume.*

For Question C, discuss the difference between half of a unit cube and a $\frac{1}{2}$-unit cube.

- *Are two $\frac{1}{2}$-unit cubes the same volume as a 1-unit cube? Why or why not?*
- *How many $\frac{1}{2}$-unit cubes fit inside a 1-unit cube?*

Question D encourages students to move from concrete situations to more abstract situations. Again, notice which formulas they tend to use.

- *What is being measured when you determine how much tape is needed?*
- *What is your process for finding how much tape is needed for each prism?*

Note that, unlike with surface area, if students add the perimeters of all the faces, they do not get the total edge length. They actually get twice the total edge length.

Summarize

Repeat some of the questions from Question B in the Explore.

- *What is a formula for finding the volume using Kurt's method?*
- *What is a formula for finding the volume using Natasha's method?*
- *How can you find the area of the base?*
- *Is there a relationship between the two formulas?*

Use one of the boxes from Problem 4.1 and ask different students to demonstrate Kurt's strategy and Natasha's strategy and explain why they work.

- *Dushane said that we should use the formula ℓ^3. Would this would work for rectangular prisms?*
- *How does having a fractional dimension affect how you find the volume of a rectangular prism?*

As a final check, you could hold up a juice or cereal box and ask students how they could find the volume.

ACE

Assignment Guide for Problem 4.2

Applications: 15–30 | Extensions: 56–65

Answers to Problem 4.2

A. 1. Prism I: length = 5 cm, width = 4 cm, height = 1 cm; Prism II: length = 5 cm, width = 4 cm, height = 2 cm; Prism III: length = 5 cm, width = 4 cm, height = 5 cm

2. Prism I: volume = 20 cm^3
Prism II: volume = 40 cm^3
Prism III: volume = 100 cm^3

Students might have found the volume by multiplying length × width × height, or by finding the number of cubes in a layer and then multiplying the number of layers by the number of cubes in each layer.

3. Prism I: surface area $= 58\ cm^2$
Prism II: surface area $= 76\ cm^2$
Prism III: surface area $= 130\ cm^2$

Students can use many methods. They can find the surface area by drawing a net and finding the area of the net, or by finding the area of the three faces they can see and doubling that area.

B. 1. Both strategies are correct. Since the base of a rectangular prism is a rectangle and its area can be found by multiplying length × width, the two methods are the same.

2. Kurt's: $V = \ell \times w \times h$, Natasha's: $V = B \times h$, where $V =$ volume, $\ell =$ length, $w =$ width, $h =$ height, and $B =$ area of base. The formulas are the same since $V = (\ell \times w) \times h = (B) \times h$.

Note that often an upper-case "B" is used to represent the area of a base, whereas a lower-case "b" is used to identify the base segment in a triangle or parallelogram.

3. $840\ cm^3$

4. The formula, volume $= \ell^3$, is the same as the other two formulas. The height, width, and length of a cube are the same, so they can be represented by the same letter or variable in the volume formula.

C. 1. Durian is correct that he cannot fit unit centimeter cubes exactly into the box, but he is incorrect in thinking that none of the formulas for volume work. Durian could find the correct volume by using a different sized cube to fill the box, or he could use the volume formula by multiplying the fraction edge lengths. Suppose the unit is a centimeter cube. Students can argue that they can either use parts of a 1-centimeter cube, or they can change the size of the unit cube to be $\frac{1}{4}$-centimeter cube or $\frac{1}{2}$-centimeter cube. This is an example of proportional reasoning, which is discussed in more depth in Grade 7 during *Stretching and Shrinking* and *Filling and Wrapping.*

2. $63.75\ cm^3$. If they use 1-centimeter cubes, the volume is $63.75\ cm^3$. If they use $\frac{1}{2}$-centimeter cubes, the volume is 8(63.75), or 510, $\frac{1}{2}$-centimeter cubes (which is equal to $63.75\ cm^3$ since there are eight $\frac{1}{2}$-centimeter cubes in one 1-centimeter cube). If they use $\frac{1}{4}$-centimeter cubes, the volume is 64(63.75), or 4,080, $\frac{1}{4}$-centimeter cubes (which is equal to $63.75\ cm^3$ since there are sixty-four $\frac{1}{4}$-centimeter cubes in one 1-centimeter cube).

