At a Glance

Problem 1.1 Pacing $1\frac{1}{2}$ Days

1.1 Getting Ready to Ride: Data Tables and Graphs

Focus Question How can you construct a graph from a table of data that depicts change over time? How is this pattern of change represented in the graph?

Launch

Tell the class about bicycles and the yearly bicycle tour across Iowa.

Ask for some predictions about the jumping jack stamina experiment:

- *How many jumping jacks do you think you could complete in 2 minutes?*
- *How do you think your jumping jack rate would change over the 2-minute test?*

Materials

Labsheets

- 1.1A: Jumping Jack Fitness Test
- 1.1B: Jumping Jack Tables and Graphs
- Graph Paper
- stopwatches (1 per group)

Explore

Check in with each group to be sure they are plotting pairs correctly.

- *What patterns do you see in the graph? Explain.*
- *What would it take to have the data points lie in a straight-line?*
- *What pattern of growth do you observe between adjacent points?*

Summarize

The core scientific issue in this Problem is how performance rates change over time.

- *Could you have chosen a different time interval for recording data in a table?*
- *How would your choice have affected your observations in Question B?*
- *What does the experiment suggest about bicycle-riding speed over time?*

Pick a point on one of the graphs and ask:

- *What are its coordinates? What information do the coordinates provide?*

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Assignment Guide for Problem 1.1

Applications: 1–3 | Connections: 14–15
Extensions: 20

Answers to Problem 1.1

A. Student data will vary. In one class, several students started jumping at a rate of 10 jumping jacks for every 10 seconds. After 1 minute, they started to slow down slightly. Many had data entries of 107 and 108 jumping jacks for 120 seconds.

B. Some students will have data that show their jumping jack rate decreases as time passes. Even though the total number of jumps increases for each 10-second interval in the table, the rate decreases since the number of jumps in each 10-second interval decreases as time passes.

C. **1.** The most likely pattern of jumping jack data is greater numbers in the early 10-second intervals than in the later intervals. Since the directions ask students to record the total number of jumping jacks at the end of each 10-second interval (not the number during the preceding 10 seconds), the total will grow more rapidly at first than later. The difference between two adjacent table entries (divided by 10 to get a rate per second) will tell the rate of jumping jacks.

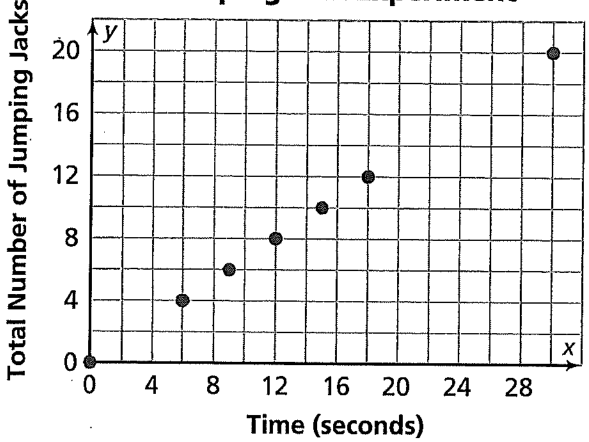
2. On a graph, greater rates will be shown by bigger steps upward from one data point to the next.

D. It seems likely that students will find that their rate of jumping jacks slows near the end of the experimental time period. The analogy to bike riding would suggest that the speed of riding will slow as the day wears on.

1. **a.** If a student jumped at a steady pace of 8 jumping jacks for every 10 seconds, the table of sample *time* and *jumps* data would look like this: (See Figure 1.)

   b. A plot of the points corresponding to (*time, jumping jack total*) pairs in the table will produce a linear pattern with the points rising up 8 for every 10 over.

Jumping Jack Experiment



3. **a.** If a student jumped at a steady pace of 4 jumping jacks for every 6 seconds, the table of sample *time* and *jumps* data would look like this:

Jumping Jack Experiment

Time (seconds)	0	6	9	12	15	18	30
Total Number of Jumping Jacks	0	4	6	8	10	12	20

   b. A plot of the points corresponding to (*time, jumping jack total*) numbers in the table of part (a) would produce a linear pattern with the points rising up 4 for every 6 over. As in part (1), it is a linear pattern except it is not as steep.

