ACE

Assignment Guide for Problem 2.1

Applications: 1–6 | Connections: 25–26
Extensions: 31–33

Answers to Problem 2.1

A. 1. Different partitioning strategies will lead to different forms of this unit rate. Some students may see this as an implied division with six segments divided up among the four people. (See Figure 1.)

2. Different partitioning strategies will lead to different forms of this unit rate. (See Figure 2.)

B. 1. Different partitioning strategies will lead to different forms of this rate. (See Figure 3, next page.)

2. Different partitioning strategies will lead to different forms of this rate. (See Figure 4, next page.)

C. 1. Several answers are possible. The picture will lead many students to say that could be 8 people in her group.

2. There are multiple answers. For example, there could be 4 people in her group. There could be 2 people in her group. It would be unusual to share the chewy worm between 2 people for that drawing.

Figure 1

Each person gets $1\frac{1}{2}$ segments.

1	2	3	4	1	2	3	4

Each person gets $\frac{3}{2}$ segments.

1	2	3	4	1	2	3	4	1	2	3	4

Each person gets $\frac{6}{4}$ segments.

1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4

Figure 2

Each person gets $1\frac{1}{3}$ segments.

1	2	3	4	5	6	1	2	3	4	5	6

Each person gets $\frac{4}{3}$ segments.

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6

3. The answer depends on the number of people in the group. If 8 people share, there would be $\frac{6}{8}$ or $\frac{3}{4}$ of a segment per person. If 4 people share, there would be $\frac{6}{4}$ or $1\frac{1}{2}$ segments per person. If 2 people share, there would be 3 segments per person. If 3 people share, there would be 2 segments per person. If 6 people share, there would be 1 segment per person.

4. There are multiple answers. **Note:** The unit rate becomes the number of pieces each person gets (numerator), and the number of segments becomes the whole (denominator). If 8 people share, there would be $\frac{6}{8}$ or $\frac{3}{4}$ of a segment per person. Each person is then getting $\frac{\frac{3}{4}}{6}$ of the chewy worm. If 4 people share, there would be $1\frac{1}{2}$ segments per person. Each person is then getting $\frac{\frac{6}{4}}{6}$ or $\frac{1\frac{1}{2}}{6}$ of the chewy worm. If 2 people share, there would be 3 segments per person. Each person is then getting $\frac{3}{6}$ or $\frac{1}{2}$ of the chewy worm. If 3 people share, there would be 2 segments per person. Each person is then getting $\frac{2}{6}$ or $\frac{1}{3}$ of the chewy worm. If 6 people share, there would be 1 segment per person. Each person is then getting $\frac{1}{6}$ of the chewy worm.

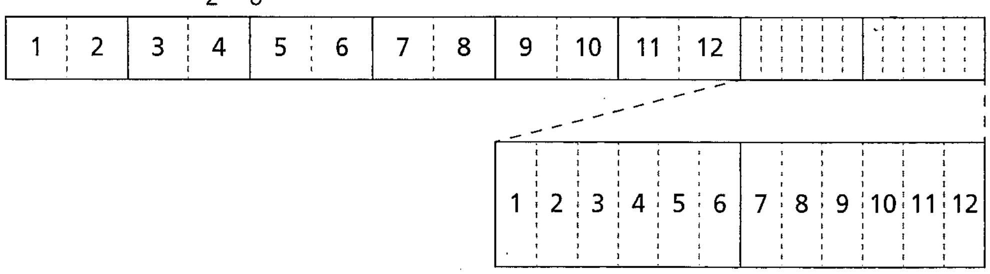
D. Answers will vary. If the worms are the same size, sharing a 6-segment worm among 4 people gives a bigger share: $\frac{6}{4}$ of a worm instead of $\frac{12}{8}$ of a worm. If the segments are the same size, then the worms are different sizes and the shares would be equal segments per person.

E. Possible answer: Every time I found a per-person amount, I found a unit rate. This told me how many segments of the chewy worm each person got. This happened in all of Questions A–D.

AT A GLANCE 2

Figure 3

Each person gets $\frac{1}{2} + \frac{1}{6}$ of a segment.



