ACE

Assignment Guide for Problem 2.1

Applications: 1–15 | Connections: 35–39
Extensions: 44–58

Answers to Problem 2.1

A. The multiples of 20 are 20, 40, 60, 80, 100, 120, 140, etc. The multiples of 60 are 60, 120, 180, 240, 300, 360, etc. The common multiples are 60, 120, 180, etc. The least common multiple is 60. So 60 seconds pass before they are both at the bottom again. Take time to have students mentally compute the common multiple answers into minutes. For example: "The siblings will both be at the bottom every minute, with the big wheel going one revolution and the small one going around three times."

B. The multiples of 50 are 50, 100, 150, 200, 250, 300, 350, etc. The multiples of 30 are 30, 60, 90, 120, 150, 180, 210, 240, etc. The least common multiple is 150. So 150 seconds pass before they are both at the bottom again. If needed, you could again ask the students to recompute in minute language: "The siblings will both be at the bottom of the Ferris wheel every two and a half minutes (or 2 minutes, 30 seconds). The big wheel would have gone around three times and the small one five times."

C. The multiples of 20 are 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300, etc. The multiples of 11 are 11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132, 143, 154, 165, 176, 187, 198, 209, 220, etc. The least common multiple is 220. So 220 seconds, or 3 minutes and 20 seconds, pass before they are both at the bottom again.

Note: If needed, repeat the idea of conversion to minutes. You might also discuss with the students if it seems reasonable for a Ferris wheel to be able to make one revolution in 20 or 11 seconds.

D. Question A: In 60 seconds, the large Ferris wheel makes 1 revolution, and the small Ferris wheel makes 3 revolutions.

Question B: In 150 seconds, the large Ferris wheel makes 3 revolutions, and the small Ferris wheel makes 5 revolutions.

Question C: In 260 seconds, the large Ferris wheel makes 13 revolutions, and the small Ferris wheel makes 20 revolutions.

Assignment Guide for Problem 2.2

Applications: 16–29 | Connections: 40–41
Extensions: 59–61

Answers to Problem 2.2

A. The multiples of 13 are 13, 26, 39, 52, 65, 78, 91, 104, 117, 130, 143, 156, 169, 182, 195, 208, 221, 234, etc.

The multiples of 17 are 17, 34, 51, 68, 85, 102, 119, 136, 153, 170, 187, 204, 221, 238, etc.

The least common multiple is 221. So the 13-year and 17-year cicadas will next appear together in 221 years.

B. The multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192, 204, 216, 228, 240, 252, 264, 276, 288, 300, 312, 324, 336, etc.

The multiples of 14 are 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196 210, 224, 238, 252, 266, 280, 294, 308, 322, 336, etc.

The multiples of 16 are 16, 32, 48, 64, 80, 96, 112, 128, 144, 160, 176, 192, 208, 224, 240, 256, 272, 288, 304, 320, 336, etc.

The least common multiple of 12, 14, and 16 is 336. So the 12-year, 14-year, and 16-year cicadas will all appear together again in 336 years.

C. Stephen's method only works when the cycles are relatively prime. Sample: 4 and 6 do not work because the product of 4 and 6 is 24, but the cicadas will next come after 12 years.

D. In Question A, the answer is equal to the product of the two cycles.

In Question B, the answer is less than the product of the three cycles.

- *Which part called just for common factors?*
- *How did you know that finding factors could help?*
- *What about the Problem signals that you need to analyze what numbers divide the amounts given?*
- *When is the greatest common factor of two numbers one of the numbers? What is special about these two numbers? Give another example that fits this pattern.*
- *When is the greatest common factor of two numbers 1? What is special about these two numbers? Give another example that fits this pattern.*
- *When is the greatest common factor of two numbers less than both numbers but greater than 1? What is special about these two numbers? Give another example that fits this pattern.*

Throughout this Investigation, Summarize discussions have included questions such as, "When is the least common multiple the product of the two numbers?" and "When is the greatest common factor equal to 1?" You may want to take this time to formalize the relationships between the answers to such questions.

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Assignment Guide for Problem 2.3

Applications: 30–34 | Connections: 42–43
Extensions: 62–69

Answers to Problem 2.3

A. **1.** By using a listing method list, we can see all of the factors of each number. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. So the common factors of 24 and 36 are 1, 2, 3, 4, 6, and 12. Therefore, Jane can make 1 huge snack pack (with 24 apples and 36 bags of trail mix), 2 packs (12 apples, 18 bags per pack), 3 packs (8 apples, 12 bags), 4 packs (6 apples, 9 bags), 6 packs (4 apples, 6 bags), or 12 packs (2 apples and 3 bags).

2. The greatest common factor of 24 and 36 is 12. This means that each of the numbers given in the problem can be divided into 12 equal parts and thus make 12 snack packs. So the greatest number of snack packs Jane can make is 12.

3. Now we have 24 apples and 30 packages of trail mix. The greatest common factor of 24 and 30 is 6, so the greatest number of snack packs Jane can now make is 6.

B. Now we have 18 cans of juice, 24 apples, and 36 packages of trail mix. The greatest common factor of 18, 24, and 36 is 6. This means that each of the numbers given in the problem can be divided into 6 equal parts and thus make 6 snack packs: 4 apples, 5 bags of trail mix, and 3 cans of juice. So the greatest number of snack packs Jane can make is 6.

C. Common multiples were used in Problem 2.1 and Problem 2.2; these problems were solved by finding the least common multiple. Common factors were used in Problem 2.3; this problem was solved by finding the greatest common factor. When the problem involves finding when two or more cyclical events will occur at the same time, you should use common multiples. When the problem involves sharing different amounts equally, you should use common factors.