Jumping Jack Experiment

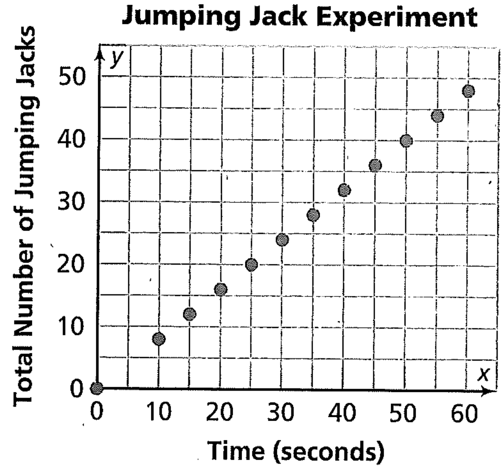


Figure 1

Jumping Jack Experiment

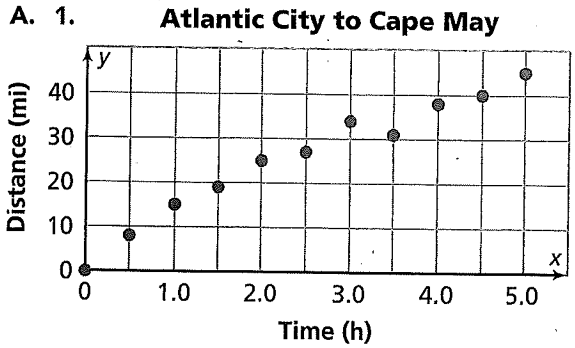
Time (seconds)	0	10	15	20	25	30	35	40	45	50	55	60
Total Number of Jumping Jacks	0	8	12	16	20	24	28	32	36	40	44	48

Assignment Guide for Problem 1.2

Applications: 4–9 | Connections: 16–19
Extensions: 23

Answers to Problem 1.2

A. 1.



2–4. Students will have a variety of observations about the pattern of travel shown in the table and the graph. They will probably notice that the cyclists went faster in the first hour than in later time periods, though the speed from hour 2.5 to hour 3 is equal to that during hour 0.5 to hour 1.0. This kind of observation is easiest to quantify by finding the difference of adjacent distance values in the data table. However, it is shown visually by greater jumps in the plotted points of the graph. Students might notice that there is a 'dip' in the graph between hour 3 and hour 4 and wonder what this could mean. One possible explanation would be that the cyclists made a wrong turn and had to backtrack to get on course. Or, one of the cyclists might have dropped something and had to turn back to retrieve it.

B. 1–3. Adjacent distance values in the table and the amount of jump between adjacent points on the graph show the speed. The data shows greatest speed in the first half-hour (8 miles covered or 16 miles per hour). There are several other half-hour time intervals in which the cyclists covered 7 miles (14 miles per hour).

The slowest riding happens between 2.0 and 2.5 hours (2 miles or 4 miles per hour). The slowdown might have been caused by a break for snack or rest. In some sense, the half hour of backtracking (from 3.0 to 3.5 hours) might be said to be slowest, with a rate of –6 miles per hour. However, the actual riding speed in that half hour was 6 miles per hour.

C. The match of stories and graph connections would be 1(V), 2(I), 3(III), 4(IV), 5(II).

D. Students who choose the table may say that it gives exact distances for given times. Students who choose the graph may say that it shows the "picture" of all the data for the day. It is easy to see from the graph when the change in distance is great, when it is small, and when it does not change. A disadvantage is that you need to estimate some values from a graph, rather than knowing exact values.

- *There are three breaks: at mid-morning, at lunch time, and at around 2:00 P.M. (or around hour 6.0).*
- *The class might assume that when the cyclists load their bicycles in the van, they will cover a greater distance in a shorter time than when they were pedaling.*

End the Summarize by discussing the advantages and disadvantages of each representation for looking at patterns of change in distance over time.

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Assignment Guide for Problem 1.3

Applications: 10–11 | Connections: 21
Extensions: 24–26

Answers to Problem 1.3

A. Here is one possible table:
(See Figure 1.)

B. The data given in Question A will yield a graph like that below. In this situation, it does make sense to connect the dots because riding was continuous over time. However, the way that the dots would be connected is somewhat arbitrary. Failing specific guidance, it usually makes sense to connect the dots with line segments. The result will often reveal an overall pattern that is not as clear with only discrete points plotted.
(See Figure 2.)

