Focus Question: What is a formula for finding the area of a triangle?

Resources

Labsheets

- Labsheet 2.1A: Triangles A–F
- Labsheet 2.1B: Triangles
- Labsheet 2.1C: Triangle Measurements (accessibility)
- Labsheet 2.1D: Areas (accessibility)
- Labsheet 2ACE: Exercises 1–6
- Labsheet 2ACE: Exercises 27–32
- Centimeter Grid Paper

At a Glance and Lesson Plan

- At a Glance: Problem 2.1 Covering and Surrounding
- Lesson Plan: Problem 2.1 Covering and Surrounding

Technology

- Areas and Perimeters of Shapes and Images

Answers

Problem 2.1

A. **1.** Triangle A: about 18.8 cm
Triangle B: about 29.2 cm
Triangle C: about 19.5 cm
Triangle D: about 24.2 cm
Triangle E: about 21.2 cm
Triangle F: about 23 cm
Strategies will vary but should include that the length of a slanted side was measured with a centimeter ruler.

2. Triangle A: $15\ \text{cm}^2$
Triangle B: $35\ \text{cm}^2$

Triangle C: 12 cm^2

Triangle D: 27 cm^2

Triangle E: 21 cm^2

Triangle F: 24 cm^2

Strategies will vary but may include cutting and rearranging the triangle into a rectangle, or surrounding a triangle with a rectangle, and then finding the area of the rectangle and dividing it by 2.

B. **1.**

Design	Area of Rectangle (cm^2)	Area of Triangle (cm^2)
A	30	15
B	70	35
C	24	12
D	54	27
E	42	21
F	48	24

2. The area of a rectangle is twice the area of a triangle or the area of the triangle is half the area of the rectangle.

C. **1.** Possible answer: $(b \times h) \div 2$ (**Note:** $(\ell \times w) \div 2$ is acceptable at this time.)

2. $A = \frac{1}{2}\left(8 \times 3\frac{1}{2}\right) = 14 \text{ in.}^2$

Corresponding ACE Answers

Applications

1. $A = (5 \times 5) \div 2 = 12.5$ square units

$P \approx 5 + 5 + 7 = 17$ units

Possible explanation: To find area, count the number of square units covering the figure, use the rule, or find half of the area of the smallest rectangle that surrounds the triangle. To find perimeter, measure around the edges of the triangle with a string and compare the length marked off on the string to the units on the grid or measure the length of each edge of the triangle and add the measurements.

2. $A = (7 \times 6) \div 2 = 21$ square units

$P \approx 7 + 7.2 + 6.8 = 21$ units

3. $A = (3 \times 7) \div 2 = 10.5$ square units

$P \approx 3 + 7.25 + 7.25 = 17.5$ units

4. $A = (4 \times 7) \div 2 = 14$ square units

$P \approx 7.3 + 7.3 + 4 = 18.6$ units

For possible explanation, see Exercise 1.

5. $A = (8 \times 7) \div 2 = 28$ square units
   $P \approx 7 + 9.5 + 8.2 = 24.7$ units
   For possible explanation, see Exercise 1.

6. $A = (2 \times 8) \div 2 = 8$ square units
   $P \approx 2 + 8 + 8.2 = 18.2$ units

Connections

27. $A = 28\,\text{cm}^2$; $P \approx 22.6\,\text{cm}$

28. $A = 28\,\text{cm}^2$; $P \approx 23\,\text{cm}$
    Possible explanation: To find area, divide shape into a 5-by-4 rectangle with two triangles on either side. The two triangles can come together to make a 2-by-4 rectangle. So, $20\,\text{cm}^2 + 8\,\text{cm}^2 = 28\,\text{cm}^2$. To find perimeter, measure around the edges of the shape with a string and compare the length marked off on the string to the units on the grid.

