At a Glance

Problem 1.1 Pacing $1\frac{1}{2}$ Days

1.1 Getting Ready to Ride: Data Tables and Graphs

Focus Question How can you construct a graph from a table of data that depicts change over time? How is this pattern of change represented in the graph?

Launch

Tell the class about bicycles and the yearly bicycle tour across Iowa.

Ask for some predictions about the jumping jack stamina experiment:

- *How many jumping jacks do you think you could complete in 2 minutes?*
- *How do you think your jumping jack rate would change over the 2-minute test?*

Materials

Labsheets

- 1.1A: Jumping Jack Fitness Test
- 1.1B: Jumping Jack Tables and Graphs
- Graph Paper
- stopwatches (1 per group)

Explore

Check in with each group to be sure they are plotting pairs correctly.

- *What patterns do you see in the graph? Explain.*
- *What would it take to have the data points lie in a straight-line?*
- *What pattern of growth do you observe between adjacent points?*

Summarize

The core scientific issue in this Problem is how performance rates change over time.

- *Could you have chosen a different time interval for recording data in a table?*
- *How would your choice have affected your observations in Question B?*
- *What does the experiment suggest about bicycle-riding speed over time?*

Pick a point on one of the graphs and ask:

- *What are its coordinates? What information do the coordinates provide?*

ACE Assignment Guide for Problem 1.1

Applications: 1–3 | Connections: 14–15
Extensions: 20

Answers to Problem 1.1

A. Student data will vary. In one class, several students started jumping at a rate of 10 jumping jacks for every 10 seconds. After 1 minute, they started to slow down slightly. Many had data entries of 107 and 108 jumping jacks for 120 seconds.

B. Some students will have data that show their jumping jack rate decreases as time passes. Even though the total number of jumps increases for each 10-second interval in the table, the rate decreases since the number of jumps in each 10-second interval decreases as time passes.

C. **1.** The most likely pattern of jumping jack data is greater numbers in the early 10-second intervals than in the later intervals. Since the directions ask students to record the total number of jumping jacks at the end of each 10-second interval (not the number during the preceding 10 seconds), the total will grow more rapidly at first than later. The difference between two adjacent table entries (divided by 10 to get a rate per second) will tell the rate of jumping jacks.

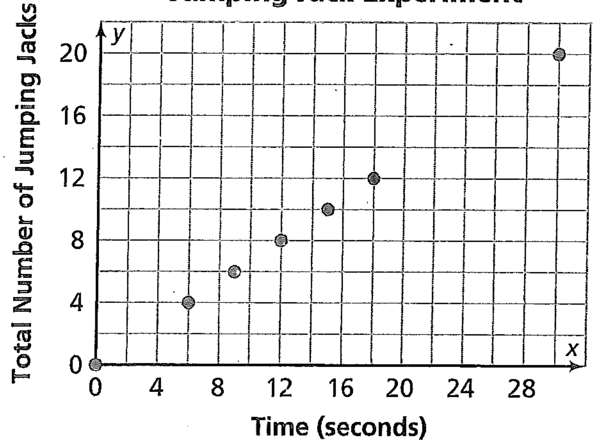
2. On a graph, greater rates will be shown by bigger steps upward from one data point to the next.

D. It seems likely that students will find that their rate of jumping jacks slows near the end of the experimental time period. The analogy to bike riding would suggest that the speed of riding will slow as the day wears on.

1. a. If a student jumped at a steady pace of 8 jumping jacks for every 10 seconds, the table of sample *time* and *jumps* data would look like this: (See Figure 1.)

b. A plot of the points corresponding to (*time, jumping jack total*) pairs in the table will produce a linear pattern with the points rising up 8 for every 10 over.

Jumping Jack Experiment



3. a. If a student jumped at a steady pace of 4 jumping jacks for every 6 seconds, the table of sample *time* and *jumps* data would look like this:

Jumping Jack Experiment

Time (seconds)	0	6	9	12	15	18	30
Total Number of Jumping Jacks	0	4	6	8	10	12	20

b. A plot of the points corresponding to (*time, jumping jack total*) numbers in the table of part (a) would produce a linear pattern with the points rising up 4 for every 6 over. As in part (1), it is a linear pattern except it is not as steep.

Jumping Jack Experiment

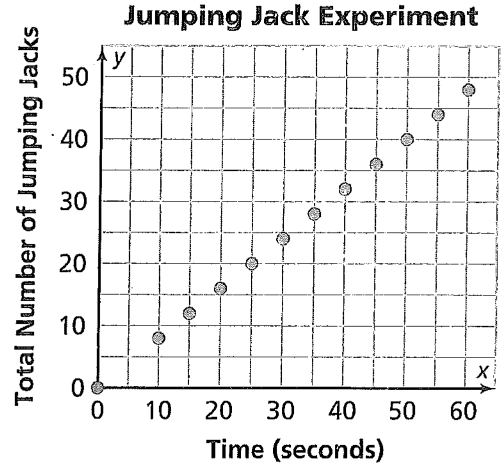


Figure 1

Jumping Jack Experiment

Time (seconds)	0	10	15	20	25	30	35	40	45	50	55	60
Total Number of Jumping Jacks	0	8	12	16	20	24	28	32	36	40	44	48

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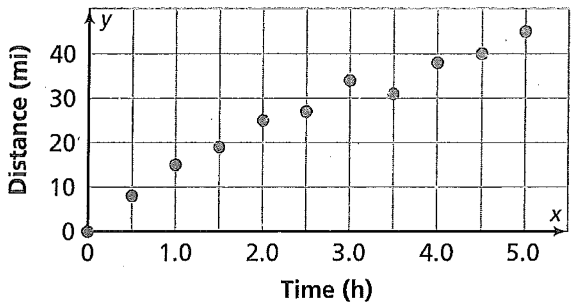
Assignment Guide for Problem 1.2

Applications: 4–9 | Connections: 16–19
Extensions: 23

Answers to Problem 1.2

A. 1. **Atlantic City to Cape May**



2–4. Students will have a variety of observations about the pattern of travel shown in the table and the graph. They will probably notice that the cyclists went faster in the first hour than in later time periods, though the speed from hour 2.5 to hour 3 is equal to that during hour 0.5 to hour 1.0. This kind of observation is easiest to quantify by finding the difference of adjacent distance values in the data table. However, it is shown visually by greater jumps in the plotted points of the graph. Students might notice that there is a 'dip' in the graph between hour 3 and hour 4 and wonder what this could mean. One possible explanation would be that the cyclists made a wrong turn and had to backtrack to get on course. Or, one of the cyclists might have dropped something and had to turn back to retrieve it.

B. 1–3. Adjacent distance values in the table and the amount of jump between adjacent points on the graph show the speed. The data shows greatest speed in the first half-hour (8 miles covered or 16 miles per hour). There are several other half-hour time intervals in which the cyclists covered 7 miles (14 miles per hour).

The slowest riding happens between 2.0 and 2.5 hours (2 miles or 4 miles per hour). The slowdown might have been caused by a break for snack or rest. In some sense, the half hour of backtracking (from 3.0 to 3.5 hours) might be said to be slowest, with a rate of –6 miles per hour. However, the actual riding speed in that half hour was 6 miles per hour.

C. The match of stories and graph connections would be 1(V), 2(I), 3(III), 4(IV), 5(II).

D. Students who choose the table may say that it gives exact distances for given times. Students who choose the graph may say that it shows the "picture" of all the data for the day. It is easy to see from the graph when the change in distance is great, when it is small, and when it does not change. A disadvantage is that you need to estimate some values from a graph, rather than knowing exact values.