D. 1. Prism I: volume $= 32$ in.^{3}
Prism II: volume $= 326.4\ cm^3$
Prism III: volume $= 67.5$ in.^{3}

Methods will vary. One common method will be to multiply length, width, and height.

2. Prism I: surface area $= 64$ in.^{2}
Prism II: surface area $= 308.8\ cm^2$
Prism III: surface area $= 133.5$ in.^{2}

Students may use different strategies for finding surface area. One common strategy is to find the area of the three unique faces of the rectangular prism, add those three faces' areas, then multiply that total by 2.

3. Prism I: tape $= 40$ in.
Prism II: tape $= 88.8$ cm
Prism III: tape $= 64$ in.

Find the perimeter of the base of the prism and multiply that perimeter by 2. This will be the amount of tape needed to cover the edges of the top and bottom bases. Then, multiply the height by 4. This will be the amount of tape needed to cover the vertical edges. Add two numbers (the doubled perimeter and the quadrupled height) together to find the total amount of tape needed to cover all the edges of the prism. Another way to find it is by recognizing that there are 4 edges with measure h units, 4 with measure ℓ units, and 4 with measure w units. Therefore, the amount of tape needed is $4(h + \ell + w)$.

AT A GLANCE 4

Assignment Guide for Problem 4.3

Applications: 31–46 | Connections: 52–55
Extensions: 66–71

Answers to Problem 4.3

A. **1.** Box 1: Rectangular prism
Faces are rectangles
Dimensions: four faces—1 cm × 8 cm; two faces—1 cm × 1 cm

Box 2: Pyramid (tetrahedron)
Faces are equilateral triangles
Dimensions: four faces—each edge = 4 cm, height is about 3.5 cm

Box 3: Triangular Prism
Faces are isosceles triangles and rectangles
Dimensions: two isosceles triangles—base = 6 cm,
height = 4 cm;
one rectangular face—2 cm × 6 cm;
two rectangular faces—2 cm × 5 cm

Box 4: Rectangular prism.
Faces are rectangles
Dimensions: two faces—3 cm × 4 cm;
two faces—2 cm × 3 cm;
two faces—2 cm × 4 cm

2. Box 1: 34 cm^2; Box 2: about 28 cm^2; Box 3: 56 cm^2; Box 4: 52 cm^2

3. One method for finding the surface area of each box is to find the area of each face of the three-dimensional figure. Another method is to find the area of the net, either by finding the area of each face or by finding areas of larger combined shapes (e.g., for Box #3, calculate the combined area of the three rectangles, then double the area of one of the triangular faces, then add those amounts together).

4. Box 1: 40 cm; Box 2: 24 cm; Box 3: 38 cm; Box 4: 36 cm

Method #1: Fold up the net and measure each edge using grid paper or a centimeter ruler.

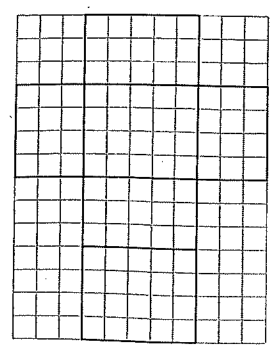
Method #2: Mark each segment that is an outside edge, but be careful not to double-count edges. Then, mark each segment in the interior of the net. Add the lengths of the marked edges.

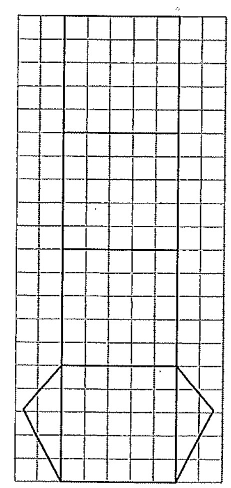
Method #3: Find the perimeter of each face. Add all the perimeters, and then divide by 2. Every edge on the object will be the edge of exactly two shapes of the net, so adding all the perimeters is the same as adding the lengths of all the edges twice. Dividing by 2 gives the amount of tape needed.