29. $A = 6\,\text{cm}^2$; $P \approx 11.4\,\text{cm}$

30. $A = 27\,\text{cm}^2$; $P \approx 24.4\,\text{cm}$

31. $A = 31.5\,\text{cm}^2$; $P \approx 25.8\,\text{cm}$
    Possible explanation: Base = 9 cm, height = 7 cm; so $A = \frac{1}{2}(9)(7) = 31.5\,\text{cm}^2$. To find perimeter, measure around the edges of the shape with a string and compare the length marked off on the string to the units on the grid.

32. $A = 15\,\text{cm}^2$; $P \approx 18.9\,\text{cm}$
    Possible explanation: Base = 5 cm, height = 6 cm; so $A = \frac{1}{2}(5)(6) = 15\,\text{cm}^2$. To find perimeter, measure around the edges of the shape with a string and compare the length marked off on the string to the units on the grid.

ACE Answers: Inv. 2 Covering and Surrounding

Focus Question: *Does it make any difference which side is used as the base when finding the area of a triangle?*

Resources

Labsheets

Labsheet 2.2A: Two Triangles

Labsheet 2.2B: Two Shaded Triangles (accessibility)

Labsheet 2ACE: Exercise 18 (accessibility)

Teaching Aids

Teaching Aid 2.2A: Triangle Height

Teaching Aid 2.2B: Triangles D and E

Teaching Aid 2.2C: Triangles on a Grid

At a Glance and Lesson Plan

At a Glance: Problem 2.2 Covering and Surrounding

Lesson Plan: Problem 2.2 Covering and Surrounding

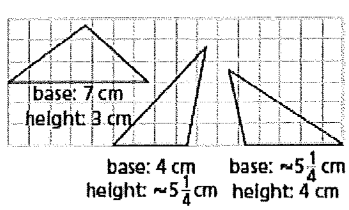
Technology

Areas and Perimeters of Shapes and Images

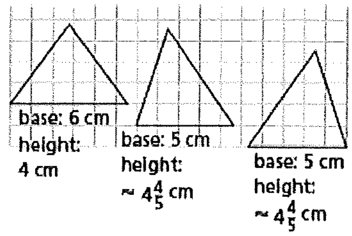
Answers

Problem 2.2

A. **1.** Possible choices for positioning and labeling Triangle 1:



Possible choices for positioning and labeling Triangle 2:



2. The following are based on approximate measurements. This is why different measurements give different areas for each triangle. Some students may count the squares. Some may imbed each triangle in a rectangle and subtract off excess area. Some may use the height–base relationship.

Triangle *D*

Base (cm)	Height (cm)	Area (cm²)
7 (longest side)	3	$10\frac{1}{2}$
4 (shortest side)	≈$5\frac{1}{4}$	≈$10\frac{1}{2}$
≈$5\frac{1}{4}$ (other side)	4	≈$10\frac{1}{2}$

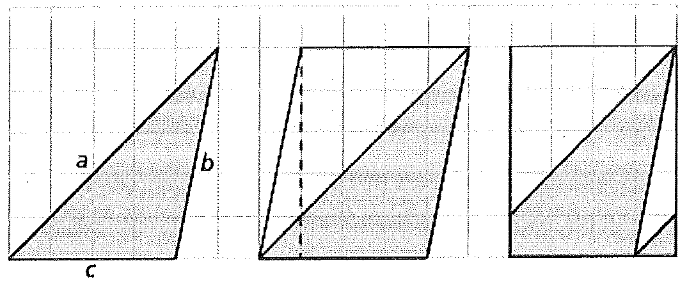
Triangle *E*

Base (cm)	Height (cm)	Area (cm²)
6 (longest side)	4	12
5 (shortest side)	≈$4\frac{4}{5}$	≈12
5 (other side)	≈$4\frac{4}{5}$	≈12

B. 1. Theoretically, the answer is no. Changing the side of the base should not change the area of the triangle. However, as students are measuring, some error will occur, so their calculations may be slightly different.