- *There are three breaks: at mid-morning, at lunch time, and at around 2:00 P.M. (or around hour 6.0).*
- *The class might assume that when the cyclists load their bicycles in the van, they will cover a greater distance in a shorter time than when they were pedaling.*

End the Summarize by discussing the advantages and disadvantages of each representation for looking at patterns of change in distance over time.

ACE Assignment Guide for Problem 1.3

Applications: 10–11 | Connections: 21
Extensions: 24–26

Answers to Problem 1.3

A. Here is one possible table:
(See Figure 1.)

B. The data given in Question A will yield a graph like that below. In this situation, it does make sense to connect the dots because riding was continuous over time. However, the way that the dots would be connected is somewhat arbitrary. Failing specific guidance, it usually makes sense to connect the dots with line segments. The result will often reveal an overall pattern that is not as clear with only discrete points plotted.
(See Figure 2.)

C. The data given in Question A and the graph in Question B match the story in the following respects:

- The trip covers 80 miles in 9 hours.
- The break for lunch occurs at noon (four hours after the 8 A.M. start) and no distance is covered in the next hour.
- The speed increases after 2 hours, when the wind shifts to the cyclists' backs.
- The swim break occurs at hour 6 (about 2 P.M.), so the cyclists cover less distance in that hour.
- The cyclists slow down in the afternoon, until they pack their bikes and ride in the van for the last hour to cover 15 miles.

D. Students will have different ideas about which presentation of the data is more informative.

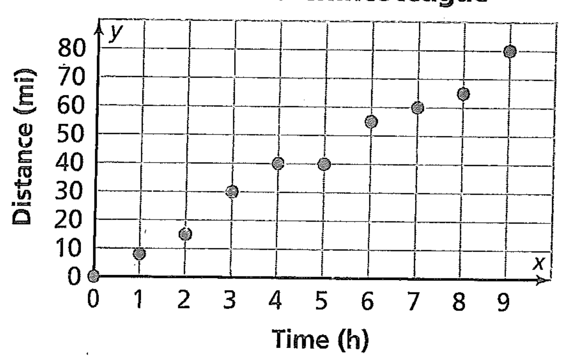
Figure 1

Lewes to Chincoteague

Trip Time (h)	0	1	2	3	4	5	6	7	8	9
Distance (mi)	0	8	15	30	40	40	55	60	65	80

Figure 2

Lewes to Chincoteague



- *How did you decide when the cyclists put their bikes in the van and rode in the van?*
- *How can you tell from the graph where the fastest speed is occurring?*

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Assignment Guide for Problem 1.4

Applications: 12–13
Extensions: 22

Answers to Problem 1.4

A. A table of (*time*, *distance*) data might look like this: (See Figure 1.)

1. The point (3, 25) is fourth from the left. It tells us that after 3 hours the cyclists had covered 25 miles.
2. The points (9, 60) and (10, 110) are respectively 10th and 11th from the left. They tell the distance covered after 9 hours (60 miles) and 10 hours (110 miles). Together, they tell that during the tenth hour, the cyclists (now in the van) traveled at an average speed of 50 miles per hour.
3. The average speed for the trip was approximately 13.2 miles per hour. You can find this from the graph or the table by using the point (11, 145) and finding 145 ÷ 11 (the distance divided by the time).

B. Speeds for the biking and van riding parts of the trip:

1. Speed increases dramatically after 9 hours, because the van moves much faster than the bikes.
2. Average bike riding speed was 60 ÷ 9 or approximately 6.7 miles per hour.
3. Average van speed was 85 ÷ 2 or approximately 42.5 miles per hour.
4. The difference between typical biking and van speed is shown by the fact that the last two data points on the graph jump much higher in one hour than do the earlier data points representing distance traveled by bicycle.

C. 1. If a professional cyclist covered 145 miles in 8 hours, his/her average speed would be 145÷8 or approximately 18.1 miles per hour.

2. Traveling at a constant speed would be reflected in a graph with points that lie in a straight line from (0, 0) to (8, 145).

Figure 1

Chincoteague to Williamsburg

Trip Time (h)	0	1	2	3	4	5	6	7	8	9	10	11
Distance (mi)	0	10	20	25	30	30	40	50	55	60	110	145

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Assignment Guide for Problem 2.1

Applications: 1–3 | Connections: 17–19

Answers to Problem 2.1

A. **1.** For 20 bikes, Rocky will charge $770 and Adrian will charge $600.

2. For 40 bikes, Rocky will charge $1,140 and Adrian will charge $1,200.

3. For 32 bikes, Rocky will charge $1,013 and Adrian will charge $960 (assuming linear interpolation between given data points).

B. **1.** If a group has $900 to spend, it can rent 26 bikes from Rocky or 30 from Adrian.

2. If a group has $400 to spend, it can rent 5 bikes from Rocky or 13 from Adrian.

C. Students should realize that for both shops rental cost increases steadily as number of bikes increases.

1. The cost for renting from Rocky starts higher but increases more slowly than cost for renting from Adrian; Rocky's charge per bike decreases as the number of bikes increases. In fact, if one makes a coordinate graph of the costs for renting from Rocky, the pattern will resemble the data for jumping jacks or riding bicycles, in which the data points rise rapidly at first, but that rate of increase drops off.

2. If students study the data very carefully, they'll discover that Adrian charges a flat rate of $30 per bike.

D. Students probably will not use the word *interpolate*, but they should understand that estimating costs for numbers of bicycles not on the graph or in the table depends on assuming given points provide upper and lower bounds for the estimate.

E. Some students may point out that costs for specific numbers of bicycles are easier to read from a table. The general trend in pricing is easier to read from a graph.

F. Students will have different opinions about which data representation—table or graph is most useful in making a decision and in presenting the case for one's choice. Given the data in the table and the graph, students should decide that Adrian offers a better deal for a number of bikes less than about 36 and Rocky offers the better deal for more than 36 bikes (though interpolation between 35 and 40 bikes is not certain in Rocky's offer).

2.2 Finding Customers: Linear and Nonlinear Patterns

Focus Question How are the relationships between independent and dependent variables in this Problem different from those in Problem 2.1? How are the differences shown in tables and graphs of data?

Launch

Review the economic terms involved—price is what each customer will pay to take part in the Bike Tour and income is the money that the tour business will take in from those customers.

Explore

Keep an eye on how students choose the independent and dependent variables. Also encourage students to describe exactly what happens to the number of customers as the price increases to $150, $200, and so on. Ask them to provide reasons.

You want students to think hard about how to choose independent and dependent variables and scales for a graph.

Summarize

The key objectives of this Problem are to illustrate examples of a decreasing linear relationship and a quadratic relationship that has a maximum value for the dependent variable (a concave down parabola graph).

- *For each price, how much money would be collected if the people who said they would pay that price actually do so?*
- *How does the number of customers change as the price increases?*
- *How is that pattern shown in the table and the graph?*
- *How is the relationship between price and income different from all preceding relationships? How is that difference shown in the table and graph of (price, income) data?*

Key Vocabulary

- income

Materials

Labsheets
- 2.2A: Finding Customers
- 2.2B: Predicting Income

- grid paper
- Coordinate Grapher Tool
- Data and Graphs Tool

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Assignment Guide for Problem 2.2

Applications: 4–7 | Connections: 20–21
Extensions: 22

Answers to Problem 2.2

A. **1.** The number of people who said they would take the tour depends on the price. In this Problem, price is the dependent variable and the number of the customers is the independent variable. (See Figure 1.)

