Applications

1. **a.** Divide 24 by 12 to see if you get a whole number. Since $12 \times 2 = 24$ or $24 \div 12 = 2$, 12 is a factor.

   b. Divide 291 by 7 to see if the answer is a whole number. Since $291 \div 7 = 41.571429\ldots$, 7 is not a factor of 291.

2. **a.** 4
   b. 9
   c. 8
   d. 9

3. **a.** 21 has the most factors; 1 and 19 are the factors of 19; 1, 3, 7, and 21 are the factors of 21; 1 and 23 are the factors of 23; 1, 5, and 25 are the factors of 25.

   b. Answers will vary. Possible answer is 36; 1, 2, 3, 4, 6, 9, 12, 18, and 36 are the factors of 36.

   c. Answers will vary. Possible answer is 18; 1, 2, 3, 6, 9, and 18 are the factors of 18.

4. **a.** 6; Since the result is a whole number, 14 is a factor of 84.

   b. 5.6; Since the result is not a whole number, 15 is not a factor of 84.

5. **a.** Yes; $18 \div 6 = 3$

   b. No; $6 \div 18 = 0.3333\ldots$

6. 2, 8, 16 are divisors of 64

7. **a.** $n = 8$

   b. $n = 12$

   c. $n = 12$

   d. $n = 20$

   e. $n = 3$

8. **a.** (See Figure 1.)

   b. Keiko picked 28. Cathy scored 21 points for 7 and 14, the factors of 28 that were not already circled. Here is the game board at this stage in the game: (See Figure 2.)

   c. Cathy picked 18 or 27. Keiko scored 9 points by circling 9 (the factor of 18 or 27 that is not already circled).

   d. Cathy scores 30 points for 5, 10, and 15. (the factors of 30 that are not already circled).

   e. The only numbers remaining with uncircled factors are 22 (11) and 26 (13). Cathy should choose 26 because she will score 13 more points than Keiko.

9. Check every number beginning with 1 until you begin to get the same factors over again. With each small number factor you find, you will find a second factor when you divide. Factors of 110 are: 1, 2, 5, 10, 11, 22, 55, 110. I know I have found all of the factors because I checked all the numbers from 1 to 10, and then started getting repeats.

10. **a.** 30 is the least possibility. Others are all multiples of 30. Common factors of any two such numbers must include 1, $2 \times 3 = 6$, $2 \times 5 = 10$, $3 \times 5 = 15$ and $2 \times 3 \times 5 = 30$ in addition to 2, 3, and 5

    b. 8 is the least possibility. Others are all multiples of 8. Common factors of any three such numbers must include 1 in addition to 2, 4, and 8.

Figure 1

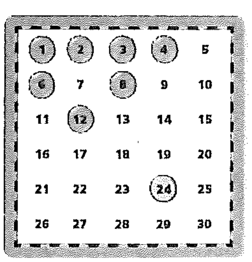
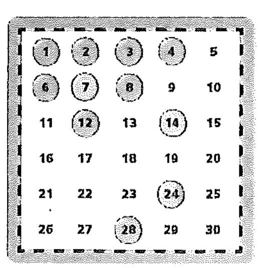


Figure 2



11. a. In the Factor Game, your opponent scores points for proper factors of the number you choose. The only proper factor of prime numbers, such as 2, 3, or 7, is 1.

b. Some numbers, such as 12, 20, and 30, have many proper factors that would give your opponent more points. (Some numbers, such as 9, 15, and 25, have fewer factors and would give you more points.)

