 Problem 4.1 Pacing 1 Day

4.1 Who Is the Best? Making Sense of Percents

Focus Question How is a percent bar useful in making comparisons with decimals?

Launch

Help students understand what it means to change a fraction to a percent. Introduce percent notation and reinforce that the percent sign indicates "out of 100".

One idea to develop is that the percent bar (or tape diagram) works like two parallel number lines. The top line shows the raw data, and the bottom line shows scaled percentages of the raw data.

Key Vocabulary

- percent

Materials

Labsheet

- 4.1: Making Sense of Percents
- Number Line Tool

Explore

Introduce the two sets of percent benchmarks. As students work, remind them of their work on partitioning in Investigation 1. It is important that students be able to use the percents they have found to predict the answer when there is a different amount for the whole.

Summarize

Focus on the various ways students used benchmarks to find the percent of a number. One goal is to develop strategies for finding percents. A second goal is to have students talk about why these strategies make sense and how to use them.

ACE Assignment Guide for Problem 4.1

Applications: 1–5, 20 | Connections: 26–31
Extensions: 34–39

Answers to Problem 4.1

A. **1.** (See Figure 1.)

2. On Team 1's bar, the mark should be a bit closer to 95 than to 126, because 108 is about 13 more than 95 and 18 less than 494. A reasonable estimate based on this fact would be slightly less than half way between the 75% and 100% marks, say about 85%. In fact, it is approximately 85.7%. On Team 2's bar, the mark should be a tiny bit to the right of 193 free throws, because 195 is 2 more than 193, but 62 less than 257. This means his percentage is a tiny bit more than 75%. In fact, it is approximately 75.9%. (See Figure 2.)

3. Some students will use the percent bars to estimate the number of successful throws in the next 200. The total is now 200 so the challenge is to mark the answers from Question A, part (2), 85% for Team 1 and 75% for Team 2, on new percent bars, and then find the numbers of free throws that are equivalent to these marks. For Team 1 it is about halfway between 150 and 200, slightly closer to 150 or approximately 170. For Team 2 it is slightly above 150 (151). (See Figure 3.)

B. Team 1's free-throw percentage was greater than 80% and Team 2's free-throw percentage was less than 80%. (See Figure 4.)

C. Will uses 25%, 50%, 75% and 100% as benchmarks that correspond to $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$. Alisha uses benchmarks that correspond to tenths: 10%, 20%, etc.

D. 1. Using Will's benchmarks, Angela and Christina have very close free-throw percentages. Both girls have higher percentages than Emily. (See Figure 5.)