2. The number of customers decreases steadily as the price increases—10 lost customers for every increase of $100 in price.

3. The change in number of customers is shown in the table by the decrease in that row. The change is shown in the graph by the downward (left-to-right) slope of the points and the connecting line.

4. You could find the number of customers for a price between two entries in the table by proportional reasoning. For example, if the price in the middle of the interval is half of the way from one price to the next higher price in the table, then the number of customers will be half of the way from one number to the next lower number in the table. For $175, one would expect 32.5 customers (of course, you can't have exactly 32.5 customers). For $325 one would expect 17.5 customers.

B. 1. (See Figure 2.)

2. (See Figure 3.)

3. As tour price increases, tour income increases until it reaches a maximum of $6,250 at a tour price of $250. Then it begins to decrease until it reaches $0 when the tour price is set at $500. (There are no willing customers.)

Based on the data and graph in parts (1) and (2), it looks like a tour price of about $250 is optimal. It will yield tour income of $6,250.

Figure 1

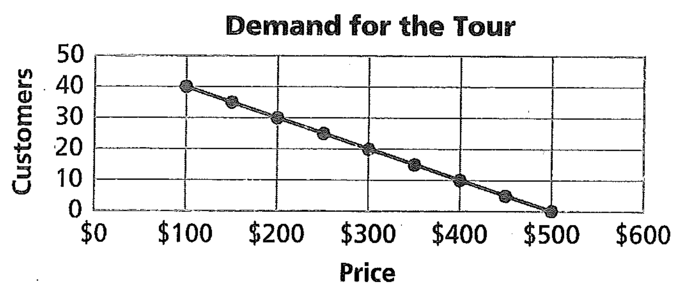
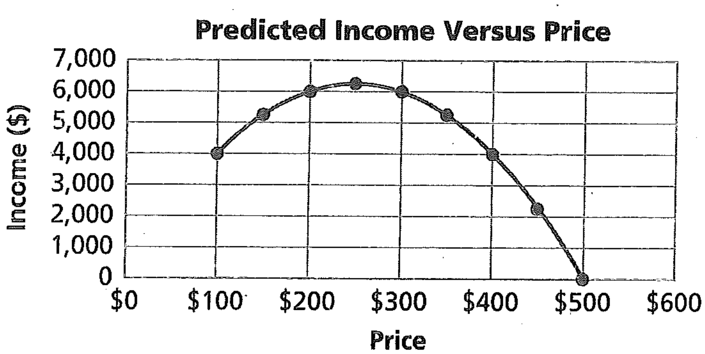


Figure 2

Predicted Tour Income

Tour Price	$100	$150	$200	$250	$300	$350	$400	$450	$500
Number of Customers	40	35	30	25	20	15	10	5	0
Tour Income ($)	4,000	5,250	6,000	6,250	6,000	5,250	4,000	2,250	0

Figure 3



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Assignment Guide for Problem 2.3

Applications: 8–9 | Connections: 23

Answers to Problem 2.3

A. 1. (See Figure 1.)

2. (See Figure 2.)

3. a. The pattern in the table and graph shows business profit starting as a negative value (or loss) for a tour price of only \$100. Then profit increases rapidly to a maximum of a bit more than \$3,000 for tour prices around \$325. Then profit declines as prices increase beyond \$350 per customer.

b. The pattern should make some sense. If the price is too low, there will be many customers and large operating costs, but too little income to pay those costs (thus the loss of \$2,000 for tour price of \$100). As the price increases, the number of customers decreases, but so does the operating cost per customer. However, when the price rises too far, it leads to a loss of customers and income that overwhelms the associated decrease in operating costs.

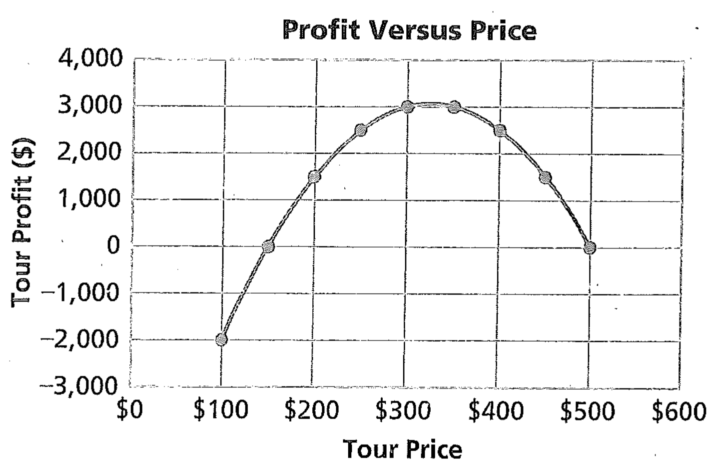
c. Based on the analysis of profit predictions, a tour price of about \$325 seems best because both the table and graph suggest this will yield maximum profit.

Figure 1

Predicted Tour Profit

Tour Price	$100	$150	$200	$250	$300	$350	$400	$450	$500
Customers	40	35	30	25	20	15	10	5	0
Tour Income ($)	4,000	5,250	6,000	6,250	6,000	5,250	4,000	2,250	0
Operating Cost ($)	6,000	5,250	4,500	3,750	3,000	2,250	1,500	750	0
Tour Profit or Loss	–2,000	0	1,500	2,500	3,000	3,000	2,500	1,500	0

Figure 2



B. **1.** The x-values represent the day the temperature occurs. The point (0,−1) corresponds to the day 0, or the day the partners are checking the forecasts. The value −1 for x represents the day before, and the value +1 for x represents the predicted temperature for the next day, etc.

2. (See Figure 3.)

3. There is no predictable pattern. The students might say that the temperatures are cold.

C. **1.** **a.** To reach (−3, 4) from (3, 4), move directly across the y-axis horizontally.

b. To reach (−3, −4) from (3, 4), move directly through the origin to a point equidistant on the other side of (0, 0).

c. To reach (3, −4)from (3, 4), move directly across the x-axis vertically

d. To reach (1.5,−2) from (3, 4), move 1.5 units to the left and 6 units down.

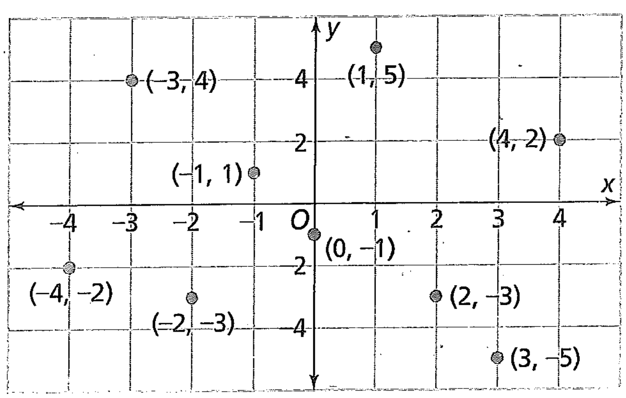
e. To reach (−1.5, 2) from (3, 4), move 4.5 units to the left and 2 units down.

f. To reach (−2.5 − 3.5) from (3, 4), move 5.5 units to the left and 7.5 units down.

D. **1.** Jakayla is correct. (−3, 4) is a reflection of (3, 4) across the y-axis, since the x-coordinate changes from 3 to 3. Similarly, (3, −4) is a reflection of (3, 4) across the x-axis. (−3, −4) is a reflection of (3, 4) across the origin.