5. Answers will vary. Students may state advantages or disadvantages related to appearance, volume, how many boxes fit on a single sheet of grid paper (for example, many pyramids can be made from the same sheet), how much material is needed to construct the box, the strength of the box's structure, etc.

B. **1.** To find the surface area, students might draw a net, and then find the area of the net. Another way students might find the surface area is to find the area of each of the faces separately and then add the areas together. The surface area of the right rectangular prism (on the left) is 94 cm^2. The surface of the right triangular prism is 72 cm^2.

2. Nets will vary. Samples:





ACE Answers Investigation 4

Applications

1–4. Patterns 2 and 4 *can* fold to form closed boxes. Patterns 1 and 3 *cannot* fold to form closed boxes.

5. a. Figures 1 and 2 *can* be folded to form a closed box. Pattern C *cannot*.

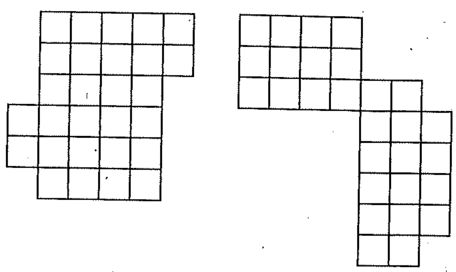
b. Figure 1: 1 unit × 1 unit × 4 units
Figure 2: 1 unit × 2 units × 4 units

c. Figure 1: 18 sq. units
Figure 2: 28 sq. units

d. Figure 1: 4 cubes
Figure 2: 8 cubes

6. a. 2 cm × 4 cm × 1 cm

b. Possible answers:



c. All nets for this box have an area of 28 cm^2.

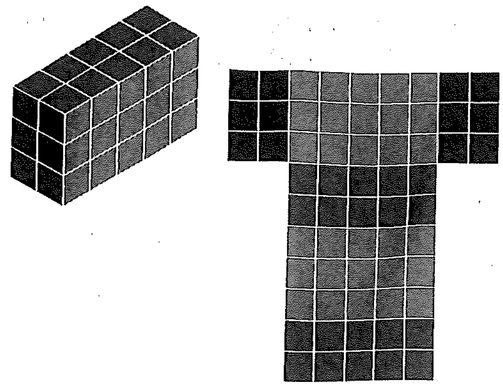
d. There are two faces with an area of 8 cm^2, two with an area 2 cm^2, and two with an area of 4 cm^2 for a total of 28 cm^2. This is the same as the area of the net.

7. Figures 1, 3, 4 and 5 *will not* fold into a box, 2 and 6 *will*. Figures 2 and 6 fold to form boxes because they have edges that will match up fully and evenly when folded. The other figures will not fold to form boxes because edges that are supposed to line up with one another have different lengths, so there will either be overlaps or spaces.

8. a, b, e, f

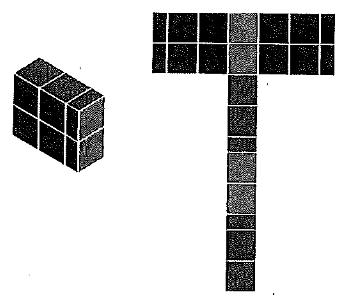
9. This net *will* fold into an open cubic box. The two triangles will meet to become one end of the box.

10. Sketch of box and possible net:



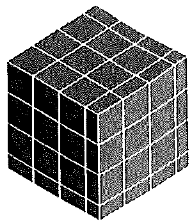
There are two of each of these faces:
2 cm × 3 cm (area is 6 cm^2);
2 cm × 5 cm (area is 10 cm^2);
3 cm × 5 cm (area is 15 cm^2).
The sum of the area of the faces is 62 cm^2.