Each person gets $\frac{2}{3}$ of a segment.

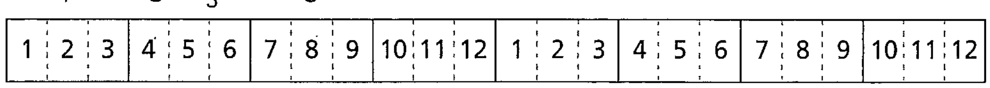
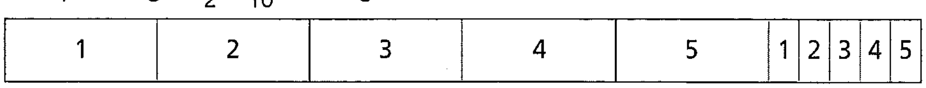
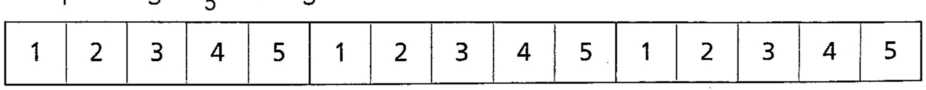


Figure 4

Each person gets $\frac{1}{2} + \frac{1}{10}$ of a segment.



Each person gets $\frac{3}{5}$ of a segment.



At a Glance

Problem 2.2 Pacing 1 Day

2.2 Unequal Shares: Using Ratios and Fractions

Focus Question How are part-to-part ratio relationships related to part-to-whole fractions?

Launch

The context of this problem is different from that of Problem 2.1. Children are not sharing equally; they are sharing according to their ages.

Students should understand that there is a connection between equivalence of fractions and equivalence of ratios, but this connection does not need to rely on writing the ratios as fractions.

Materials

Accessibility Labsheet

- 2.2: Fraction Strips

Explore

As students work, listen for their ability to describe why their ratios are equivalent.

To help students who may incorrectly think that "getting more pieces" means that the ratio is becoming more favorable, remind them that the ratio is the comparison between the numbers. In the case of Jared and Peter, Peter gets $1\frac{1}{2}$ times the segments that Jared gets.

To help students focus on equivalence, show them diagrams of chewy fruit worms that are the same size but that have different-sized segments. Students will probably relate these diagrams to Investigation 1. They will see that computing the scaled-up ratio is much like finding equivalent fractions.

Summarize

Focus on equivalent ratios and their relationship to equivalent fractions. Ask students to describe the ways in which they found equivalent ratios. You could begin with the common factor strategy described in the Explore section by saying, *We know that Crystal and Alexa can share a 9-segment worm. I talked to a group that found a lesser number of segments that Crystal and Alexa could easily share. I want them to talk to us about how they found that number.*

ACE

Assignment Guide for Problem 2.2

Applications: 7–15 | Connections: 27–28

Extensions: 34–35

Answers to Problem 2.2

A. Crystal and Alexa could share a 3-segment worm without cutting; Crystal gets 2 segments and Alexa gets 1. They could share without further cutting worms with 6 segments, 9 segments or any number of segments that is a multiple of 3.

AT A GLANCE 2

Some possible answers include:

Crystal's Segments	Alexa's Segments	Total Segments
2	1	3
4	2	6
6	3	9
8	4	12
10	5	15
12	6	18
14	7	21

B. 1. They could share a 25-segment worm (Jared gets 10 segments, Peter gets 15), or a 5-segment worm (Jared gets 2 segments, Peter gets 3), or any multiple of 5 segments.

Some possible answers include:

Jared's Segments	Peter's Segments	Total Segments
2	3	5
4	6	10
6	9	15
8	12	20
10	15	25
12	18	30
14	21	35

2. The ratio of the number of parts Jared gets to the number of parts Peter gets is 10 to 15, or 2 to 3.

Some possible answers include:

Jared's Segments	Peter's Segments	Total Segments	Ratio for Jared to Peter
2	3	5	2 to 3
4	6	10	4 to 6
6	9	15	6 to 9
8	12	20	8 to 12
10	15	25	10 to 12
12	18	30	12 to 18
14	21	35	14 to 21

3. Yes. In both cases, the relationship between the boys' shares is the same. For every 2 segments Jared gets, Peter gets 3 segments. In the case of 10 to 15, Jared gets 2 segments 5 times, and Peter gets 3 segments 5 times.