2. Mitch is correct. The x-axis and y-axis are like mirrors. The image, or reflection, of (−3, 4) over the y-axis is (3, 4). The same is true for the other points. The mirror image is also a reflection.

Figure 3



Assignment Guide for Problem 2.4

Applications: 10–16 | Extensions: 24

Answers to Problem 2.4

A. Graph 2: The number of students who go on a school trip depends on the price of the trip for each student. The pattern in graph 2 seems most likely to represent the relationship of those variables because it shows number of students going on the trip declining as price increases.

B. Graph 6: When a skateboard rider goes down one side of a half pipe ramp and up the other side, her speed increases until she reaches the bottom and then decreases as she rides up the other side. Despite the appeal of graph 5 (it shows a half-pipe shape), the graph best showing speed over time in this case is graph 6.

C. Graph 1 is probably best for showing how water level changes over time when someone fills a tub, takes a bath, and empties the tub. Graph 1 shows water entering the tub at a constant rate for a time; then a short interval when the bather allows the water to cool a bit; then a jump in water level when the bather enters the tub; then a period of constant depth; then a quick drop when the bather gets out of the tub; and finally a steady decline as water is drained out of the tub.

D. Graph 4: The waiting time for a popular ride at an amusement park is related to the number of people in the park. Graph 4 shows that waiting time increases as the number of people increases.

E. Graph 8: The daily profit or loss of an amusement park increases as the number of paying customers increases. Graph 8 shows this. It starts with a negative value for profit because there will still be expenses if there are no customers.

F. Graph 3: As the seasons change, the number of hours of daylight changes in a periodic pattern illustrated best by Graph 3.

G. Graph 7: The daily profit or loss of an outdoor skating rink depends on the daytime high temperature. For temperatures that are either too high or too low, the number of customers and thus the daily profit will decline and even become a loss. For temperatures in the cool-but-not-too-cold range, the rink will make a profit. This pattern is shown best by Graph 7.

H. Graph 2: Weekly attendance at a popular movie declines as time passes from the date the movie first appears in theaters. The most common pattern is that shown in Graph 2.

You might extend students' understanding by asking them to evaluate the expression for the dependent variable for a given value of the independent variable.

Have students use the equations they wrote to answer each of the following questions:

- *In Question A, what is the discounted price if the regular price is $166?*
- *In Question B, what is the sales tax on a tour that costs $200?*
- *In Question C, how far can you bike at 20 miles per hour if you travel 7.5 hours?*
- *In Question D, how long will it take to drive 350 miles if the van is traveling at 55 miles per hour?*

Because students will probably not give sophisticated answers to the Focus Question, you might actually pose the Focus Question in more concrete form with a specific value of k. For example, you might say, "What can you tell about how two variables are related if the equation $y = x + 3$ is given?" and expect answers like, "To get the value of y you always add 3 to the value of x" or "The value of y is always 3 more than the value of x."

ACE

Assignment Guide for Problem 3.1

Applications: 1 | Connections: 22–25

Answers to Problem 3.1

A. **1.** With a $50 discount, the new prices would be $350, $450, and $600 respectively.

2. $D = P - 50$

B. **1.** With 6% sales tax rate, the taxes would be $24, $30, and $39 respectively. (**Note:** It might be useful to point out that sales tax calculations that do not yield exact dollars and cents (e.g. 6% of $3.05 = $.183) are generally rounded to the next higher penny, not the nearest penny.)

2. $T = 0.06P$

C. **1.** Travelling at 20 miles per hour, the cyclist would cover 40 miles in 2 hours, 60 miles in 3 hours, and 70 miles in 3.5 hours.

2. There is a constant rate of increase in the distance traveled depending on time. An equation is $d = 20t$.

3. As time increases by 1 hour, distance increases by 20 miles.

D. **1.** Time will be 8.75 hours (8 hours and 45 minutes) at 40 miles per hour, 7 hours at 50 miles per hour, and 5.8333... hours (5 hours and 50 minutes) at 60 miles per hour.

2. $t = \frac{350}{s}$ or $t = 350 \div s$. It might be useful to point out the use of fraction notation for division as in $t = \frac{350}{s}$.

Ask the following questions for Question C.

- *Use the equation from Question C to find the number of inches that correspond to 10 centimeters and the number of centimeters that correspond to 10 inches. How do these values relate to the data in a table or graph?*

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Assignment Guide for Problem 3.2

Applications: 2–5 | Connections: 26–30
Extensions: 36

Answers to Problem 3.2

A. **1.** (See Figure 1.)

2. Equations that show how distance d and time t are related would be $d = 50t$, $d = 55t$, and $d = 60t$.

3. (See Figure 2.)

4. **a.** The pattern relating distance and time in the table shows constant hourly increases in distance equal to the speed; the graph is a linear pattern with the line representing the greatest speed being steepest.

b. Yes, Theo is correct. Average speed provides information about the change in distance for every 1-hour increase. These coefficients can also be considered as unit rate because they indicate change in distance for every 1 hour.

5. Table: The pattern relating distance and time in the table shows constant hourly increases in distance equal to the speed. You can add twice the rate of change to the last entry in order to find the distance for 6 hours, or you can add two more rows for $t = 5$ hours and $t = 6$ hours.

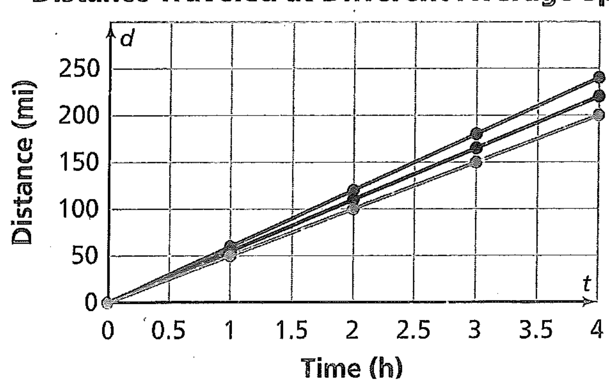
Figure 1

Distance Traveled at Different Average Speeds

Time (h)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0	0	0
1	50	55	60
2	100	110	120
3	150	165	180
4	200	220	240

Figure 2

Distance Traveled at Different Average Speeds



Graph: You can expand the graph to $x = 6$ and read the corresponding y-value.

a. Equation: You can substitute 6 for t in each equation.

(See Figure 3.)

If you substitute 6 for t in $d = 50t$, you get $d = 50(6) = 300$ miles.

If you substitute 6 for t in $d = 55t$, you get $d = 55(6) = 330$ miles.

If you substitute 6 for t in $d = 60t$, you get $d = 60(6) = 360$ miles.

b. Equation: You can substitute 275 for d in each equation and use fact families or guess-and-check to find t.

For $d = 50t$, when $d = 275$, $t = 5.5$.

For $d = 55t$, when $d = 275$, $t = 5$.

For $d = 60t$, when $d = 275$, $t \approx 4.58$.

B. 1. a.

Smartphone Monthly Charges

Number of Text Messages	Cost
0	$0
500	$15
1,000	$30
1,500	$45
2,000	$60
2,500	$75

b. The cost for 1,000 messages is $30. For 1,725 messages, the cost is the cost for 1,500 messages, $45, plus $\$.03 \times 225 = \$6.75$. So the cost of 1,725 messages is $51.75.

c. Using the table, the charge for 2,500 messages is $75. The charge for 2,000 messages is $60. $18 is more than the charge for 500 messages and less than the charge for 1,000 messages. The charge for 600 messages is $18.