Figure 1

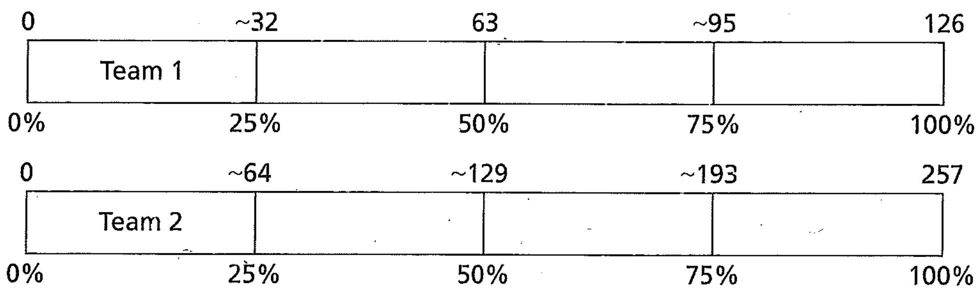


Figure 2

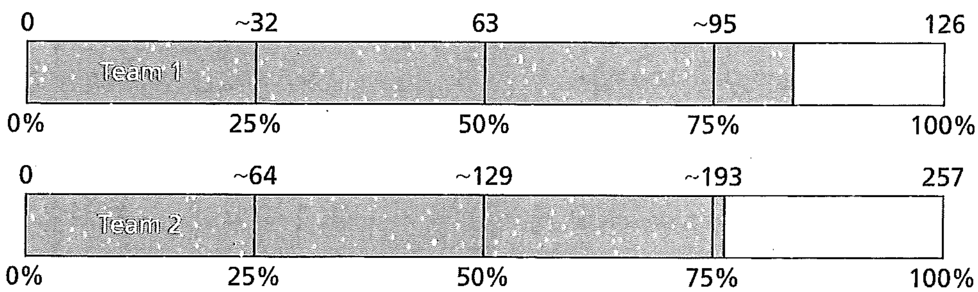


Figure 3

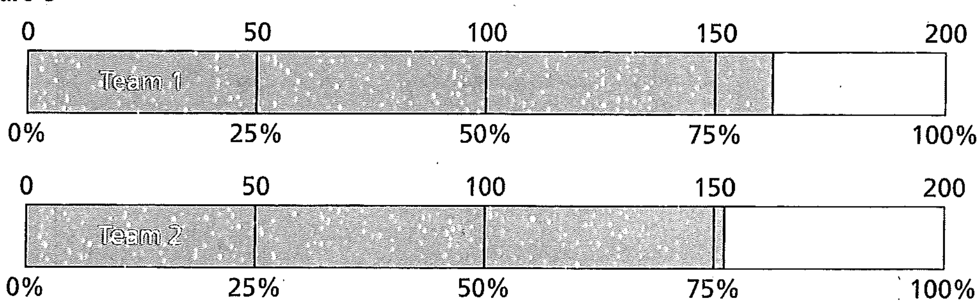
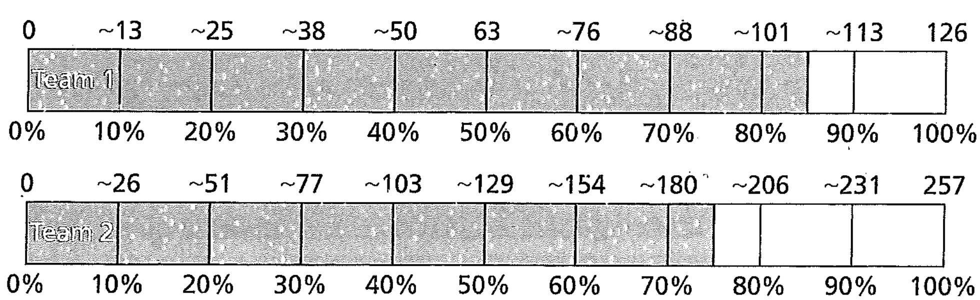


Figure 4



Using Alisha's benchmarks, Angela's free-throw percentage is 80%, while Christina's is slightly greater than 80%. (See Figure 6.)

Christina has the highest free-throw percentage; Angela's is next, and Emily's is the least.

2. Students may make percent bars, ratios, fractions, or decimals. Using ratios to predict, Angela will make 24 baskets (12 : 15 = 24 : 30), Emily will make 22 or 23 (15 : 20 = 7.5 : 10 = 22.5 : 30), and Christina will make 24. (This one is hard to rename with 30 as the second part of the ratio. Using a decimal approximation for $\frac{13}{16}$, 0.81, as a unit rate, we get 30 × 0.81 or about 24.)

E. Using the example of free throws, percents are like fractions in that they express part of the whole. The part is the number of free throws that were successful; the whole is the number that were attempted. Percents are like ratios in that they tell us how many free throws were made for every 100 attempted.

Will and Alisha are both correct. If we emphasize part of a whole, we are thinking about fractions. If we emphasize a comparison between the two quantities, we are thinking about ratios.