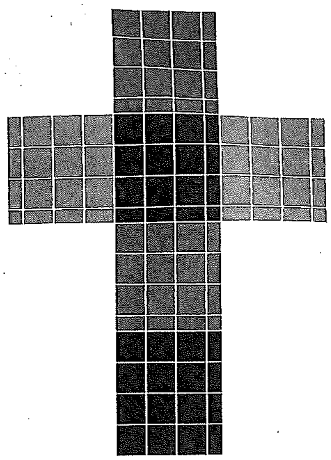
11. Sketch of box and possible net:



There are two of each of these faces:
2 cm × 1 cm (area is 2 cm^2)
2 cm × $2\frac{1}{2}$ cm (area is 5 cm^2)
1 cm × $2\frac{1}{2}$ cm (area is $2\frac{1}{2}$ cm^2).
The sum of the areas of the faces is 19 cm^2.

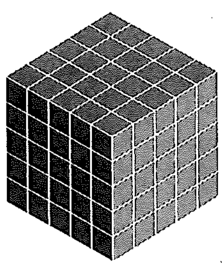
12. Sketch of box and possible net:

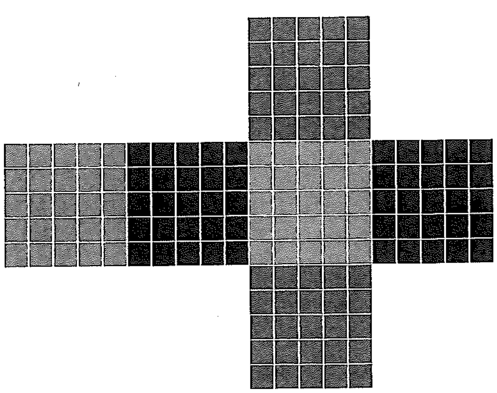




There are six faces. Each is $3\frac{2}{3}$ in. $\times$ $3\frac{2}{3}$ in. (each face has area $13\frac{4}{9}$ in.^{2}). The sum of the areas of the faces is $80\frac{2}{3}$ in.^{2}.

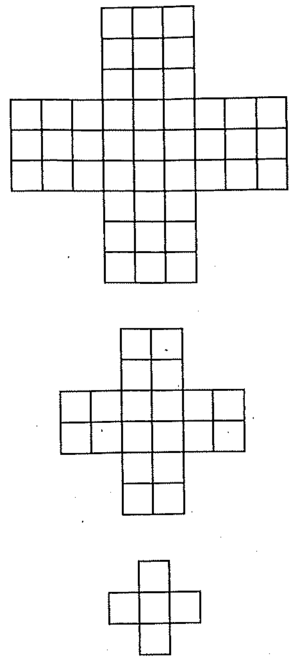
13. Sketch of box and possible net:



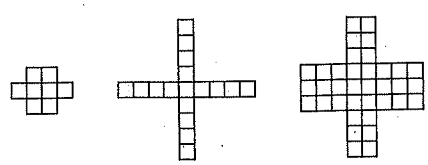


There are six faces. Each is 5 cm $\times$ 5 cm (area is 25 cm^2). The sum of the areas of the faces is 150 cm^2.

14. a. Possible nets:



b. Possible nets:



c. Answers will depend on answers to parts (a) and (b). For the examples given above, the areas (in order) are: (a) 45 square units, 20 square units, 5 square units, and (b) 8 square units, 17 square units, and 36 square units.

15. $\ell = \frac{1}{2}$ in., $w = 2$ in., $h = \frac{1}{2}$ in.

volume = 12 cubic $\frac{1}{2}$-inches

volume = $1\frac{1}{2}$ cubic inches

surface area = $9\frac{1}{2}$ in.^{2}

16. $\ell = 1\frac{1}{2}$ in., $w = 2\frac{1}{2}$ in., $h = 1\frac{1}{2}$ in.
volume = 45 cubic $\frac{1}{2}$-inches
volume = $5\frac{5}{8}$ cubic inches.
surface area = $19\frac{1}{2}$ in.^{2}