Smartphone Monthly Charges

Number of Text Messages	Cost
500	$15
600	$18
700	$21
800	$24
900	$27
1,000	$30

2. a. The monthly charge B is related to the number of text messages n by the equation $B = 0.03n$.

b. For 1,250 text messages, the cost is $B = \$.03 \times 1{,}250 = \$37.50$.

(See Figure 3.)

Figure 3

Distance Traveled at Different Average Speeds

Time (h)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0	0	0
1	50	55	60
2	100	110	120
3	150	165	180
4	200	220	240
5	250	275	300
6	300	330	360

AT A GLANCE 3

3. a. (See Figure 4.)

   b. You can use the graph by finding 1,000 (or 1,725 or 1,250) on the x-axis, finding the corresponding point on the line, and then looking at the y-axis for the cost.

C. 1. 5 inches ≈ 12.5 centimeters, 12 inches ≈ 30 centimeters, and 7.5 inches ≈ 18.75 centimeters.

   2. $C = 2.5l$

      $C = 2.5(12) = 25$ centimeters

   3. 10 centimeters ≈ 4 inches, 30 centimeters ≈ 12 inches, and 100 centimeters ≈ 40 inches.

4. For part (1), you can use the graph by finding 5 (or 12 or 7.5) on the x-axis, finding the corresponding point on the line, and then looking at the y-axis for the length in centimeters.

   For part (3), you can use the graph by finding 10 (or 30 or 100) on the y-axis, finding the corresponding point on the line, and then looking at the x-axis for the length in inches.

   Students might construct a rule for unit conversion in the opposite direction: $l = \frac{C}{2.5}$. In that case, their graphs will have axis units swapped, and the steepness of the line will be different.

   (See Figure 5.)

Figure 4

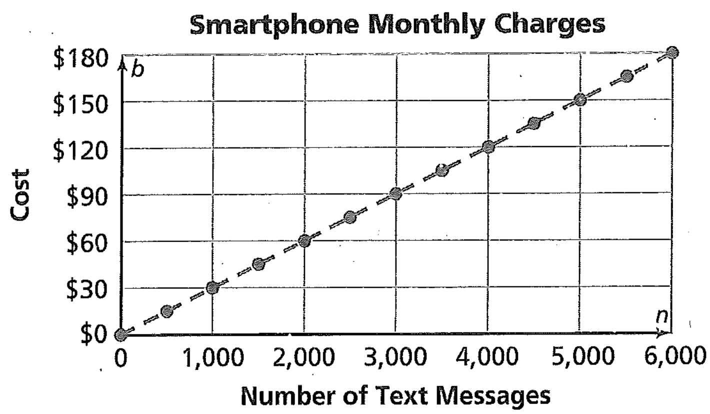
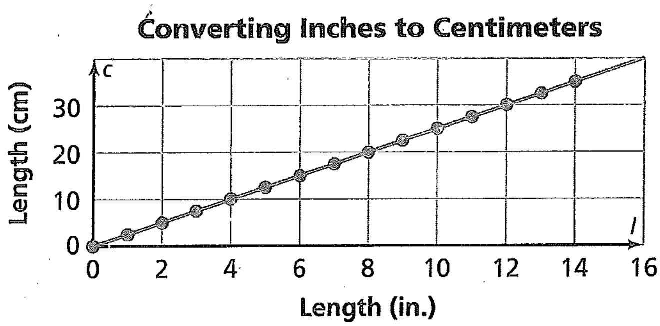
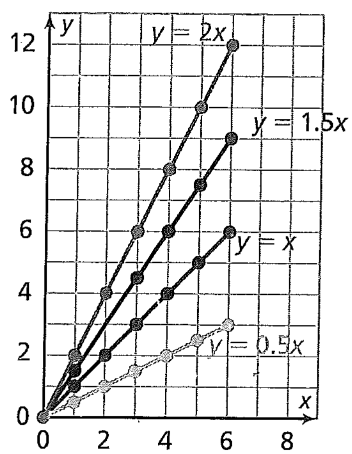


Figure 5



D. 1.

x	0	1	2	3	4	5	6
$y \cdot 2x$	0	2	4	6	8	10	12
$y \cdot 0.5x$	0	0.5	1	1.5	2	2.5	3
$y \cdot 1.5x$	0	1.5	3	4.5	6	7.5	9
$y \cdot x$	0	1	2	3	4	5	6



2. The pattern of change in each table is that each increase of 1 in the value of x leads to an increase of m in the value of y. The graphs are all linear patterns (and it does make sense to connect the points) with slope indicated by the size of the coefficient m. Greater values of m lead to steeper slopes.

3. **a–b.** Table: You can look in the column for $x = 2$ to find the value for y for each equation.

   $y = 4$, 1, 3, and 2 for equations $y = 2x$, $y = 0.5x$, $y = 1.5x$, and $y = x$, respectively.

   Graph: You can trace the y-value for $x = 2$ for each graph.

   Equation: You can substitute 2 for x in the equation.

   Table: Similarly, you can use the table to find the value of x in each equation by seeing where $y = 6$. You will have to extend the table for the equation $y = 0.5x$.

   Graph: You can trace the x-value for $y = 6$ for each graph.

   Equation: You can substitute 6 for y in the equation and use fact families or guess-and-check to solve for x.

4. A problem context for $y = 0.5x$ might be something like this: "If the operator of a juice bottle machine makes 50 cents profit on each sale, the profit in dollars on sale of x bottles will be given by $y = 0.5x$"

   A problem context for $y = 1.5x$ might be something like this: "If it takes 1.5 yards of material to make a decorative flag, the equation $y = 1.5x$ gives the amount of material required to make x flags."

   A problem context for $y = 2x$ might be something like this: "A National Hockey League team earns 2 points for every game it wins, so the team's point total (from wins) will be given by $y = 2x$ (though teams can earn 1 point for losses that occur in overtime)."

   A problem context for $y = x$ might be something like this: "If Mike runs 1 meter per second, how far y does Mike run in x seconds?"

E. All relationships can be given by equations in the general form $y = mx$. The pattern of growth in the dependent variable is an increase of m for each increase of 1 in the value of the independent variable. All graphs have a linear pattern of points that are steeper for greater values of m.

During the discussion of Question C, you might ask:

- *How does this equation compare to the equations you wrote for Questions A and B?*
- *How does this graph compare to the graphs you made in Questions A and B?*
- *If the value of one variable is known, how can you find the value of the other variable?*
- *What are the advantages and disadvantages of each representation?*

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Assignment Guide for Problem 3.3

Applications: 6–9 | Connections: 31–35

Answers to Problem 3.3

A. 1. a. (See Figures 1 and 2.)

b. The table and graph both show a steady rate of increase. The pattern of change in the table is that each increase of 4 in the number of people in a group leads to a constant increase in the group cost. The graph shows a linear pattern with a start value of 50.

2. a. You can calculate the admission price for a group with any number of people by multiplying the number of people by 10 and adding 50 to that result.

b. The equation $p = 10n + 50$ relates admission price p to group size n.