C. The data given in Question A and the graph in Question B match the story in the following respects:

- The trip covers 80 miles in 9 hours.
- The break for lunch occurs at noon (four hours after the 8 A.M. start) and no distance is covered in the next hour.
- The speed increases after 2 hours, when the wind shifts to the cyclists' backs.
- The swim break occurs at hour 6 (about 2 P.M.), so the cyclists cover less distance in that hour.
- The cyclists slow down in the afternoon, until they pack their bikes and ride in the van for the last hour to cover 15 miles.

D. Students will have different ideas about which presentation of the data is more informative.

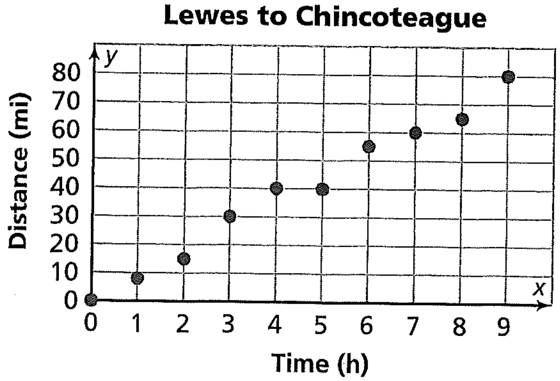
Figure 1

Lewes to Chincoteague

Trip Time (h)	0	1	2	3	4	5	6	7	8	9
Distance (mi)	0	8	15	30	40	40	55	60	65	80

Figure 2

Lewes to Chincoteague



- *How did you decide when the cyclists put their bikes in the van and rode in the van?*
- *How can you tell from the graph where the fastest speed is occurring?*

ACE Assignment Guide for Problem 1.4

Applications: 12–13
Extensions: 22

Answers to Problem 1.4

A. A table of (*time, distance*) data might look like this: (See Figure 1.)

1. The point (3, 25) is fourth from the left. It tells us that after 3 hours the cyclists had covered 25 miles.
2. The points (9, 60) and (10, 110) are respectively 10th and 11th from the left. They tell the distance covered after 9 hours (60 miles) and 10 hours (110 miles). Together, they tell that during the tenth hour, the cyclists (now in the van) traveled at an average speed of 50 miles per hour.
3. The average speed for the trip was approximately 13.2 miles per hour. You can find this from the graph or the table by using the point (11, 145) and finding $145 \div 11$ (the distance divided by the time).

B. Speeds for the biking and van riding parts of the trip:

1. Speed increases dramatically after 9 hours, because the van moves much faster than the bikes.
2. Average bike riding speed was $60 \div 9$ or approximately 6.7 miles per hour.
3. Average van speed was $85 \div 2$ or approximately 42.5 miles per hour.
4. The difference between typical biking and van speed is shown by the fact that the last two data points on the graph jump much higher in one hour than do the earlier data points representing distance traveled by bicycle.

C. 1. If a professional cyclist covered 145 miles in 8 hours, his/her average speed would be $145 \div 8$ or approximately 18.1 miles per hour.

2. Traveling at a constant speed would be reflected in a graph with points that lie in a straight line from (0, 0) to (8, 145).

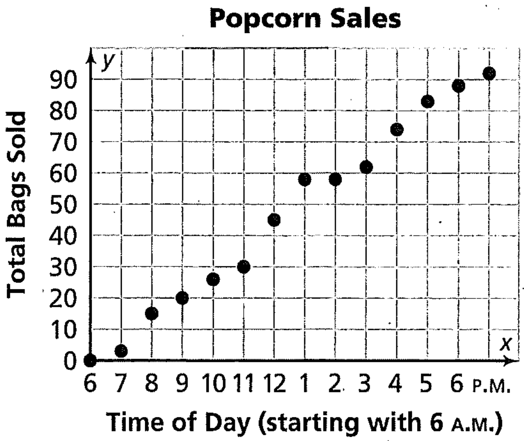
Figure 1

Chincoteague to Williamsburg

Trip Time (h)	0	1	2	3	4	5	6	7	8	9	10	11
Distance (mi)	0	10	20	25	30	30	40	50	55	60	110	145

Applications

1. a.