4. There are two unit rates with one of the boys getting 1 segment: Jared gets $\frac{2}{3}$ of a segment for every 1 segment for Peter. Jared gets 1 segment for every $1\frac{1}{2}$ segments for Peter.

C. 1. Possible answers: Caleb might be 8 years old and Isaiah 6 years old. They might be 4 and 3. They might be 12 and 9.

2. a. Crystal gets $\frac{2}{3}$ of the worm she shares with Alexa. Alexa gets $\frac{1}{3}$ of the worm.

b. Jared gets $\frac{2}{5}$ of the worm he shares with Peter. Peter gets $\frac{3}{5}$ of the worm.

c. Answers will vary. Possible answer: If I write the fraction of the worm each person gets using the same denominator, the ratio of the numerators is equivalent to the ratio of the number of segments each person gets.

For example, Caleb gets $\frac{8}{14}$ (or $\frac{4}{7}$) of the worm and Isaiah gets $\frac{6}{14}$ (or $\frac{3}{7}$) of the worm. The ratio of the segments is 8 : 6 (or 4 : 3). If you add the two numbers in the ratio, you get a number that can be the denominator of the fraction of a worm each person gets. For example, if the ratio of segments is 4 : 3, then one person gets 4 out of every (4 + 3) segments, or $\frac{4}{7}$ of a worm, and the other gets $\frac{3}{7}$.

Suggested Questions

- *What are some patterns you notice in the rate table in Question A? Are there more ways than one to find missing entries in the rate table?*
- *One person doubled the cost of 10 chewy fruit worms to get the cost of 20 chewy fruit worms. Another person found the cost of 1 chewy fruit worm and then multiplied by 20. Why do both strategies give the same answer?*

ACE

Assignment Guide for Problem 2.3

Applications: 16–24 | Connections: 29–30
Extensions: 36–37

Answers to Problem 2.3

A. **1.** (See Figure 1.)

2. It will cost $.30 for 3 worms. It will cost $30 for 300 worms.

3. You can buy 500 worms with $50. You can buy 100 worms with $10.

4. The unit price for one worm is $.10. The unit rate for one worm is $.10, i.e., $.10 per worm.

B. **1.** (See Figure 2.)

2. 48 cups of popcorn can be made from 12 ounces of popcorn kernels. 120 cups of popcorn can be made from 30 ounces of popcorn kernels.

3. 10 ounces of popcorn kernels are needed to make 40 cups of popcorn. 25 ounces of popcorn kernels are needed to make 100 cups of popcorn.

4. 50 ounces of popcorn kernels are needed to make 200 cups of popcorn or 100 servings.

5. $\frac{1}{4}$ ounce of popcorn kernels are needed to make 1 cup of popcorn.

C. **1.** Rate tables show that ratios can be multiplied or divided to find equivalent ratios. For example, if you know one ratio, you can find another equivalent ratio by doubling, tripling, or halving, etc.

2. Unit rates are easy to work with because you multiply them by the quantity or number of units to find an equivalent ratio. For example, if you know that the unit rate is 4 cups to 1 ounce, then for 3 ounces you will get 3(4) = 12, or 12 cups.

Figure 1

Chewy Fruit Worm Pricing

Number of Worms	1	5	10	15	30	90	150	180
Reduced Price	$.10	$.50	$1	$1.50	$3	$9	$15	$18

Figure 2

Popcorn Table

Number of Cups of Popcorn	4	8	12	16	20	24	28	32	36	40	44	48
Number of Ounces of Popcorn Kernels	1	2	3	4	5	6	7	8	9	10	11	12

ACE

Assignment Guide for Problem 3.1

Applications: 1–15 | Connections: 20–22, 24
Extensions: 89–91

Answers to Problem 3.1

A. 1. (See Figure 1.)

2. $\frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}, -\frac{5}{4}$. All those that have numerators greater than the denominators.