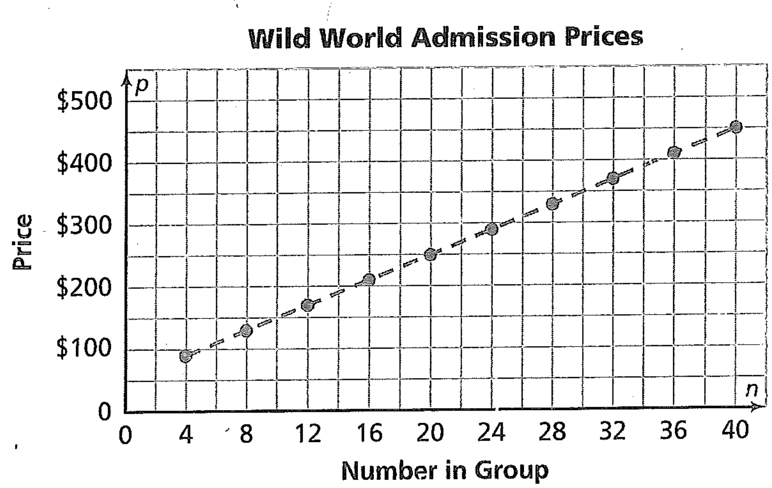
c. Both Problems show a linear relationship (There is a constant rate of change.). Both graphs are straight lines, indicating that the change is constant. In Problem 3.2, you can express the relationship between the dependent and independent variables with one operation. In this Problem, you need two operations to express the relationship.

Figure 1

Wild World Admission Prices

Number in Group	4	8	12	16	20	24	28	32	36	40
Price	$90	$130	$170	$210	$250	$290	$330	$370	$410	$450

Figure 2



3. a. Table: 18 is halfway from 16 to 20. The group cost for 16 is \$210, and for 20, it is \$250. So the group cost for 18 people will be \$210 + \$20 = \$230.

Graph: You can trace the y-value for $x = 18$.

Equation: You can substitute 18 for the n-value in the equation and solve for p.

$$p = 10 \times 18 + 50 = 230$$

b. Table: 350 is halfway from 330 to 370. 28 people cost \$330, and 32 people cost \$370. So 30 people will cost \$ 350.

390 is halfway from 370 to 410. 32 people cost \$370, and 36 people cost \$410. So 34 people will cost \$390.

Graph: You can trace the x-value for $y = 350$.

You can trace the x-value for $y = 390$.

B. 1. (See Figure 3.)

2. To find your bonus card balance after any number of rides, you multiply the number of rides by 6 and subtract that result from 100.

3. $P = 100 - 6r$, where P is the number of points left and r is the number of rides taken.

4. Cost per ride appears as subtraction because there is a decrease in the value of the bonus card balance for each ride.

100 bonus points at the start appears as addition in the equation.

5. (See Figure 4.)

The values of the dependent variable decrease at a constant rate as the values of the independent variable increase.

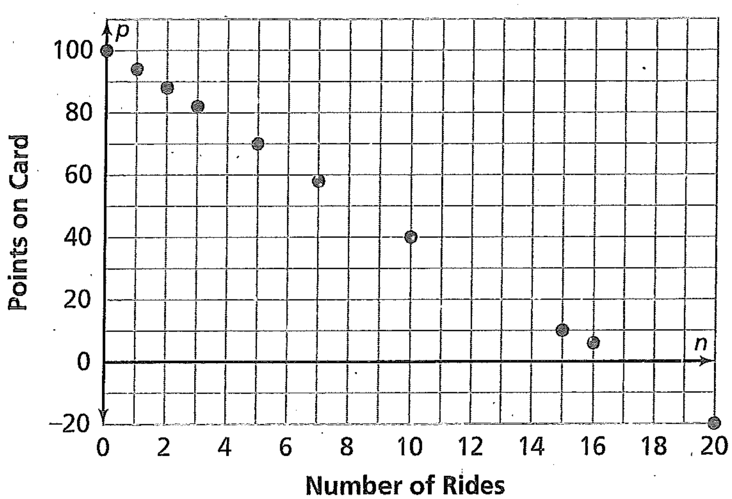
Figure 3

Bonus Card Balance

Number of Rides	0	1	2	3	5	7	10	15
Points on Card	100	94	88	82	70	58	40	10

Figure 4

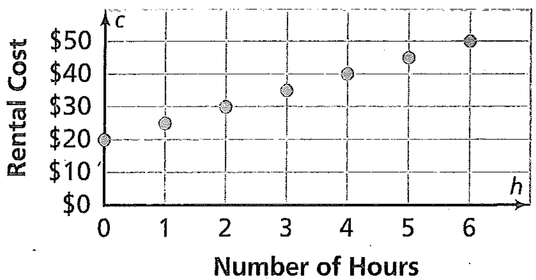
Bonus Card Balance



C. **1.** The variable h represents hours of rental time and the variable c represents rental cost. The 20 represents the minimum rental charge of \$20 and the 5 says that each hour of time used adds \$5 to the cost of the rental.

2. (See Figure 5.)

Golf Cart Rental



In this case, it is not clear whether (or how) one should connect the points on the graph. Rentals like this often charge a full hour for any part of an hour used, so the proper graph would be a step function that is level between whole hours.

3. The cost per hour shows up in the table and graph as a steady rate of increase of 5 dollars for every 1 hour. It shows up in the equation as the coefficient 5.

4. The 20 is the entry in the table when the number of hours is 0. The 20 is also where the graph intersects the y-axis.

5. If you substitute 0 for x, $y = 20 + 5(0) = 25$. Therefore, neither (0, 4) nor (0, 20) satisfy the relationship represented by the equation.

If you substitute 7 for x, $y = 20 + 5(7) = 55$. Therefore, (7, 55) does satisfy the relationship represented by the equation.

You could also use a graph or a table to check for these points.

Figure 5

Golf Cart Rental

Number of Hours	0	1	2	3	4	5	6
Rental Cost	\$20	\$25	\$30	\$35	\$40	\$45	\$50

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Assignment Guide for Problem 3.4

Applications: 10–21 | Connections: 37–42

Answers to Problem 3.4

A. Group prices are \$100 for 5 members, \$160 for 11 members, and \$280 for 23 members.

B. Points left are 82 after 3 rides, 58 after 7 rides, and 16 after 14 rides.

C. Predicted trip times are about 9.8 hours at 45 miles per hour, 8.4 hours at 55 miles per hour, and 7.4 hours at 65 miles per hour.

D. Profit will be \$600 for 8 customers; \$1,400 for 12 customers; \$3,000 for 20 customers; and \$5,000 for 30 customers.

E. **1.** If each edge of the cubical workshop is 4.25 meters long, the total surface area of the floor, walls, and ceiling is $6(4.25)^2 = 108.375$ m^2. (**Note:** Squaring comes before multiplication.)

2. The volume of the workshop is $(4.25)^3 = 76.765625$ m^3.

At a Glance

Problem 4.1 Pacing 1 Day

4.1 Taking the Plunge: Equivalent Expressions I

Focus Question Is it possible to have two different, but equivalent, expressions for a given situation? Explain.

Launch

In this Problem students will explore situations that lead to two or more equivalent expressions for the dependent variable.

- *What are the variables in each situation?*
- *How many pieces do you need to make a ladder of n squares?*
- *How many pieces would be needed to make a tower of n cubes?*

Materials

Labsheet
- 4.1 Equivalent Expressions I
- poster paper (optional)

Explore

If you notice students examining the picture, ask the following questions.

- *What patterns do you see in the picture?*
- *How do you know that you are adding three each time?*
- *What patterns do you see in the table?*

Summarize

The big idea of Problem 4.1 is that algebraic expressions can often be written in quite different but mathematically equivalent forms.

