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Assignment Guide for Problem 2.1

Applications: 1–3 | Connections: 17–19

Answers to Problem 2.1

A. **1.** For 20 bikes, Rocky will charge \$770 and Adrian will charge \$600.

2. For 40 bikes, Rocky will charge \$1,140 and Adrian will charge \$1,200.

3. For 32 bikes, Rocky will charge \$1,013 and Adrian will charge \$960 (assuming linear interpolation between given data points).

B. **1.** If a group has \$900 to spend, it can rent 26 bikes from Rocky or 30 from Adrian.

2. If a group has \$400 to spend, it can rent 5 bikes from Rocky or 13 from Adrian.

C. Students should realize that for both shops rental cost increases steadily as number of bikes increases.

1. The cost for renting from Rocky starts higher but increases more slowly than cost for renting from Adrian; Rocky's charge per bike decreases as the number of bikes increases. In fact, if one makes a coordinate graph of the costs for renting from Rocky, the pattern will resemble the data for jumping jacks or riding bicycles, in which the data points rise rapidly at first, but that rate of increase drops off.

2. If students study the data very carefully, they'll discover that Adrian charges a flat rate of \$30 per bike.

D. Students probably will not use the word *interpolate*, but they should understand that estimating costs for numbers of bicycles not on the graph or in the table depends on assuming given points provide upper and lower bounds for the estimate.

E. Some students may point out that costs for specific numbers of bicycles are easier to read from a table. The general trend in pricing is easier to read from a graph.

F. Students will have different opinions about which data representation—table or graph is most useful in making a decision and in presenting the case for one's choice. Given the data in the table and the graph, students should decide that Adrian offers a better deal for a number of bikes less than about 36 and Rocky offers the better deal for more than 36 bikes (though interpolation between 35 and 40 bikes is not certain in Rocky's offer).

 Problem 2.2 Pacing 1 Day

2.2 Finding Customers: Linear and Nonlinear Patterns

Focus Question How are the relationships between independent and dependent variables in this Problem different from those in Problem 2.1? How are the differences shown in tables and graphs of data?

Launch

Review the economic terms involved—price is what each customer will pay to take part in the Bike Tour and income is the money that the tour business will take in from those customers.

Explore

Keep an eye on how students choose the independent and dependent variables. Also encourage students to describe exactly what happens to the number of customers as the price increases to $150, $200, and so on. Ask them to provide reasons.

You want students to think hard about how to choose independent and dependent variables and scales for a graph.

Summarize

The key objectives of this Problem are to illustrate examples of a decreasing linear relationship and a quadratic relationship that has a maximum value for the dependent variable (a concave down parabola graph).

- *For each price, how much money would be collected if the people who said they would pay that price actually do so?*
- *How does the number of customers change as the price increases?*
- *How is that pattern shown in the table and the graph?*
- *How is the relationship between price and income different from all preceding relationships? How is that difference shown in the table and graph of (price, income) data?*

Key Vocabulary

- income

Materials

Labsheets
- 2.2A: Finding Customers
- 2.2B: Predicting Income

- grid paper
- Coordinate Grapher Tool
- Data and Graphs Tool

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Assignment Guide for Problem 2.2

Applications: 4–7 | Connections: 20–21
Extensions: 22

Answers to Problem 2.2

A. **1.** The number of people who said they would take the tour depends on the price. In this Problem, price is the dependent variable and the number of the customers is the independent variable. (See Figure 1.)

2. The number of customers decreases steadily as the price increases—10 lost customers for every increase of $100 in price.

3. The change in number of customers is shown in the table by the decrease in that row. The change is shown in the graph by the downward (left-to-right) slope of the points and the connecting line.

4. You could find the number of customers for a price between two entries in the table by proportional reasoning. For example, if the price in the middle of the interval is half of the way from one price to the next higher price in the table, then the number of customers will be half of the way from one number to the next lower number in the table. For $175, one would expect 32.5 customers (of course, you can't have exactly 32.5 customers). For $325 one would expect 17.5 customers.

B. **1.** (See Figure 2.)

2. (See Figure 3.)

3. As tour price increases, tour income increases until it reaches a maximum of $6,250 at a tour price of $250. Then it begins to decrease until it reaches $0 when the tour price is set at $500. (There are no willing customers.)

Based on the data and graph in parts (1) and (2), it looks like a tour price of about $250 is optimal. It will yield tour income of $6,250.