B. 1. (See Figure 2.)

2. $1\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, -1\frac{1}{3}$, and $-1\frac{2}{3}$. All those that are whole or mixed numbers.

C. 1. $-\frac{1}{2}$

2. $\frac{1}{2}$

3. 0

4. $-\left(\frac{1}{2}\right) = -\frac{1}{2}$

$-\left(-\left(\frac{1}{2}\right)\right) = -\left(-\frac{1}{2}\right) = \frac{1}{2}$

$-(0) = 0$

D. 1. 1 and −1

2. Two numbers: $\frac{5}{4}$ and $-\frac{5}{4}$

3. One number, 0

E. 1. a. This is possible. In the image on the left, the temperature was 10° C; in the photo on the right, the temperature was −10° C. This means that each day's temperature was 10° from freezing—one day was above freezing; the other day was below. The two temperatures are 20 degrees apart.

b. Yes. If the bird and the fish are equidistant from sea level, the height of the bird's position would be the positive value above sea level and the depth of fish's position would be the negative value below sea level.

2. a. Aaron is correct. If he gets the answer right, he will have 300 points. If he gets the answer wrong, he will have −300 points. The absolute value of each of these numbers of points is 300.

b. The point values of the questions could be any pair of opposites.

Figure 1

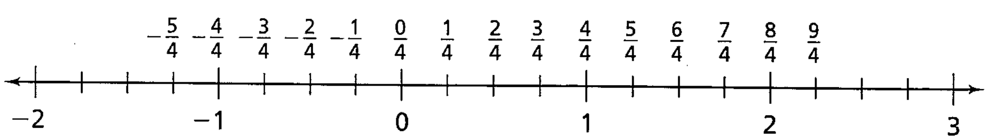
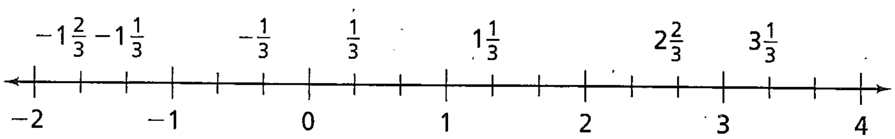


Figure 2



At a Glance

Problem 3.2 Pacing 1 Day

3.2 Estimating and Ordering Rational Numbers

Focus Question When comparing two rational numbers, what are some useful strategies for deciding which is greater?

Launch

One way to estimate the size of a fraction is to compare it to benchmarks.

Suggested Questions

- *Name a fraction close to, but not equal to $\frac{1}{2}$.*
- *Is it greater than or less than $\frac{1}{2}$?*
- *Name a fraction close to, but not equal to $-\frac{1}{2}$.*
- *Is it greater or less than $-\frac{1}{2}$?*

Key Vocabulary

- benchmark

Materials

Labsheets

- 3.2A: Fraction Benchmarks
- 3.2B: Fraction Benchmarks
- rulers
- unlined paper
- Fraction Shapes Tool
- Number Line Tool

Explore

If you think of a fraction as a part–whole relationship, then if two fractions have common denominators, their pieces are the same size. The numerator tells how many pieces are in each fraction, so the fraction with the greater numerator is greater.

If you think of a fraction as a part–whole relationship, then you can compare it to benchmarks by determining how far it is from the benchmark.

Summarize

Discuss how students used the benchmarks and equivalent fractions to order the rational numbers.

Discuss where $\frac{3}{12}$ and $\frac{3}{4}$ belong. Students often want to "round up" and say that these numbers are closer to the right-hand benchmark. Take some time to discuss why rounding is not always the most effective strategy.

ACE Assignment Guide for Problem 3.2

Applications: 16–19 | Connections: 23, 25–52
Extensions: 94–96, 98–99

Answers to Problem 3.2

A. (See Figure 1, next page.)

B. **1.** $-\frac{5}{2} < 3$; because a negative number always less than a positive number.

2. $0 > -3$; because zero is always greater than a negative number.

3. $-\frac{5}{3} > -\frac{11}{2}$; because $-\frac{5}{3}$ or $-1\frac{2}{3}$ is closer to zero than $-\frac{11}{2}$ or $-5\frac{1}{2}$.

AT A GLANCE 3