17. $\ell = 2\frac{1}{2}$ in., $w = 2\frac{1}{2}$ in., $h = 3\frac{1}{2}$ in.
volume = 175 cubic $\frac{1}{2}$-inches
volume = $21\frac{7}{8}$ cubic inches
surface area = $47\frac{1}{2}$ in.^{2}

Note: There are eight cubes measuring $\frac{1}{2}$ inch on each side (8 cubic $\frac{1}{2}$-inches) in 1 in.^{3}. Therefore, if you multiply the volume in cubic inches by 8, you will get the volume in cubic $\frac{1}{2}$-inches.

18. No, Keira does not have enough paper. The surface area of the package is 792 in.^{2}, which is greater than the amount of wrapping paper she has.

19. **a.** square

b. No, you do not have enough information to find the surface area of the pyramid, because the height of the triangular faces is not known.

20. S.A. = 288 m^2; V = 256 m^3

21. S.A. = 864 cm^2; V = 1,728 cm^3

22. S.A. = 301 in.^{2}; V = 294 in.^{3}

23. S.A. = $234\frac{3}{8}$ ft^2; $V = 235\frac{1}{8}$ ft^3

24. Brenda is incorrect, because the units are different for height. She could find the volume in cubic inches by multiplying $36 \times 48 \times 3 = 5{,}184$ in.^{3}. Alternatively, she could find the volume in cubic feet by multiplying $3 \times 4 \times \frac{1}{4} = 3$ ft^3.

25. **a.** 144 in.^{3}

b. 12 board-feet = 1 ft^3. Stacking 12 board-feet together would result in a stack of wood that is 1 ft by 1 ft by 12 in.

c. 6 in. $\times$ 6 in. $\times$ 48 in. = 1,728 in.^{3}, which is greater than a board-foot (which has a volume of 144 in.^{3}). The difference between the two volumes is 1,728 − 144 = 1,584 in.^{3}.

Note: If students multiply $6 \times 6 \times 4$, they will get 144, but the unit will not be cubic inches, since they have not converted the measurement of 4 ft to 48 in.

26. **a.** 576 in.^{3}

b. Some possible dimensions include:
1 in. $\times$ 1 in. $\times$ 576 in.;
2 in. $\times$ 3 in. $\times$ 96 in.; 24 in. $\times$ 3 in. $\times$ 8 in.;
18 in. $\times$ 2 in. $\times$ 16 in.; any factor triple of 576 works.

c. No, the surface areas are different. In general, objects with different dimensions typically have different surface areas.

27. **a.** Andrea can use $\frac{1}{2}$-inch blocks or $\frac{1}{4}$-inch blocks.

b. For $\frac{1}{2}$-inch blocks, Andrea would need 5 blocks in a row, 7 blocks in a column, and 8 in a stack. In total, she would need 280 $\frac{1}{2}$-inch blocks. For $\frac{1}{4}$-inch blocks, Andrea would need 10 blocks in a row, 14 blocks in a column, and 16 in a stack. In total, she would need 2,240 quarter-inch blocks.

c. Yes, it is possible to describe the volume as 35 in.^{3}.

28. A (volume of A = 35 in.^{3}; volume of B = 27 in.^{3}; volume of C = 32 in.^{3}; volume of D = 27 in.^{3})

29. One method is to calculate the volumes of all the prisms. A second method is to notice whether or not there is one common dimension in all prisms. That way, you can find the greatest product of the dissimilar dimensions.

30. a. 8 blocks

b. $2\frac{1}{4}$ in. long; $\frac{3}{4}$ in. tall; 2 in. wide

c. $2\frac{1}{4}$ in. $\times$ $\frac{3}{4}$ in. $\times$ 2 in. $= 3\frac{3}{8}$ in.^{3}

d. Possible solutions: 36 blocks by 1 block; 18 blocks by 2 blocks; 12 blocks by 3 blocks; 9 blocks by 4 blocks; 6 blocks by 6 blocks

31. a. Net 1: S.A. $\approx$ 21.3 square units
Net 2: S.A. = 48 square units
Net 3: S.A. = 32 square units
Net 4: S.A. = 104 square units

b. Only Nets 3 and 4 fold up to be a right rectangular prism.