Figure 5

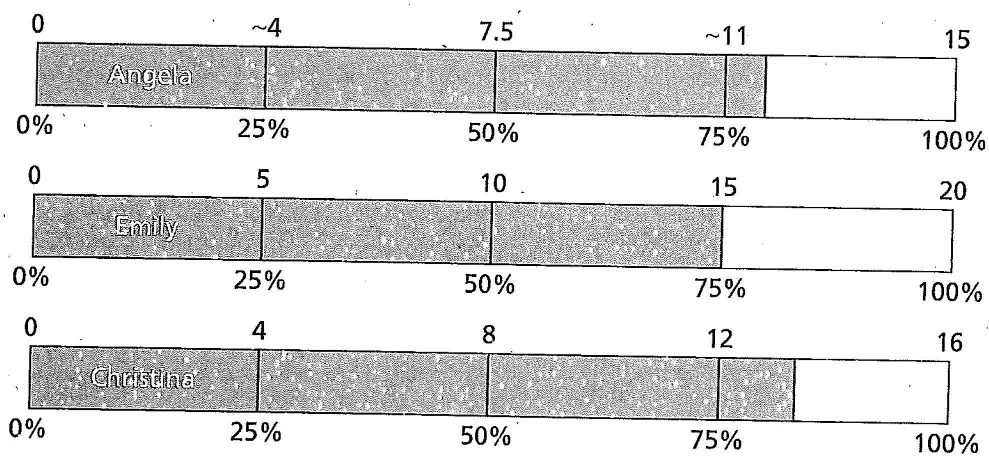
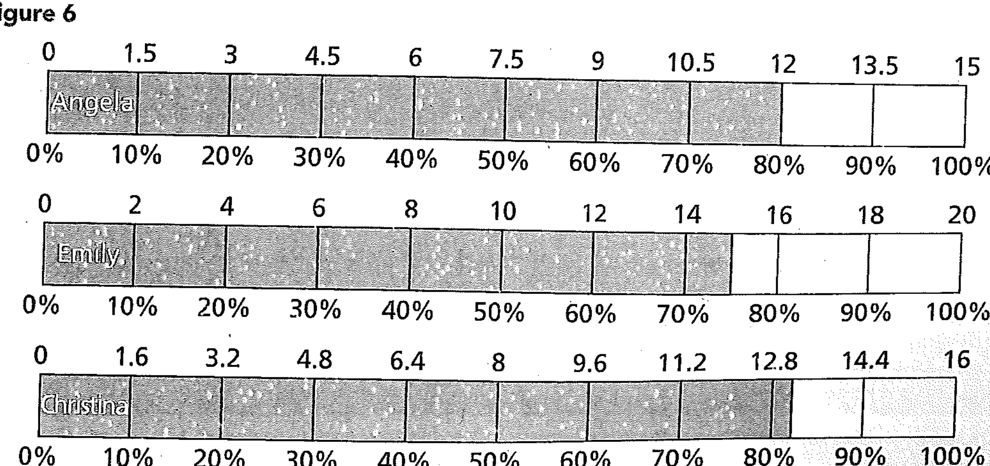


Figure 6



At a Glance

Problem 4.2 Pacing 1 Day

4.2 Genetic Traits: Finding Percents

Focus Question How can partitioning be used to express one number as a percent of another number?

Launch

Make sure students know that their goal in this problem is to improve their estimation strategies from the previous Problem in order to get more precise answers.

Remind students of their work in Problem 4.1, where they used 25% and 10% as benchmarks. Ask how they might get better estimates than the estimates they get with these benchmarks. If students don't suggest it, you can suggest cutting a 10% benchmark in half to get 5%, and cutting 10% into 10 parts to get 1%, just as they cut 100% into 10 parts.

Materials

Labsheets

- 4.2: Genetic Traits
- 4ACE: Exercises 15–16

Explore

Introduce the context and the math with data on genetic traits gathered from a middle school classroom.

Make the important connection between using a percent bar and using a rate table to answer the question "How many?" You may have to help students see similarities.

Summarize

Bring out both partitioning strategies—partitioning according to percents and partitioning according to the total.

ACE Assignment Guide for Problem 4.2

Applications: 6–19 | Connections: 32–33
Extensions: 40

Answers to Problem 4.2

A. 1. **Classroom Trait Survey**

Trait	Yes	No	Total
Attached Earlobes	12	18	30
Dimples	7	23	30
Straight Hair	24	6	30
Widow's Peak	17	13	30

2. Percent bars and estimates will vary. Possible estimates include: Attached earlobes 40% (actual value: 40%), Dimples 25% (actual value: $23\frac{1}{3}$%), Straight hair 80% (actual value: 80%), and Widow's peak 60% (actual value: $56\frac{2}{3}$%). (See Figure 1.)

3. At this rate (about 60%) about 300 students out of 500 will have a widow's peak. Strategies may vary. Students might make a new percent bar with 500 matched with 100%, and partition to find the number that goes with 60%. They might think of a percent as a ratio and rename 60 : 100 as 300 : 500. They might possibly make a rate table.

AT A GLANCE 4

B. 1. There are 34 students in Marjorie's class.

2. Answers will vary. Students might say that to get the number of students at 10% we have to figure 34 ÷ 10, because 10% is a tenth of 100%. This will produce 34 ÷ 10 or 3.4. To get the number of students at 40% they might multiply 3.4 by 4, or they might think that 40% is $\frac{4}{10}$ or $\frac{2}{5}$ and find a fifth of 34, i.e., 34 ÷ 5 = $6\frac{4}{5}$ or 6.8, and then double this.

3. 15 people in Marjorie's class have dimples.