Possible answer: The scale on the y-axis should range from 0 to 100 because 100 is the greatest number of bags. **Note:** Some students may use a scale from 0 to 14 on the x-axis and say that is easier to write than actual times (e.g., 6:00 A.M.).

b. Sales increase rather steadily throughout the day, with somewhat greater sales between hours 5 (11 A.M.) and 7 (1 P.M.), (the lunch hour), and somewhat slower sales for the following two hours.

c. Greatest sale (15 bags) was in the sixth hour (11 A.M. to 12 P.M.); least sales (0 bags) in the eighth hour (1 P.M. to 2 P.M.).

2. Jamil's graph shows a common student error/misconception. He has plotted the data points at equal intervals on the x-axis, even though the data do not represent population at equally spaced time intervals.

a. Ming's graph and interpretation are accurate. The population has grown much more rapidly (approximately double the earlier rate) in the past 15 years.

b. Jamil's graph and interpretation are flawed because he has failed to notice that the last three data points should not be equally spaced horizontally.

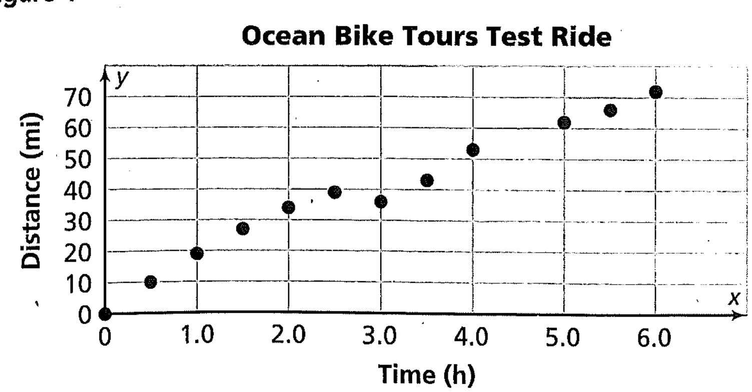
3. **a.** High sales probably occur just before school starts (when many kids are eating breakfast at school), at lunch, and just as school ends. Low sales probably occur while most students are in class.

b. It probably does make sense to connect the points because sales are likely distributed throughout the hours (albeit not uniformly so).

4. **a.** (See Figure 1.)

b. Fastest riding was in the first half-hour and the half-hour between 3.5 and 4.0 hours. In those time intervals, the riders covered 10 miles for an average speed of 20 miles per hour. By comparing adjacent entries in the data table, you can find the greatest increase in a half-hour. The greatest increases are shown on the graph by the largest jumps upward from one point to the next.

Figure 1



c. Slowest riding was in the half-hour between 2.5 and 3.0 hours (when the riders were actually backtracking on their trip.) In that time interval, the riders covered 3 miles for an average speed of 6 miles per hour. By comparing adjacent entries in the data table, you can find the least increase in a half-hour. The least increases are shown on the graph by the smallest jumps upward from one point to the next. In this case, the jump is actually down, but the absolute value of the distance is 3.

d. The overall pattern in the data (excepting the backtrack interval) shows gradual slowing over the first half of the trip and again over the second half of the trip. This pattern is more apparent in the table than the graph. **Note:** There is a missing data point at time 4.5 hours.

e. The dip might be caused by a wrong turn that required backtracking or by something dropped and only realized later down the road, requiring backtracking to pick up the lost item.

5. a. No, Ken is not correct. His graph only seems to go higher because he has chosen a different scale on the *y*-axis. Both Andrea and Ken completed 90 sit-ups.

b. These data show a gradual slowing of the rate of sit-ups, like what you would see (and expect) in the bike ride.

c. Overall, both Andrea and Ken had the same pace. Andrea's pace = $\frac{90 \text{ sit-ups}}{10 \text{ minutes}}$ = 9 sit-ups per minute. Ken's pace = $\frac{90 \text{ sit-ups}}{10 \text{ minutes}}$ = 9 sit-ups per minute.

d. Ken's pace in the first two seconds (approximately) = $\frac{38 \text{ sit-ups}}{2 \text{ minutes}}$ = 19 sit-ups per minute.

Ken's pace in the last two seconds (approximately) = $\frac{10 \text{ sit-ups}}{2 \text{ minutes}}$ = 5 sit-ups per minute.

6. a. (See Figure 2.)