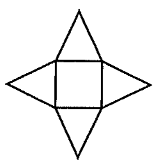
32. A and E; B and F; C and D

33. 6 rectangles

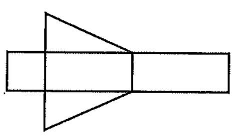
34. 3 rectangles, 2 triangles

35. 1 square, 4 triangles

36.



37.



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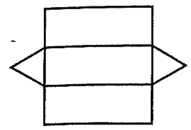

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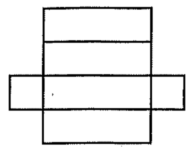


40. 96 in.^{2}

41. 184 m^2

42. 243 cm^2

43. 144 ft^2

44. 60 ft

45. a. Two faces are 4 ft $\times$ 6 ft, two faces are 4 ft $\times$ 12 ft, and two faces are 6 ft $\times$ 12 ft.

b. 24 ft^2; 48 ft^2; 72 ft^2

c. 288 ft^2

46. a. Z

b. T

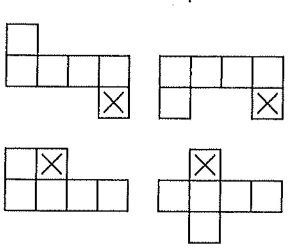
c. M

Connections

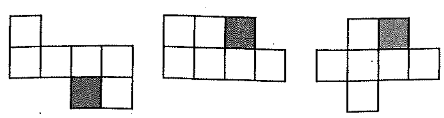
47. A, B, C, and E all have a perimeter of 14 units; D a has perimeter of 12 units.

48. B and E

49. Any of hexominos B, C, D, or E can have one square removed to form a net for an open cubic box. Examples:

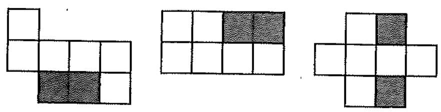


50. Hexominos B, C, D, and E can all have one square added without changing the perimeter. The perimeter does not change if you add the new square to a corner—the square covers two units of perimeter while adding two new units. In the examples below, the shaded square has been added.

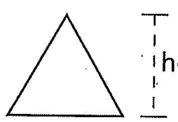


ACE ANSWERS 4

51. Hexominos B, D, and E can have two squares added without changing the perimeter. For Hexominos B and D, a rectangle with a perimeter of 6 units has been added to a corner. Three units are added to the perimeter at the same time that three are being covered. Hexomino E has two units added to different corners, each unit covering two units of perimeter while adding two new units. In the examples below, the shaded squares have been added.



52. a.



height = $559\frac{9}{20}$ feet

base = 646 feet

b. $\approx 180{,}702.35$ ft^3

c. 646 ft $\times$ 3 = 1,938 ft

53. a. Both students are correct because in a right rectangular prism, the area of the base is equivalent to length times width.

b. Yes, all are correct. Method 1 is the definition of surface area. Methods 2 and 3 calculate surface area and are equivalent by the Distributive Property.

c. volume = $242\frac{2}{3}$ in.^{3}

surface area = $264\frac{1}{6}$ in.^{2}

Likely formulas or methods include the ones given in part (b), and $V = \ell \times w \times h$.

d. Answers will vary. Sample answer: To find the volume, I chose to multiply length times width times height. I chose this method because it uses the three dimensions given in the Problem and I could easily substitute in the values I was given. To find the volume, I chose Method 2, S.A. = $2(\ell w \times \ell w \times hw)$.

I chose this formula because, again, I could easily substitute in the values of length, width, and height. Also, it seemed faster to multiply by 2 once at the end than to multiply by 2 three times throughout the calculation, as in Method 3.

