Assignment Guide for Problem 4.1

Applications: 1–6 | Connections: 66
Extensions: 80–87

Answers to Problem 4.1

A. **1.** The sum of two even numbers is even. (Two rectangles with height 2 can be put together to form a larger rectangle with height 2.)

2. The sum of two odd numbers is even. (The tile models for the odd numbers each have an extra square. Combining the models pairs the extra squares to form a rectangle with height 2.)

3. The sum of an odd number and an even number is odd. (Models for even numbers are rectangles with height 2. Models for odd numbers are rectangles with height 2 but with an extra square. Combining the models still gives an extra square.)

4. The product of two even numbers is even. (Combining an even number of even rectangles gives another rectangle with height 2.)

5. The product of two odd numbers is odd. (Combining an odd number of rectangles with height 2 and an extra square gives another rectangle with an extra square.)

6. The product of an even number and an odd number is even. (Combining an odd number of even rectangles gives another rectangle with height 2.)

B. 127 + 38 is odd. You check to see whether each addend is odd or even and then use the conjectures that we have shown to be true.

C. Zero is an even number. Students could argue that it has to be even because of the way odd and even numbers are distributed on the number line. They could also argue that 0 is divisible by 2, so it must be even. Finally, some students could point out that a "rectangle" of height 2, made with zero tiles, would have no extra tile.

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Assignment Guide for Problem 4.2

Applications: 7–23 | Connections: 67–74
Extensions: 88–90

Answers to Problem 4.2

A. **1.** 5(4 + 7); 5(4) + 5(7)

2. (5 + 3)2; (5)2 + (3)2

3. 5(20 + 4); 5(20) + 5(4)

4. (3 + 2)8; (3)8 + (2)8

B. Possible dimensions: 1 × (4 + 24), 2 × (2 + 12), 4 × (1 + 6). Each of these gives the small rectangle an area of 4 square units and the other rectangle an area of 24 square units, for a total area of 28 square units. Students may also write their answers in the form 1 by 28, or 1 by (4 + 24).

C. **1.** 6(5) + 6(9); See Figure 1.

2. 4(7 + 3); See Figure 2.

D. **1.** See Figure 3.

Sample explanation for first row of table: 1(36 + 48) = 1 × 36 + 1 × 48
Let 1 be the width of the space. Then 36 is the length of Mr. Wei's space, 48 is the length of Mrs. Johnson's space, 36 + 48 is the length of their spaces together, 1 × 36 is the area of Mr. Wei's space, and 1 × 48 is the area of Mrs. Johnson's space.

2. The greatest width is 12, the greatest common factor of 36 and 48. The corresponding length would be (4 + 3) = 7. The expressions 12(3 + 4) and 12 × 3 + 12 × 4 represent these dimensions.

Figure 1

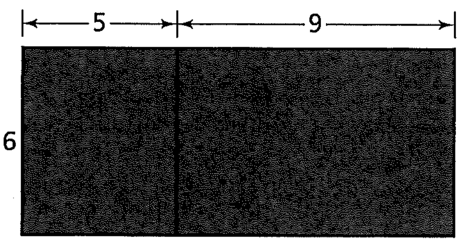


Figure 2

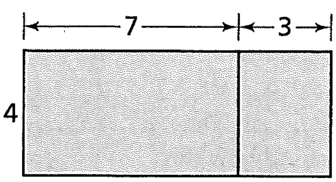


Figure 3

Possible Whole Number Dimensions

If the dimensions of Mr. Wei's space are...	Then the dimensions of Mrs. Johnson's space are...
1 × 36	1 × 48
2 × 18	2 × 24
3 × 12	3 × 16
4 × 9	4 × 12
6 × 6	6 × 8
12 × 3	12 × 4

3. 3 is the length of Mr. Wei's space and 4 is the length of Ms. Johnson's space. 3 and 4 do not have any common factors. 12 is the greatest common factor, because there are no more factors in common.

4. $a = 8$, $b = 2$, $c = 3$

E. **1.** The area of the largest rectangle is the sum of the two smaller rectangles: $8 \times 27 = (8 \times 20) + (8 \times 7)$. Place value shows the equivalence between 8×27 and $(8 \times 20) + (8 \times 7)$. The Distributive Property helps clarify the connection between addition and multiplication: $8 \times (20 + 7) = (8 \times 20) + (8 \times 7)$ and $8 \times 27 = (8 \times 20) + (8 \times 7)$. Therefore, $8 \times 27 = 8 \times (20 + 7)$.

2a. The missing areas are 600 and 8. The area of the entire rectangle is equal to 32×24, which can be written as $(30 + 2) \times (20 + 4)$ using place values. The Distributive Property results in the equivalent expression $(30 + 2) \times (20) + (30 + 2) \times (4)$. Applying it again gives $(30 \times 20) + (2 \times 10) + (30 \times 4) + (2 \times 4)$. Of the terms in this last expression, 30×4 and 2×20 are labeled, while the missing areas are $30 \times 20 = 600$ and $2 \times 4 = 8$.