- *If $n = 7$, do you get the same value in each of the equations for the ladder?*
- *If $n = 7$, do you get the same value in each equation for the tower?*
- *What can you say about these expressions for the dependent variable?*

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Assignment Guide for Problem 4.1

Applications: 1–4 | Connections: 21–25

Answers to Problem 4.1

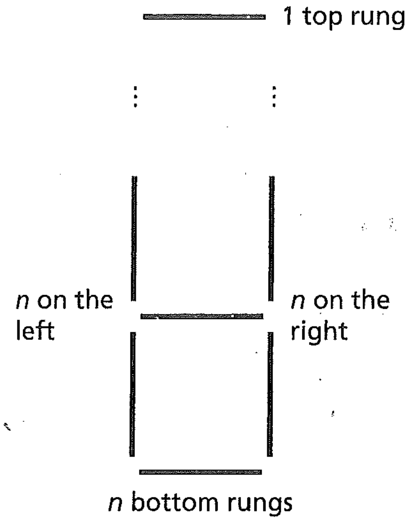
A. **1.** (See Figure 1.)

2. There are many different expressions showing how to calculate the number of pieces needed to make a ladder of n squares. The first instinct of many students will be to notice that increasing the number of squares by 1 increases the number of pieces by 3. We want them also to recognize that this results in an equation such as $P = 3n + 1$ or some equivalent closed form. Other reasonable conjectured expressions are introduced in Problem 4.2:

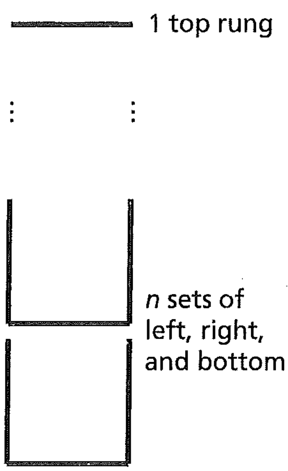
Figure 1

Number of Squares	1	2	3	4	5	10	20
Number of Pieces	4	7	10	13	16	31	61

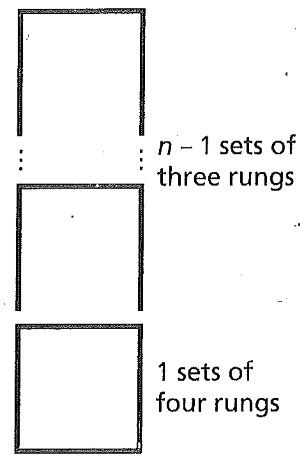
Tabitha's equation: $P = n + n + n + 1$



Chaska's equation: $P = 1 + 3n$



Eva's equation: $P = 4 + 3(n - 1)$



B. **1.** The second row of a table that counts pieces used to make a tower with n cubic levels would look like Figure 2.

2. There are many possible equivalent expressions that show how to calculate the number of pieces in a tower of n cubes. One natural example is $P = 8n + 4$.

Figure 2

Number of Cubes	1	2	3	4	5	10	20
Number of Pieces	12	20	28	36	44	84	164

Assignment Guide for Problem 4.2

Applications: 5–6 | Connections: 26–38

Answers to Problem 4.2

A. 1. Tabitha might have reasoned that a ladder of height n will use n pieces up one side, n pieces up the other side, and $n + 1$ cross pieces.

Chaska might have reasoned that a ladder of height n will have one cross piece at the bottom and then completion of each of the n squares above that base will require addition of 3 pieces.

Latrell took a superficial way of thinking and reasoned that each square has four sides, so n sides will require $4n$ pieces. This reasoning ignores the fact that each cross piece (except the bottom and top pieces) is both top of one square and bottom of the next.

Eva might have reasoned that when you start with one square at the bottom (4 pieces), you then need to add 3 pieces above it to get the next square, and so on up to the top. There will be $(n - 1)$ of those additions to get a ladder of height n squares.

2. The expressions of Tabitha, Latrell, and Eva all give $B = 4$ when $n = 1$; $P = 16$ when $n = 5$; $P = 31$ when $n = 10$; and $P = 61$ when $n = 20$.

3. Based on work in Questions A and B, the expressions written by Tabitha, Chaska, and Eva are equivalent to each other, but not to that written by Latrell.

4. Answers to this question will vary, depending on what students came up with in work on Problem 4.1.

B. 1. Student ideas about the number of pieces to make a full tower of n cubes will depend on whether they assume that the sides of one ladder face can serve as sides of the adjacent ladder face or not. In the simplest assumption that all faces can stand alone, the rule relating number of pieces for a tower to number of cubes in the tower is $4B$, where P is the number of pieces in one ladder. For Tabitha, this means $4(n + n + n + 1)$; for Chaska, this means $4(1 + 3n)$; for Latrell, this means (incorrectly) $4(4n)$; for Eva, this means $4(4 + 3(n - 1))$

If students reason that the vertical sides of adjacent ladder faces are actually the same, the number in the whole tower will be given by $4 + 8n$ or something equivalent (for example, $12 + 8(n - 1)$).

2. The graphs and tables for any equivalent expressions are the same.

Assignment Guide for Problem 4.3

Applications: 7–11 | Connections: 39–52
Extensions: 65, 67

Answers to Problem 4.3

A. 1. $B = 30n$

2. $F = 120n$

3. $R = 1000$

B. 1. a. The first two equations are correct expressions of the total cost relationship. Celia simply adds the three component cost expressions to get a total. Theo seems to have combined bike rental and food and campsite cost rates before multiplying by the number of customers. That makes sense. Liz' equation is not correct because it combines fixed and variable costs to form one large variable cost.

b. In Celia's equation the $30n$ represents total cost for all bike rentals, the $120n$ represents the total cost for all food and campsite fees, and the 1,000 represents the cost of the rental of the bus and trailer for bikes. In Theo's equation, the 150n represents the total cost for bike rentals, food, and campsite fees, and the 1,000 is again the rental cost for bus and trailer.

2. (See figure 1.)

The table suggests that the top two equations are equivalent because the total costs are the same for the given number of customers.

3. (See figure 2.)

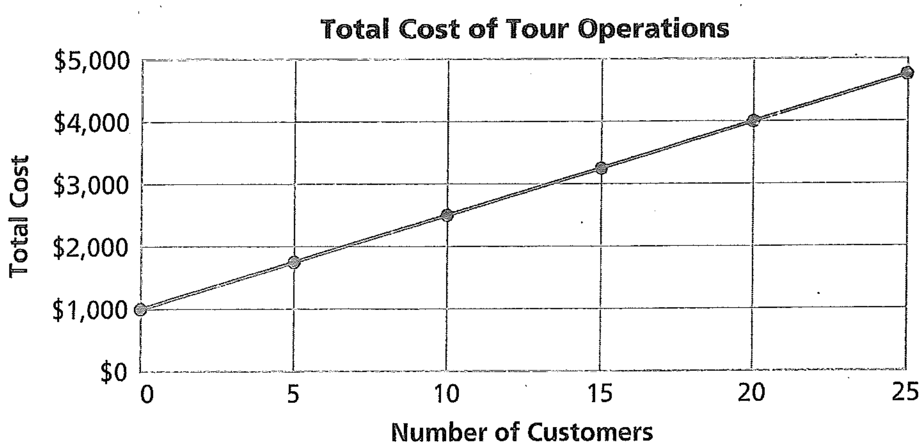
The graph of Liz's cost equation (given below) will have a much steeper slope and it will pass through the origin (0, 0). (See Figure 3.)