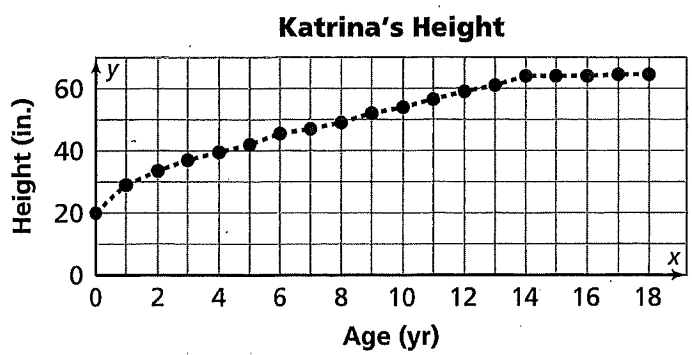
b. between birth and age 1 (9 inches)

c. from age 14 to 16 and from age 17 to 18

d. It makes sense to connect the points because growth occurs between birthdays. **Note:** The question of how these points should be connected, by line segments or a curve, is another point of discussion.

e. Answers will vary. The exact change in height is easier to read from the table. However, students may argue that the graph provides a better overall picture.

Figure 2



7. a. 6 hours after midnight, or 6:00 A.M.; 16.2 meters

   b. noon; 10.0 meters

   c. Water depth changes most rapidly (by 1.7 meters per hour) between 2 A.M. and 3 A.M., between 8 A.M. and 9 A.M., between 2 P.M. and 3 P.M., and 1.3 between 8 P.M. and 9 P.M. This pattern shows the physical property of tides that they move most swiftly at points halfway between high and low tides (which occur roughly every six hours).

   d. (See Figure 3.)

   e. Choice of scales is a decision between informative and cluttered. A scale of 1 on either axis would yield tick marks and grid lines that are pretty dense. A scale of 6 on the x-axis would be a bit crude for identifying interesting points in the pattern of tidal effect. However, there is no correct answer for this choice.

8. a. Each graph shows a steady growth in the Huntsville population from about 10,000 in 1996 to about 14,000 in 2004.

   b. The graphs seem to show the same data values. Only the different choices of scales cause the different appearances of the graphs. However, by making irregular y-axis scales, Graphs A and B give impressions that are misleading.

9. a. (See Figure 4.)

   b. The difference between the highest and lowest temperatures is about 35°F (from 50°F to 85°F).

   c. The temperature rose at a rate of about 20 degrees per hour (actually 10 degrees per half-hour) between 1.5 and 2.0 hours, between 2.0 and 2.5 hours, and again between 4.0 and 4.5 hours. The temperature fell at a rate of about 20 degrees per hour (actually 10 degrees per half-hour) in the first half-hour and again between 2.5 and 3.0 hours.

Figure 3

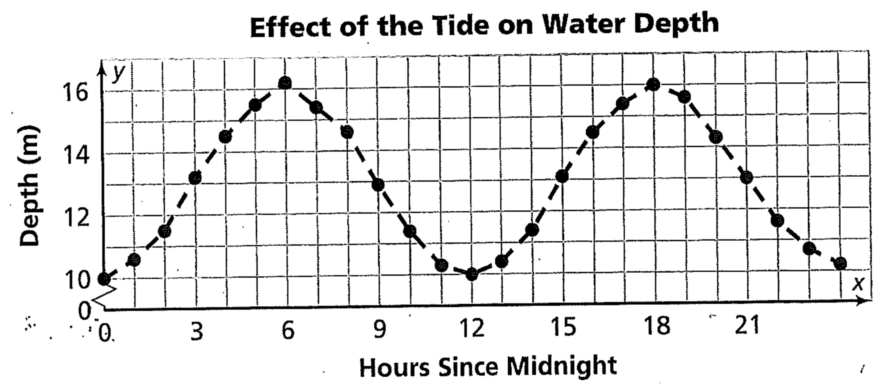


Figure 4

Temperatures for Day 1

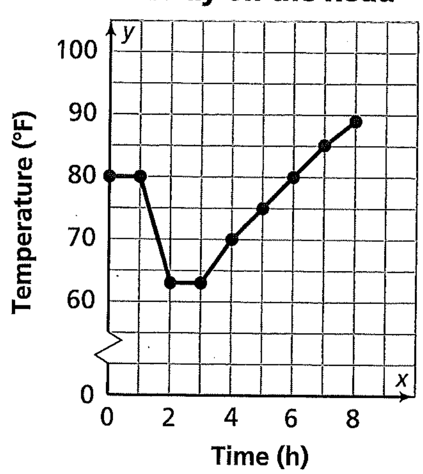
Time (h)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Temperature (°F)	60	50	55	60	70	80	70	65	70	80	85

d. Answers will vary, but students will probably express some preference for actually having numerical data to inspect for answers to questions like those in parts (b) and (c). Graphs tend to give overall pictures that are easier to remember in a global sense.

