



Assignment Guide for Problem 1.2

Applications: 8–13 | Connections: 37–38
Extensions: 42

Answers to Problem 1.2

A. 1.

First Move	Proper Factors	My Score	Opponent's Score
1	None	0	0
2	1	2	1
3	1	3	1
4	1, 2	4	3
5	1	5	1
6	1, 2, 3	6	6
7	1	7	1
8	1, 2, 4	8	7
9	1, 3	9	4
10	1, 2, 5	10	8
11	1	11	1
12	1, 2, 3, 4, 6	12	16
13	1	13	1
14	1, 2, 7	14	10
15	1, 3, 5	15	9
16	1, 2, 4, 8	16	15
17	1	17	1
18	1, 2, 3, 6, 9	18	21
19	1	19	1
20	1, 2, 4, 5, 10	20	22
21	1, 3, 7	21	11
22	1, 2, 11	22	14
23	1	23	1
24	1, 2, 3, 4, 6, 8, 12	24	36
25	1, 5	25	6
26	1, 2, 13	26	16
27	1, 3, 9	27	13
28	1, 2, 4, 7, 14	28	28
29	1	29	1
30	1, 2, 3, 5, 6, 10, 15	30	42

2. Sample answers:

1 is a factor of all numbers. Some numbers have only two factors, 1 and that number itself. All of the numbers with two factors are odd except 2. The numbers 4, 9, 16, and 25 have an odd number of factors; they alternate even/odd/even/odd, so the next number with an odd number of factors will be even.

B. 1. 29; it gives you the greatest point advantage, 28 points, over your opponent.

2. 1; if you choose 1, you lose your turn because 1 has no proper factors for your opponent to choose. Other than 1, 24 and 30 are the worst moves. Your opponent gets 12 points more than you get.

C. 1. 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29; prime numbers are good first moves. A prime number has only two factors: the number itself and 1. Since the only proper factor of a prime number is 1, your opponent scores only 1 point. Large prime numbers are the best first moves. For example, 29 gives you 29 points and your opponent only 1 point.

2. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, and 30; a composite number with many proper factors would be a bad first move, because your opponent would get many points. Composite numbers with proper-factor sums that are greater than the number are the worst first moves. For example, if you choose 30 or 24 as a first move, your opponent will score 12 points more than you will. Not all composite numbers are bad first moves. For example, if you choose 25, you will get 25 points, and your opponent will only get 6.

D. 1. two factors: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29

2. three factors: 4, 9, and 25

3. four factors: 6, 8, 10, 14, 15, 21, 22, 26, and 27

4. six factors: 12, 20, and 28

5. It is better to choose numbers with only 2 or 3 factors as a move because your opponent always scores fewer points than you.

E. 28; $1 + 2 + 4 + 7 + 14 = 28$.

F. 1. Anna is wrong. She forgot 3, 8, and 12, which are also proper factors of 24.

2. To find all the proper factors of a number n , you can check all of the whole numbers 1 to $\frac{n}{2}$. Since factors of a number occur in pairs, you can find the factor pairs starting with $(1, n)$ and stop when the factor pairs begin to repeat in reverse order. (Factor pairs occur in the next problem.)