12. a. i. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.
ii. Multiples of 3; 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30.
iii. 12, 21, 30, and 210 will all appear in the sequence, because they are all multiples of 3.

b. i. 7, 14, 21, 28, 35, 42, 49, 56, 63, 70.
ii. Multiples of 7
iii. 21 and 210 will appear in the sequence, because they are multiples of 7.

c. i. $6 \times 1 = 6$; $6 \times 2 = 12$; $6 \times 3 = 18$; $6 \times 4 = 24$; $6 \times 5 = 30$; $6 \times 6 = 36$; $6 \times 7 = 42$; $6 \times 8 = 48$; $6 \times 9 = 54$; and $6 \times 10 = 60$.
ii. Multiples of 6
iii. 12, 30, and 210 will appear in the sequence, because they are multiples of 6

13. a. (See Figure 3.)

b. 31, 37, 41, 43, and 47 are prime.

c. 36 and 49 are square numbers.

d. 47; it is the greatest prime number.

e. 48; The second player gets 76 points, which is 28 more points than the first player.

Figure 3

First Move	Proper Factors	My Score	Opponent's Score
31	1	31	1
32	1, 2, 4, 8, 16	32	31
33	1, 3, 11	33	15
34	1, 2, 17	34	20
35	1, 5, 7	35	13
36	1, 2, 3, 4, 6, 9, 12, 18	36	55
37	1	37	1
38	1, 2, 19	38	22
39	1, 3, 13	39	17
40	1, 2, 4, 5, 8, 10, 20	40	50
41	1	41	1
42	1, 2, 3, 6, 7, 14, 21	42	54
43	1	43	1
44	1, 2, 4, 11, 22	44	40
45	1, 3, 5, 9, 15	45	33
46	1, 2, 23	46	26
47	1	47	1
48	1, 2, 3, 4, 6, 8, 12, 16, 24	48	76
49	1, 7	49	8

14. a. Move the paper clip from 6 to make the products 5×1, 5×2, 5×3, 5×4, 5×5, 5×7, 5×8, and 5×9; move the paper clip from the 5 to make the products 6×1, 6×2, 6×3, 6×4, 6×6, 6×7, 6×8, and 6×9.

b. Moving the paper clip from the 6 to the 3, 4, or 9, makes 15, 20, or 45, respectively; moving the paper clip from the 5 to the 7 makes 42.

c. Moving the paper clip from the 5 to the 7 makes 6×7, which is 42.

d. Possible answer: Move the 5 to the 7 to get 42; this blocks the opponent and gets 3 in a row.

15. a. $3 \times 1 = 3$; $3 \times 2 = 6$; $3 \times 3 = 9$; $3 \times 4 = 12$; $3 \times 5 = 15$; $3 \times 6 = 18$; $3 \times 7 = 21$; $3 \times 8 = 24$; $3 \times 9 = 27$. So you can get a 3, 6, 9, 12, 15, 18, 21, 24 and 27, which are all multiples of 3.

b. $3 \times 11 = 33$; $3 \times 13 = 39$; $3 \times 17 = 51$; $3 \times 19 = 57$; $3 \times 20 = 60$; and many others.

c. There are infinitely many multiples of 3.

16. a. 2 and 9; or 3 and 6

b. 1 and 18

17. a. 36 can be found on the product game by 6×6 or 4×9. 36 is composite.

b. 5 can be found on the product game by only 1×5. 5 is prime.

c. 7 can be found on the product game by only 1×7. 7 is prime.

d. 9 can be found on the product game by 1×9 or 3×3. 9 is composite.

18. Since the numbers on the game board are multiples of the numbers given as possible factors, you could argue in support of Sal's position. The Product Game is more specific, because it implies the result of multiplying the two numbers together, while there are infinite number multiples of two chosen numbers.

19. a. 2

b. No. All other even numbers have 2 as a factor 2, in addition to 1 and themselves.

20. a. 2, 3, and 7

b. 21 is missing.

21. a. 3, 5, 6, and 7

b. 25 is missing.