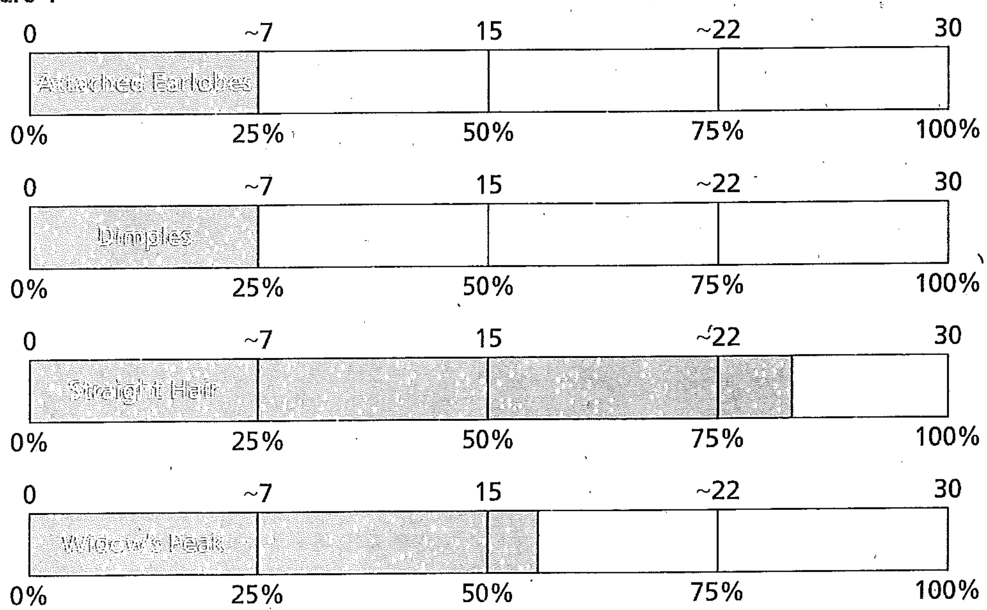
4. 44% of the people in Marjorie's class have dimples.

5. She used 10% benchmarks, then cut the interval between 40% and 50% into 10 equal spaces, each worth 1%. At the same time she cut the interval between 13.6 and 17 into 10 parts each worth 0.34. Four of these intervals of 0.34 is approximately 15, 13.6 + 1.36.

6. Answers will vary depending on the class.

C. A rate table has two labeled rows, and the data is paired in columns, with each pair of data related in the same way (or in the same ratio). The percent bar has two rows also: the top of the bar is labeled with the raw data and the bottom of the bar is labeled with the percents. These numbers are paired in the same way; that is, the relationship between 12 people and 40% is the same as 30 people and 100%.

Figure 1



Percents Encourage students to use percent bars to express themselves. Point out that their fraction work leads to the percent work.

Ratios Bring out the part-to-part comparison. Students may find the fraction of artworks chosen by the public and then the comparative fraction of 200. Take the opportunity to connect this correct strategy to the equivalent ratios strategy. In a ratio strategy, express the ratio of the two kinds of artwork, and then scale that ratio until the total of the two parts is 200.

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Assignment Guide for Problem 4.3

Applications: 21–25 | Extensions: 41–44

Answers to Problem 4.3

A. Answers will vary. Many students will guess a number close to 100. Students will want to see the right-hand side of the exhibit. Some will notice that the exhibit is cut off on the left edge of the photograph, and will want to know how many artworks are cropped out. Some students will want to know whether each frame represents a different work of art because some of the artwork appears to go together.

B. This information should greatly reduce the estimated number of works of art in the exhibit.

C. **1.** (See Figure 1.)

2. Students will likely estimate 24 works of art on the left-hand side. If this is $\frac{2}{3}$ of the total, then there should be a total of 36 works of art in this part of the exhibit. On the right, there are 8 works of art (and part of a seventh). If this is $\frac{2}{3}$ of the total, then there should be a total of 12 in this part. $36 + 12 = 48$.

D. **1.** Approximately 67% of the works were chosen by the public and approximately 33% were chosen by the curators. Students might make percent bars, equivalent ratios, or rate tables to find these percentages. If 36 pictures are 100% of the total, then 3.6 pictures are 10%, so 24 pictures are about 67% of the total.

2. If there are 200 works of art, the public chose approximately 133 and the curators chose approximately 67. Students might make percent bars, equivalent ratios, or rate tables to find these percentages. If 200 pictures are 100% of the total, then what number of pictures would match 67% chosen by the public.

E. Answers will vary. $\frac{67}{33}$ would be a name that more closely approximates the ratio of publicly chosen to curator chosen artworks.

Figure 1