54. a. Matt is correct because a cube is a special type of right rectangular prism where the length, width, and height are all equal to *s*.

b. 1,331 cm^3

c. 726 cm^2

55. a. 120 in.

b. They both are correct. Each edge is added 4 times to find the total edge length of the box. The two expressions are equivalent because of the Distributive Property.

Extensions

56. a. The perimeter of the thickest part is $2 \times 10 + 2 \times 15 = 50$ in. Taking the maximum size (108 in.) and subtracting the girth (50 in.) will give the maximum height of the box. $108 - 50 = 58$ in. The box can be at most 58 in. tall.

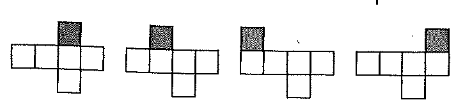
b. Yes. The length is 30 in. The girth is 78 in. because $30 + 30 + 9 + 9 = 78$. $30 + 78 = 108$, so the total size is exactly 108 in.

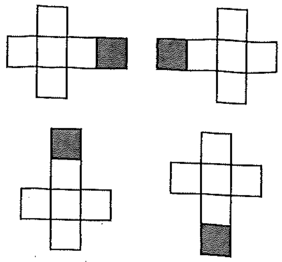
57. 4 in. wide $\times$ 3 in. tall $\times$ 5 in. deep

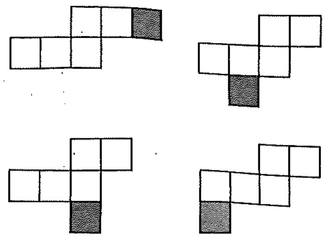
58. 4 in. wide $\times$ 3 in. tall $\times$ 5 in. deep

59. 3 in. wide $\times$ 4 in. tall $\times$ 5 in. deep

60.



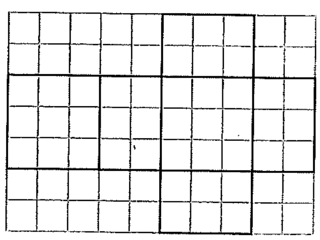
61. 

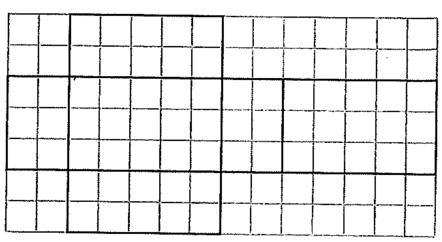
62. 

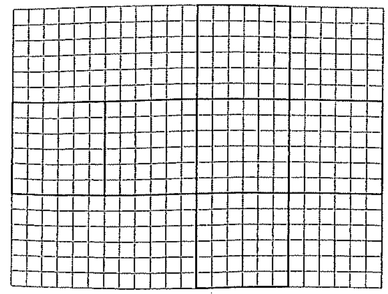
63. Abigail's prism is smaller; the area of the triangular base is less than the area of the rectangular base.

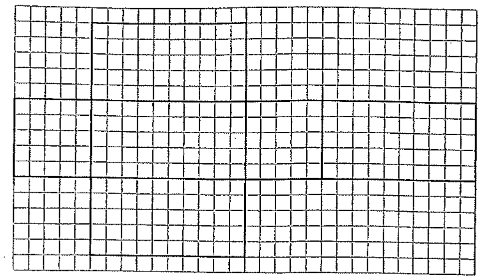
64. Charlie's prism is smaller, because they have the same base, but the height of Charlie's prism is less than the height of Diane's prism.

65. The base area of Elliot's prism is 24 cm^2. The base area of Fiona's prism is 24 cm^2. Their prisms have the same height, so their prisms also have the same volume.

66. 

67. 

68. 

69. 

70. **a.** 6 rectangles

b. 2 triangles, 3 rectangles

c. 2 pentagons, 5 rectangles

d. 2 hexagons, 6 rectangles

e. 2 decagons, 10 rectangles

f. 2 *n*-gons, *n* rectangles

71. S.A. $= \frac{1}{2} \times 13 \times 15 \times 20 = 1{,}950$ cm^2