2. In some cases, the base that is chosen may result in a more accurate measure for the base, height, or both.

C. The formula does work, but it is difficult to draw a rectangle in this case to show that the area of the triangle is half the area of rectangle. The diagram in the answer might help. At this level students can use their empirical data to confirm the conjecture that the area of triangle is the same no matter which side and corresponding height is used as a base and height.



It is easier to find the area when the height falls inside the rectangle.

It is easier to find the position that gives the nicest measurements, such as whole-number measurements.

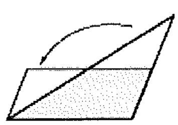
Corresponding ACE Answers

Applications

7. **a.** $\left(h \times \frac{1}{2}\right) \times b$ the area of the right triangle

b. 6 cm, $\left(10 \times \frac{1}{2}\right) \times b = 30$, $= 30 \div 5 = 6$

c. Yes, this will work for any triangle. In the case of a right triangle, the new shape is a rectangle, but in general the new shape is a parallelogram.



8. **a.** 39 cm^2

b. 7.5 cm^2

c. 40 m^2

d. 35 ft^2

9. Vashon is correct, because no matter which side of the triangle he chooses for the base, as long as he chooses the corresponding height, the area will be the same.

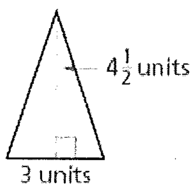
10. **a.** Dimensions for the simplest orientation of each triangle:

	Base (cm)	Height (cm)	Area (cm^2)
A	5	6	15
B	10	7	35
C	3	8	12
D	9	6	27
E	6	7	21
F	6	8	24

b. The areas should be the same. Any differences should be small and related to different approximations in measuring.

11. Talisa is correct, because it does not matter which side of the triangle you use as the base, as long as you choose the appropriate corresponding height. For example, a right triangle with a base of 4 units and a height of 5 units will have an area of 10 square units; the area would be the same if the base is 5 units and the height is 4 units.

12. **a.**



b. $6\frac{3}{4}$ square units

13. The height is 3.2 m because $0.5 \times 2.5 \times 3.2 = 4$; $h = \frac{4}{0.5 \times 2.5} = 3.2$

14. $A = 28\ cm^2$; $P = 22$ cm

15. $A = 120\ cm^2$; $P = 60$ cm

16. $A = 60\ cm^2$; $P = 36$ cm

17. $A = 7.03\ in.^2$; $P = 14.25$ in.

18. Keisha is incorrect, because these triangles are the same size and shape (congruent) and therefore have the same area. Also, they both have an area of $(3\ cm \times 4\ cm) \div 2 = 6\ cm^2$.

19. $A = 24\ cm^2$

20. $A = 24\ cm^2$

21. $A = 24\ cm^2$

Connections

Focus Question: What can you say is true and what can you say is not true about triangles that have the same base and height?

Resources

Labsheets

Centimeter Grid Paper

At a Glance and Lesson Plan

At a Glance: Problem 2.3 Covering and Surrounding

Lesson Plan: Problem 2.3 Covering and Surrounding

Technology

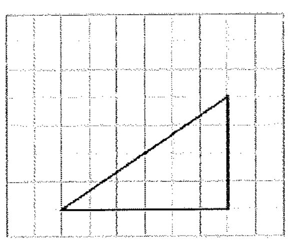
Areas and Perimeters of Shapes and Images

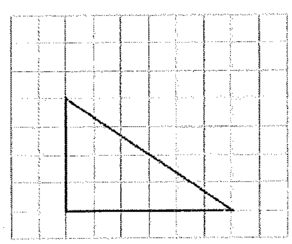
Geoboard

Answers

Problem 2.3

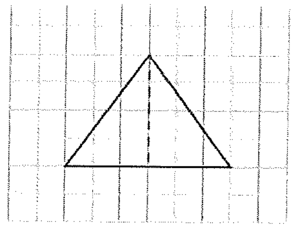
A. **1.** Possible triangles:



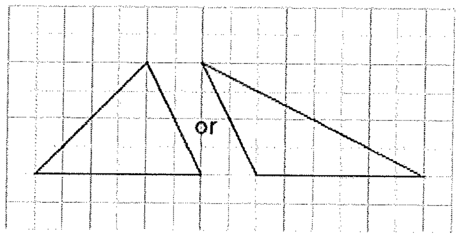


2. For possible triangles, see Question A, part (1).

3. There is one possibility:



4. Possible triangles:

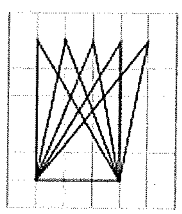


5. $12\,cm^2$

B. 1. The four triangles have the same base, height, and area.

2. Because they have the same base, height, and area.

C. 1. Answers will vary. Here is one possibility of a triangle family that has a base of 3, a height of 5, and an area of $7\frac{1}{2}$ cm^2.



2. The triangles have the same area but not necessarily the same shape. Note that this question is similar to the focus question.

Corresponding ACE Answers

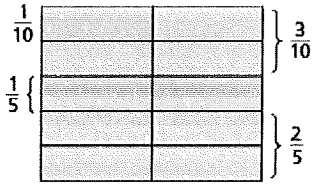
Applications

22. Marlika is correct. The base and height of a triangle determine its area, not the size of its angles.

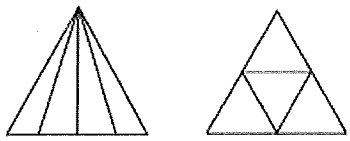
23. D

Connections

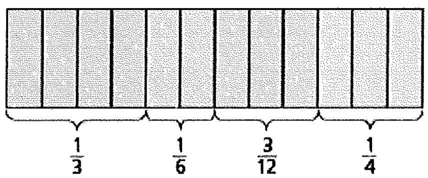
36. Answers will vary. Students may divide squares into tenths, and then mark off $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, and $\frac{4}{10}$.



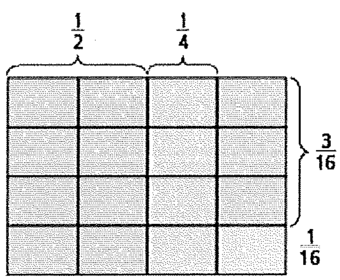
37. Answers will vary. Two possibilities:



38. Answers will vary. Students may divide the rectangle into twelfths, and then mark off $\frac{4}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, and $\frac{3}{12}$.



39. Answers will vary. Students may divide the rectangle into sixteenths, and then mark off $\frac{1}{16}$, $\frac{8}{16}$, $\frac{3}{16}$, and $\frac{4}{16}$.



ACE Answers: Inv. 2 Covering and Surrounding

***Focus Question:** What conditions for a triangle produce triangles that have the same area? Do they have the same shape? Explain.*

Resources

Labsheets

Centimeter Grid Paper

At a Glance and Lesson Plan

At a Glance: Problem 2.4 Covering and Surrounding

Lesson Plan: Problem 2.4 Covering and Surrounding

Technology

Areas and Perimeters of Shapes and Images

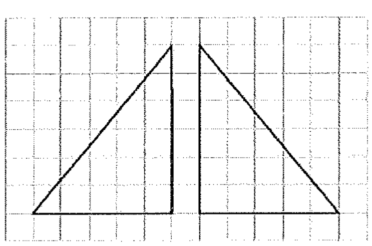
Assessments

Check Up 2

Answers

Problem 2.4

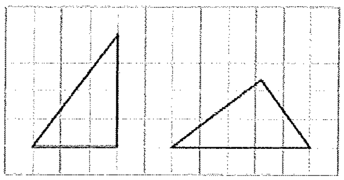
A. Drawings will vary. Students may recognize that this question is making reference to the idea of "triangle families" in Problem 2.3.You can draw many triangles (actually, an infinite number) with the same base and height. The areas of the triangles will be the same, but the perimeters will be different. Possible drawings:



B. Drawings will vary. Correct examples include a triangle with base 6 units and height 5 units, a triangle with base 10 units and height 3 units, and a triangle with base

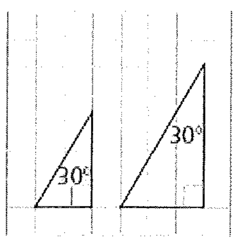
15 units and height 2 units. The perimeters will all be different. Note that any triangle from Question A will satisfy this constraint.

C. Only one triangle is possible. Of course, this triangle may be oriented in different ways.



Note: In the Grade 7 Unit *Shapes and Designs*, students learn that at most one triangle is possible from three given side lengths.

D. Drawings will vary. You can draw an infinite number of right triangles with a 30° angle. The triangles will have different areas and different perimeters, but they will all have the same angle measures and the same shape. Possible drawings:

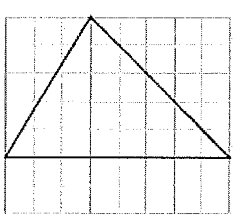


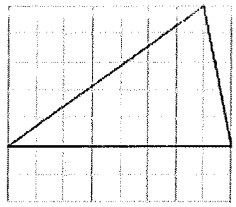
Note: The ideas of similarity are covered in the Grade 7 Unit *Stretching and Shrinking.*

Corresponding ACE Answers

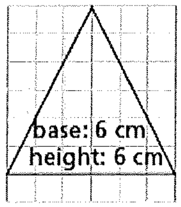
Applications

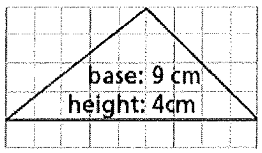
24. The triangles have the same area. Some possible triangles:



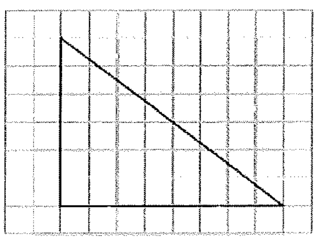


25. These triangles have the same area but not the same perimeter. Some possible triangles:





26. The only possible triangle (although it may be oriented multiple ways):



Note: With these side lengths, the only possible triangle is a right triangle. Students may not know this yet, as the Pythagorean Theorem will justify this for students in the Grade 8 Unit *Looking for Pythagoras*. Therefore, if their drawings are slightly inaccurate, they may think there is more than one. If this occurs, tell them that there is only one triangle. You could give them straws of lengths 6, 8, and 10 centimeters to demonstrate the uniqueness.

Extensions

40. Possible answer: The hexagon can be subdivided into a rectangle and two triangles. The area of each of the figures can be found easily, and then the results can be added together. For the perimeter you could measure one side and multiply it by six, because this is a regular hexagon.

41. C

42. **a.** There is enough information to find the area of the polygon in Figures 1 and 3.

b. The areas of the other patterns are

1. $32\,\text{cm}^2$

2. Not enough information.
(**Note:** Once students have studied the Pythagorean Theorem in the *Looking for Pythagoras* Unit in Grade 8, they will be able to determine that the height of each isosceles triangle is 1.5 cm. The area of each triangle is $3\,\text{cm}^2$. So, the total area is $28\,\text{cm}^2$.)

3. $48\,\text{cm}^2$

4. Not enough information.
(**Note:** The height of each triangle is $2\sqrt{3} \approx 3.5\,\text{cm}$. The area of each triangle is $4\sqrt{3} \approx 7\,\text{cm}^2$. So, the total area is $48 + 8\sqrt{3} \approx 62\,\text{cm}^2$. Again, students will be able to find the area of this polygon after the Grade 8 Unit *Looking for Pythagoras*.)

ACE Answers: Inv. 2 Covering and Surrounding