e. Connecting the points shows the temperature changing at a steady rate between half-hour marks. It makes sense to connect the points because time is a continuous variable, so there will be temperatures after 15 minutes, after 37 minutes, and so on. The information may not be completely accurate because the temperature may not have changed at a constant rate. However, it is useful for making estimates.

10. Answers will vary, but the table and the graph should show that it was warm at 8 A.M. (at time = 0 hours). Then, the temperature decreased rapidly to 63°F by midmorning and stayed constant for about an hour. After this, the temperature increased until it reached 89°F at 4 P.M. (See Figure 5.)

A Day on the Road



11. Graph I shows Amanda's hunger and Graph II shows her happiness. The increases are quite gradual with hunger, and the decreases are rather sudden when Amanda eats. The graph for Amanda's happiness shows that she can stay at the same level of happiness for a while, such as when she is having fun at basketball practice from 4 P.M. to 6 P.M.

12. a. The graph shows something moving at a constant speed over a period of time.

b. Possible answer: The graph is not reasonable for a cyclist or for the wind under normal conditions. A rider's speed can be affected by fatigue or environmental factors such as temperature, wind speed or direction, and terrain. A van could travel close to a constant speed on a flat surface. The wind usually comes in gusts. It does not seem that it would remain constant over a long period of time. **Note:** Some students might answer that they do not know what the scale is. So if a small amount of space on the y-axis means millions and millions, then this graph is possible for the cyclist, the van, or the wind because their small amount of speeding up and slowing down would not show up on the graph.

13. a. 43.125 miles per hour

b. 50 miles per hour

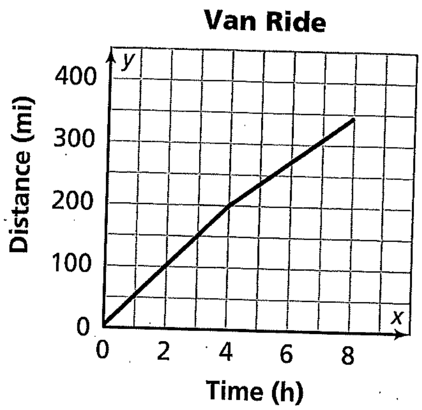
c. 36.25 miles per hour

Figure 5

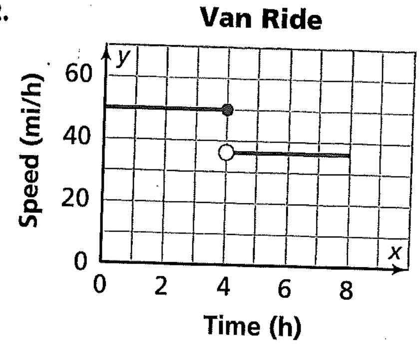
A Day on the Road

Time (h)	0	1	2	3	4	5	6	7	8
Temperature (°F)	80	80	63	63	70	75	80	85	89

d. 1.

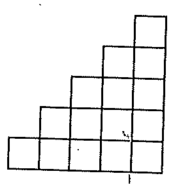


2.



Connections

14. a.



b. (See Figure 6.)

c. There are several ways to describe the pattern of increase in total number of squares. As the number of squares in the bottom row increases from n to $n + 1$, the total number of squares increases by $n + 1$. One formula for the total number of squares on a base of length n is $T = \frac{n(n+1)}{2}$ or $T = \frac{n^2}{2} + \frac{n}{2}$.
Note: It is not expected that students come up with this formal rule.

15. (See Figure 7.)

16. 0.25, 0.5, 0.75, 1.00, 1.25, 1.5, 1.75, 2.0; add 0.25.

17. $\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}$; 1; add $\frac{1}{8}$.

18. $\frac{1}{6}, \frac{1}{3}, \frac{4}{6} = \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{32}{6} = \frac{16}{3}$; multiply by 2.