2b. There are several different expressions students can write that are equivalent to 32×24. Here are two:

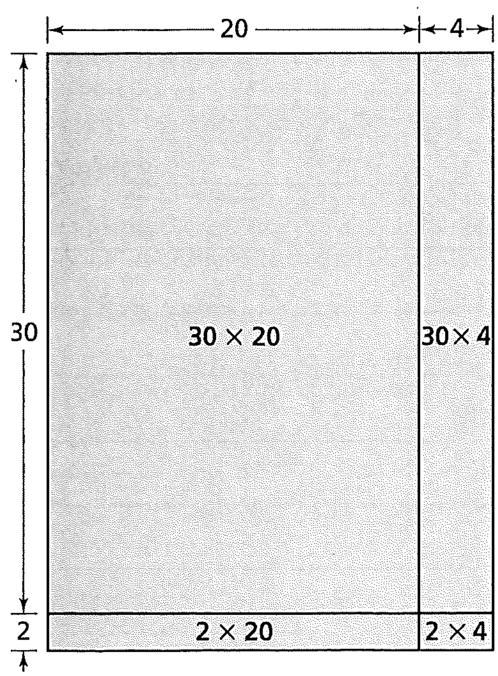
$$\begin{aligned}(30 + 2) \times (20 + 4) &= (30 + 2) \times 20 + (30 + 2) \times 4 \\ &= 30(20) + 2(20) + 30(4) + 2(4) \\ &= 600 + 40 + 120 + 8 \\ &= 768\end{aligned}$$

$$\begin{aligned}(30 + 2) \times (20 + 4) &= 30(20 + 4) + 2(20 + 4) \\ &= 30(20) + 30(4) + 2(20) + 2(4) \\ &= 600 + 120 + 40 + 8 \\ &= 768\end{aligned}$$

2c. The area of the largest rectangle, 32×24, is the same as the sum of the areas of two rectangles, 32×20 and 32×4 (or 20×32 and 4×32) or the same as the sum of the areas of four smaller rectangles, 30×20, 2×20, 30×4, and 2×4 (or 20×30, 20×2, 4×30, and 4×2). (See Figure 4.)

3. The area of a rectangle can be found in two different ways (as the product of its length and width and as the sum of the areas of smaller rectangles within it), just as the Distributive Property allows you to write a number in two different ways (as the product of two factors and as the sum of two addends).

Figure 4



ACE Assignment Guide for Problem 4.3

Applications: 24–60 | Connections: 75–79

Answers to Problem 4.3

A. **1.** \$42; $(12 \times 2) + (6 \times 3)$

2. Two expressions are $(12 \times 2) + (12 \times 3)$ and $12(2 + 3)$, both of which simplify to 60, or \$60. For a contextual reference, students can imagine the objects being sold in bundles. Suppose Nic is buying pen-and-paper sets, with one pen and one pad of paper in each set. The set costs \$2 for the pen and \$3 for the paper, but now it is clear that he has 12 things that cost \$5, because $2 + 3 = 5$.

3. Jenn can't factor out 12 and still be left with whole numbers in the parentheses, but she can factor out 6.

$$(12 \times 2) + (6 \times 3) = (6 \times 2 \times 2) + (6 \times 3)$$
$$= 6(2 \times 2 + 3)$$
$$= 6(4 + 3)$$

Here, students can imagine that the pen-and-paper sets Jenn bought include 2 pens and 1 pad of paper. So Jenn bought 6 sets, each costing \$4 for the two pens and \$3 for the pad of paper.

B. Answers will vary. Possible answers are $2 + 5 + (1)3 = 10$ and $2(5) + 1(3) = 13$.

C. **1.** $3 + 5 \times 2 + 4 = 3 + 10 + 4$
$= 17$

2. $3 + 5(2 + 4) = 3 + 5(6)$
$= 3 + 30$
$= 33$

3. $4 + (3 + 7) \div 2 - 2(4) = 4 + 10 \div 2 - 2(4)$
$= 4 + 5 - 8$
$= 1$

4. $3^3 + 5(2 + 3) - 25 = 27 + 5(5) - 25$
$= 27 + 25 - 25$
$= 27$

5. $2 + 5^3 \times 10 = 2 + 125 \times 10$
$= 2 + 1250$
$= 1252$

6. $4 \div 4 + 7^2 = 4 \div 4 + 49$
$= 1 + 49$
$= 50$

D. Students' preferred strategies will vary. The Distributive Property can be used for Question C parts (2) and (4). It could also be used for part (3), but it is unlikely, as the method would result in decimals and students have not yet been exposed to the Distributive Property of Division.

Assignment Guide for Problem 4.4

Applications: 61–65 | Extensions: 91

Answers to Problem 4.4

A. \$256; $8(15 + 17) = 8(32) = 256$, since the number of the boxes sold is $15 + 17$ and the cost of a box of fudge is \$8. Alternatively, students may write $8 \times 15 + 8 \times 17$ to represent the amount of money made each week and then combine to make the two-week total.

B. 20-yard line; $35 - 5 \times 3 = 35 - 15 = 20$, since they lose 5 yards on each play.
Note: If students solved the question as if the team were on their opponent's 35-yard line, then after 3 plays they would be on the 50-yard line.

C. The possibilities correspond to the common factors of 20 and 30, that is, 1, 2, 5, and 10. Thus, the possibilities are 1 pack (with 30 cookies and 20 apples each), 2 packs (with 15 cookies and 10 apples each), 5 packs (with 6 cookies and 4 apples each), and 10 packs (with 3 cookies and 2 apples each).

D. 31 fewer people; $114 - 83 = 31$, since 114 people returned to the mainland but only 83 people went to the island.

E. \$516; students can use one of two equivalent expressions to calculate the cost, $(25 + 18) \times 12$ or $25 \times 12 + 18 \times 12$. In the first expression, the total number of students is $43 = 25 + 18$ and the cost per person is \$12. Therefore, the total cost for the two clubs is $516 = (25 + 18) \times 12 = 43 \times 12$. In the second expression the trip cost is 25×12, or \$300 for the first club. For the second club, the cost is 18×12, or \$216. Therefore, the total cost is the sum of the two clubs' costs: $25 \times 12 + 18 \times 12 = 300 + 216 = 516$, or \$516. The Distributive Property shows the equivalency of the two methods.