22. dimensions: 1×24, 2×12, 3×8, 4×6, 6×4, 8×3, 12×2, 24×1
factor pairs: 1, 24; 2, 12; 3, 8; 4, 6

23. dimensions: 1×32, 2×16, 4×8, 8×4, 16×2, 32×1
factor pairs: 1, 32; 2, 16; 4, 8

24. dimensions: 1×48, 2×24, 3×16, 4×12 6×8, 8×6, 12×4, 16×3, 24×2, 48×1
factor pairs: 1, 48; 2, 24; 3, 16; 4, 12; 6, 8

25. dimensions: 1×45, 3×15, 5×9, 9×5, 15×3, 45×1
factor pairs: 1, 45; 3, 15; 5, 9

26. dimensions: 1×60, 2×30, 3×20, 4×15 5×12, 6×10, 10×6, 12×5, 15×4, 20×3, 30×2, 60×1
factor pairs: 1, 60; 2, 30; 3, 20; 4, 15; 5, 12 6, 10

27. dimensions: 1×72, 2×36, 3×24, 4×18 6×12, 8×9, 9×8, 12×6, 18×4, 24×3, 36×2, 72×1
factor pairs: 1, 72; 2, 36; 3, 24; 4, 18; 6, 12 8, 9

28. a. Prime numbers have only two factors, 1 and itself. Examples: 2, 3, 5, 7, 11, . . .

b. Square numbers have odd numbers of factors. Examples: 4, 9, 16, 25, 36, . . .

c. No. Because prime numbers have two factors (from a) and square numbers have an odd number of factors (from b) you could never have a prime number that is also square.

29. two

30. B

31. His number is 25. His number must be a square number because it has an odd number of factors. 16 has five factors and 36 has nine factors.

32. 100 fans in 1 row, 50 fans in 2 rows, 25 fans in 4 rows, 20 fans in 5 rows, 10 fans in 10 rows, 5 fans in 20 rows, 4 fans in 25 rows, 2 fans in 50 rows, or 1 fan in 100 rows. Answers about which arrangement to choose will vary. Sample: I would rather have one long banner that wraps around part of the stadium, so I would choose 100 fans in one row. Sample: I would rather have a big square that you could see on TV, so I would choose ten fans in ten rows.

33. **a.** 1×64, 2×32, 4×16, 8×8, 16×4, 32×2, and 64×1.

b. Answers will vary as they did in Exercise 32.

Connections

34. 5 hours $\times\ 60\frac{\text{minutes}}{\text{hour}} = 300$ minutes; $300 + 30 = 330$ minutes. $330 \div 12 = 27.5$. Therefore, they can play 27 games.

35. B; Carlos read $27 + 31 + 28 = 86$ pages the first part of the week. He had $144 - 86 = 58$ pages left for Thursday and Friday. Since he read the same number of pages each day, $58 \div 2 = 29$. Carlos read 29 pages on Thursday.

36. **a.** For $n + 3 < 50$, the possible answers of n are 0,1,2,3,4,5, . . .,45,46.

b. For $3n < 50$, the possible answers of n are 0,1,2,3,4,5, . . .,15,16.

37. 24 has many factors, so it can be divided into many equal parts. Since 23 is prime, it cannot be subdivided. The only proper factors of 25 are 1 and 5, so it can only be subdivided into 5 groups of 5.

38. Because 60 has many factors, and 59 and 61 do not.

39. **a.** Various answers; for example, group sizes 1, 2, 4, 5, 6, 3, 2, 2, 3, and 2. The goal is to find 10 numbers whose sum is 30.

b. Group sizes 3, 3, 3, 3, 3, 3, 3, 3, 3, and 3. If she does not have ten groups, she could have 1 group of 30 students, 2 groups of 15 students each, 5 groups of 6 students each, 6 groups of 5 students each, 10 groups of 3 students each, 15 groups of 2 students each, or 30 groups of 1 student each.

c. In part (a), the sum of the numbers in the ten groups must be 30. In part (b), we are considering the factors of 30.

40. 500 days

Extensions

41. **a.** $1 + 2 + 4 + 5 + 10 + 20 + 25 + 50 = 117$

b. $1 + 3 + 9 + 11 + 33 = 57$

c. 97, because it is the largest prime less than 100.