Figure 6

Squares in a Pattern

Number of Squares in Bottow Row	1	2	3	4	5	6	7	8	9	10
Total Number of Squares	1	3	6	10	15	21	28	36	45	55

Figure 7

Cubes in a Pyramid

Width of Base	3	5	7	9	11	13	15
Total Number of Cubes	10	35	84	165	286	455	680

19. **a.** (See Figure 8.)

b. Answers will vary. Some students may note that the 3-day tour is the most preferred length and surmise that a 3-day trip is the best option. However, other students may observe that half of the most popular tours are shorter than 5 days, and half are longer than 5 days, so a 5-day tour is the average length and would be a popular option.

20. **a.** Data relating approximate annual salary to level of schooling completed will look like that in the following table: (See Figure 9.)

b. The greatest increases occur after 12 and 16 years of education. This is probably because a diploma qualifies a person for higher-paying jobs. (You may want to point out to students that these are not starting salaries. Some of these people have been in their field for years. The participants of this study are people over 25.)

c. Answers will vary. It is often easier to see changes, or jumps, in a graph, but it is easier to use a table to find the exact amount of those changes.

21. Answers will vary. Samples: student's height or weight over time, number of friends over time, amount of spending money over time. The key is getting students to explain how their tables and graphs tell a story about change over time of some quantity that interests them. Check students' graphs.

Figure 8

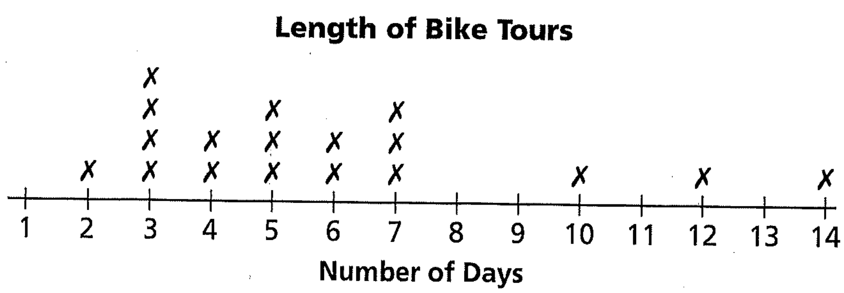


Figure 9

Education and Salary

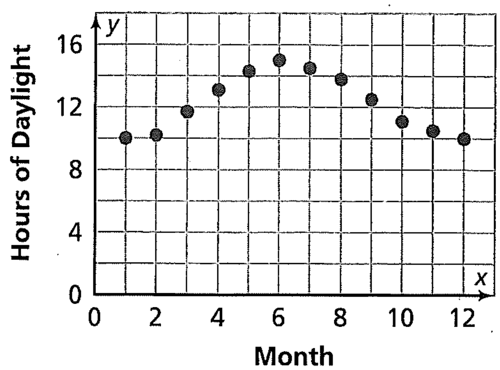
Years of Education	8	9	10	11	12	13	14	15	16
Average Salary	$12,500	$14,000	$16,500	$17,500	$28,000	$30,500	$34,000	$36,000	$49,000

Extensions

22. a. The hours of daylight increase from a minimum of 10 in January to a maximum of 15 in June (midsummer) and then symmetrically decrease to the winter minimum of 10 in December. Since the winter solstice occurs in the latter part of December and the summer solstice in the latter part of June, these patterns make sense.

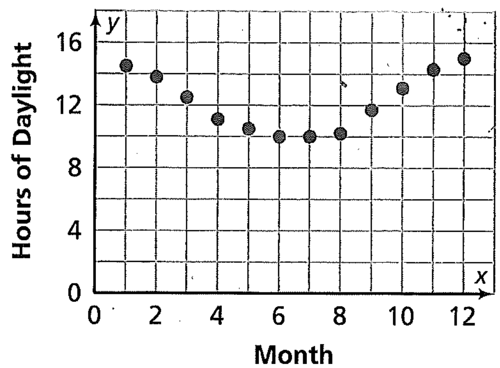
b. A graph of the (*month, daylight hours*) data illustrates the pattern described in part (a).

Daylight in Chicago



c. The daylight hour pattern that you would expect in Melbourne is symmetric to and opposite of that in Chicago. A graph would look like this:

Daylight in Melbourne



d. (See Figure 10.)