Figure 1

Operating Cost As A Function of Number of Customers

Number of Customers n	5	10	15	20	25
$C = 30n + 120n + 1{,}000$	1,750	2,500	3,250	4,000	4,750
$C = 150n + 1{,}000$	1,750	2,500	3,250	4,000	4,750
$C = 1{,}150n$	5,750	11,500	17,250	23,000	28,750

Figure 2



4. Celia's and Theo's equations are equivalent because

$$30n + 120n + 1000 = (30 + 120)n + 1000 = 150n + 1000$$

C. 1. a. terms: $5x$, x, 6; coefficients: 5, 1, 6; equivalent expression (sample) $6x + 6$ or $6(x + 1)$

b. terms: $10x$, $-2q$; coefficients: 10, -2; equivalent expression (sample) $8q$

2. Start with the right side.

$$4 + 3(n - 1) = 4 + 3n - 3$$

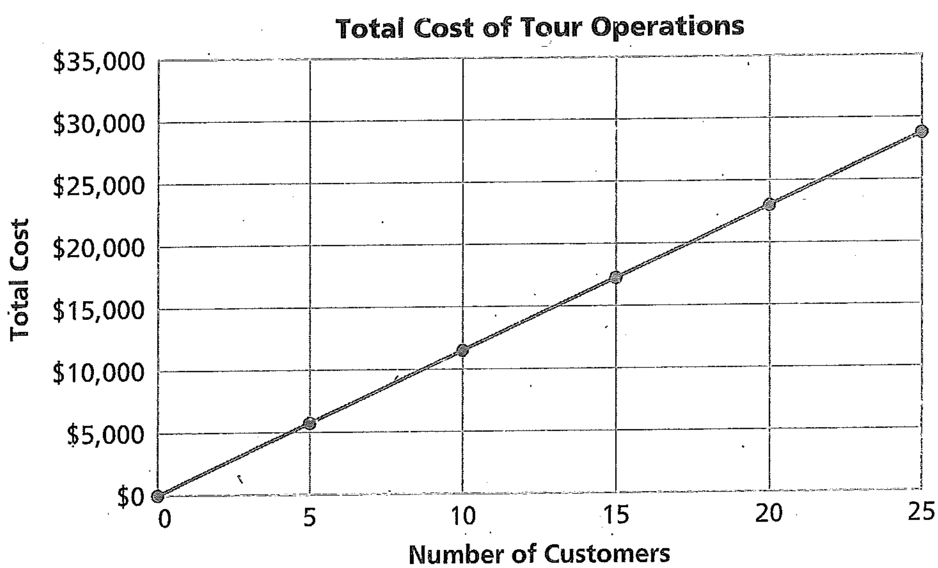
by the Distributive Property

$$= 4 - 3 + 3n$$

$$= 1 + 3n$$

D. Following Celia's idea of including each cost component explicitly, her new equation might be $C = 30n + 120n + 1000 + 50 + 10n$ (or some other order of these terms). Theo's compact form is likely to be $C = 160n + 1050$.

Figure 3



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Assignment Guide for Problem 4.3

Applications: 12–19 | Connections: 53–58
Extensions: 68

Answers to Problem 4.4

A. **1.** $9{,}450 = 21n$ has solution given by $9{,}450 \div 21 = 450$.

2. The solution by division works by reasoning from related members of a multiplication and division fact family.

3. The answer can be checked by substituting 450 for n: $21(450) = 9{,}450$, which means the answer is correct.

B. **1.** $d = p - 0.25$

2. $2.79 = p - 0.25$ has solution $p = 3.04$. Check $2.79 = 3.04 - 0.25$.

3. The solution is obtained by recognizing that the given equation is equivalent to the fact-family-related equation $p = 2.79 + 0.25$.

C. **1.** $C = n + 95.50$, where C is the cost, and n is the number of bandanas. **Note:** Students may also rewrite n as $1n$ to emphasize that they are multiplying the number of bandanas by \$1. This ensures that the units are consistent across terms (each term represents a number of dollars).

2. $C = \$50 + \$95.50 = \$145.50$ which is the cost for 50 bandanas.

$116.50 = n + 95.50$, so by fact-family reasoning $n = 116.50 - 95.50 = 21$ (21 bandanas).

D. **Note:** Properties of equality are used in *Moving Straight Ahead* to solve linear equations. At this stage, fact families, graphs, tables and guess and check suffice.

1. $x + 22.5 = 49.25$ has solution $x = 26.75$ because the original equation is equivalent to $x = 49.25 - 22.5$ by fact-family reasoning.

Check: $26.75 + 22.5 = 49.25$.

2. $37.2 = n - 12$ has solution $n = 49.2$ because the original equation is equivalent to $n = 37.2 + 12$ by fact-family reasoning.

Check: $37.2 = 49.2 - 12$.

3. $55t = 176$ has solution $t = 3.2$ because the original equation is equivalent to $t = \frac{176}{55}$ by fact-family reasoning.
Check: $55(3.2) = 176$.

4. $y = 14m$ has solution $m = \frac{y}{14}$ by fact-family reasoning. Since y must be 14 times larger than m, m must be one fourteenth as large as y.

Check: $14\left(\frac{y}{14}\right) = y$; you are multiplying y by 14 and then dividing by 14, which results in y.

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Assignment Guide for Problem 4.5

Applications: 20 | Connections: 59–64
Extensions: 66, 69

Answers to Problem 4.5

A. $35n > \$1{,}050$ has solution $n > 30$ bungee jumpers, or in other words, any number of bungee jumpers over 30. Possible answers include 31, 50, 100. These solutions work because substituting them for n satisfies the inequality:

$35(31) = \$1{,}085 > \$1{,}050$; $35(50) = \$1750$, $35(100) = \$3500$.

B. $4g \leq 17.50$ has solution $g \leq 4.375$ gallons, or any number of gallons less than or equal to 4.375 gallons. Possible answers include 4, 3, or 2 gallons.

C. To keep the costs under \$200, solve the inequality $n + 95.50 < 200$, which has solution $n < 104.50$. In this context, you cannot make half a bandana, so the number of bandanas must be 104 or less. Possible answers include 100, 90, or 10.

D. $84 < 14m$ has solution $6 < m$.

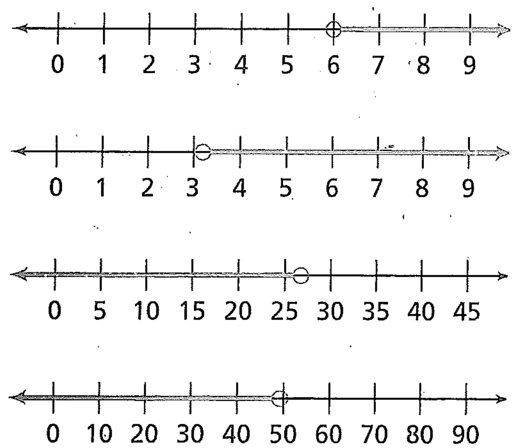
$55t > 176$ has solution $t > \frac{176}{55}$ or $t > 3.2$

$x + 22.5 < 49.25$ has solution $x < 26.75$

$37.2 > n - 12$ has solution $49.2 > n$.

E. **1.** The thicker part of the line represents all numbers that would make the inequality true.

2.



F. **1.** Answers may vary. For example, a T-shirt company charges \$50 for design and \$6 per T-shirt.

2. Any point that lies on the graph will satisfy the equation $y = 50 + 4x$. For $x = 8$, $y = 50 + 4(8) \neq 92$, so (8, 92) is not on the line. $50 + 4(15) = 110$, so (15, 110) does lie on the line.

3. Example question: How much would it cost for 15 T-shirts to be made?

4. Example question: If the total cost of T-shirts was \$110 or less, what is the number of T-shirts that could be made? $n < 15$