42. The numbers that have two odd digits (Clue 2) and give a remainder of 4 when divided by 5 (Clue 1) are 19, 39, 59, 79, and 99. Of these numbers, 19 is the only one with digits that add to 10 (Clue 3). The number is 19.

43. a. (See Figure 4.)

b. Abundant means "more than enough," which is appropriate since the sum of an abundant number's factors is more than the number. Deficient means "not enough," which is appropriate since the sum of a deficient number's factors is less than the number. Perfect means "exactly right," which is appropriate because the sum of a perfect number's factors is equal to the number.

c. abundant

d. deficient

44. a. $1 + 2 + 4 + 8 = 15$; your opponent scores one fewer point.

b. $1 + 2 = 3$; your opponent scores one fewer point.

c. 2, 8, and 32. (Any power of 2 that is less than 49.) The name near-perfect fits because the sum of the factors is 1 less than the total needed for the number to be perfect.

d. 64 and 128.
$1 + 2 + 4 + 8 + 16 + 32 = 63$ and
$1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$.

45. Some possible answers: $3 \times 10 \times 10$; $2 \times 3 \times 50$; many more.

46. a. $2 \times 294 = 588$

b. 1, 588; 2, 294; 3, 196; 4, 147; 6, 98; 7, 84; 12, 49; 14, 42; 21, 28

c. There are twice as many rectangles as factor pairs because you can reverse each factor pair to find another rectangle: 1×588; 2×294; 3×196; 4×147; 6×98; 7×84; 12×49; 14×42; and 21×28; then reverse: 28×21; 42×14; 49×12; 84×7; 98×6; 147×4; 196×3; 294×2; and 588×1.

47. a. 10×10

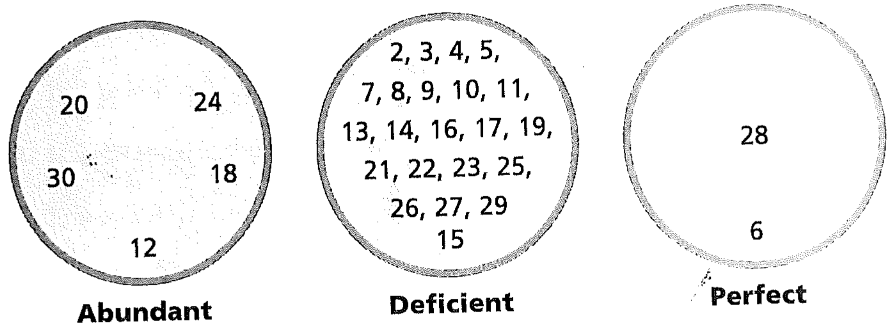
b. 9×9

c. If students know of the square root, this is where the factors reverse order. Another way to explain this is to find the factor pair that makes a square.

d. The factor pairs reverse order at the square root of the original number. Geometrically, if you use the factor pairs to build rectangles, the rectangle that is closest to a square will be the last factor pair before the pairs reverse order.

e. Yes, (similar to d). Because there are only two possible ways to write a prime number as a product of its factors, the turn around occurs after the first pair.

Figure 4



48. Students might notice that for square numbers the last factor pair is the one that uses the square root, though they may not use that vocabulary word. Extending this idea to the non-square numbers, they may say that looking for a square near the number being investigated helps; thus, 66 is close to 8×8, and 7 and 8 are not factors of 66, so there is no point in looking further than 6×11. (See Figure 5.)

49. a. Samples: $4 = 2 \times 2$, $9 = 3 \times 3$, . . .

b. Samples: $8 = 2 \times 4$, $12 = 2 \times 6$, . . .

c. Samples: $16 = 4 \times 4$, $24 = 4 \times 6$, . . .

d. No; prime numbers have only two factors, 1 and itself. They couldn't be the product of two prime numbers, a prime number and another composite number, or two composite numbers.

Figure 5

Number	16	30	36	40	50	64	66
Last Factor Pair	4 × 4	5 × 6	6 × 6	5 × 8	5 × 10	8 × 8	6 × 11