23. a. Answers will vary. Students might make a reasonable argument for any of the graphs. Yet, some graphs seem to be better than others. The following arguments assume that the intersection of the x- and y-axes is point (0, 0) on all graphs. Unlike Graph I, Graphs II, III, and IV show that there is a price that results in the maximum profit. Graph IV is a better representation because Graph III shows the unlikely event of making a profit at a very low price for each shirt. Students might draw a more detailed graph that shows a negative profit (loss) when the price is too low.

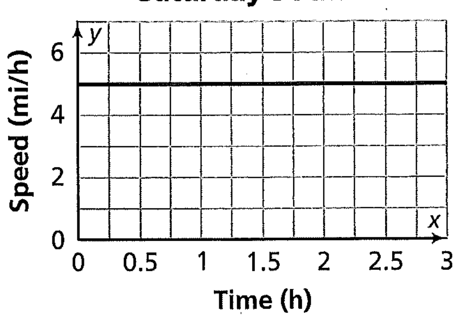
b. Possible answers: selling price, price the club must pay for the sweatshirts, the location and times the club chooses for selling sweatshirts, and customer demand (which might depend on other variables such as income and weather)

Figure 10

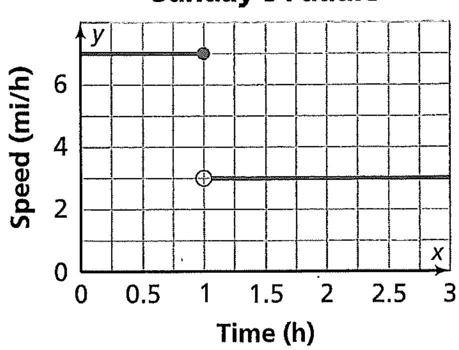
Daylight in Melbourne

Month	1	2	3	4	5	6	7	8	9	10	11	12
Daylight Hours	14.5	13.8	12.5	11.0	10.5	10.0	10.0	10.2	11.7	13.1	14.3	15.0

24. a.



b.



c. Three hours of lake paddling would carry them a total distance of 13 miles. However, the up- and downriver paddling would carry them 7 miles downstream (5 + 2 = 7 miles per hour for 1 hour) but only 6 miles back upstream (5 − 2 = 3 miles per hour for 2 hours), leaving them still 1 mile downstream from their starting point.

25. Answers will vary. Possible answers:

a. You would expect that the greater the distance from school to your home, the longer it will take to walk home from school. For example:

Walking Home From School

Distance (mi)	Time (min)
$\frac{1}{4}$	5
$\frac{1}{2}$	10
$\frac{3}{4}$	15
1	20
$1\frac{1}{2}$	30

b. You would expect the number of bags of popcorn sold at a theater to decrease as the price increases. For example:

Popcorn Sales at a Theater

Price of Popcorn at Theater	Number of Bags Sold
$2	50
$4	40
$6	30
$8	20
$10	10

c. You would expect the time it takes a plane to complete a 500-mile trip to decrease as the speed increases. For example:

Airplane Trip

Speed (mi/h)	Time (h)
100	5
125	4
150	3.33
175	2.86
200	2.5

d. You would expect a monthly cell phone bill to increase as the number of text messages sent increases. For example:

Monthly Cell Phone Bill

Number of Text Messages	Cost of Bill
1,000	$30
2,000	$40
3,000	$50
4,000	$60
5,000	$70

e. You would expect the cost of a long-distance telephone call to increase as the length of the call increases. For example:

Long–Distance Telephone Costs

Length (min)	Cost
1	$ .30
5	$1.50
10	$3.00
15	$4.50
20	$6.00

In practice, an overall use plan can set a fixed cost for any number up to a certain (usually large) number of text messages or calls.

26. **a.** after 10 seconds, 10 jumping jacks; after 20 seconds, 20 jumping jacks; after 60 seconds, 40 jumping jacks

b. Other points show 5 jumping jacks after 5 seconds, 15 after 15 seconds, 25 after 25 seconds, 30 after 35 seconds, 35 after 45 seconds.

c. Linear interpolation would predict 27.5 after 30 seconds, 32.5 after 40 seconds, and 36.3 after 50 seconds. **Note:** It is not expected that students use the term *interpolation*.

d. The rate of jumping jacks decreases as time passes.

e. Connecting the first and last points of the graph with a straight line segment would suggest a constant rate of jumping jacks, a pattern that is quite different from that shown by the actual data plot.