Figure 1

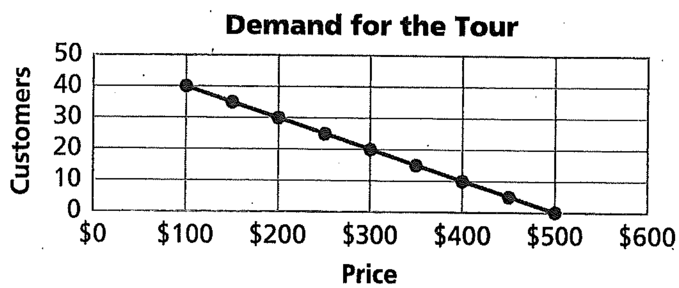
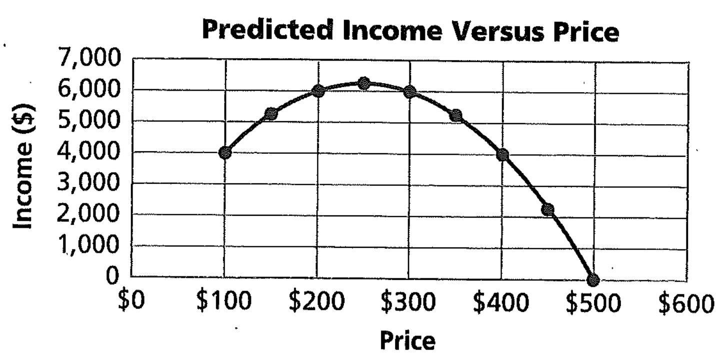


Figure 2

Predicted Tour Income

Tour Price	$100	$150	$200	$250	$300	$350	$400	$450	$500
Number of Customers	40	35	30	25	20	15	10	5	0
Tour Income ($)	4,000	5,250	6,000	6,250	6,000	5,250	4,000	2,250	0

Figure 3



Assignment Guide for Problem 2.3

Applications: 8–9 | Connections: 23

Answers to Problem 2.3

A. **1.** (See Figure 1.)

2. (See Figure 2.)

3. **a.** The pattern in the table and graph shows business profit starting as a negative value (or loss) for a tour price of only $100. Then profit increases rapidly to a maximum of a bit more than $3,000 for tour prices around $325. Then profit declines as prices increase beyond $350 per customer.

b. The pattern should make some sense. If the price is too low, there will be many customers and large operating costs, but too little income to pay those costs (thus the loss of $2,000 for tour price of $100). As the price increases, the number of customers decreases, but so does the operating cost per customer. However, when the price rises too far, it leads to a loss of customers and income that overwhelms the associated decrease in operating costs.

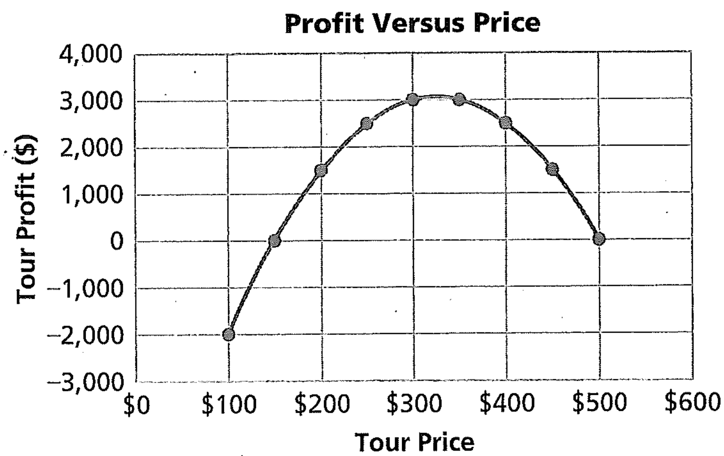
c. Based on the analysis of profit predictions, a tour price of about $325 seems best because both the table and graph suggest this will yield maximum profit.

Figure 1

Predicted Tour Profit

Tour Price	$100	$150	$200	$250	$300	$350	$400	$450	$500
Customers	40	35	30	25	20	15	10	5	0
Tour Income ($)	4,000	5,250	6,000	6,250	6,000	5,250	4,000	2,250	0
Operating Cost ($)	6,000	5,250	4,500	3,750	3,000	2,250	1,500	750	0
Tour Profit or Loss	–2,000	0	1,500	2,500	3,000	3,000	2,500	1,500	0

Figure 2



B. 1. The x-values represent the day the temperature occurs. The point (0, −1) corresponds to the day 0, or the day the partners are checking the forecasts. The value −1 for x represents the day before, and the value +1 for x represents the predicted temperature for the next day, etc.

2. (See Figure 3.)

3. There is no predictable pattern. The students might say that the temperatures are cold.

C. 1. a. To reach (−3, 4) from (3, 4), move directly across the y-axis horizontally.

b. To reach (−3, −4) from (3, 4), move directly through the origin to a point equidistant on the other side of (0, 0).

c. To reach (3, −4)from (3, 4), move directly across the x-axis vertically

d. To reach (1.5,−2) from (3, 4), move 1.5 units to the left and 6 units down.

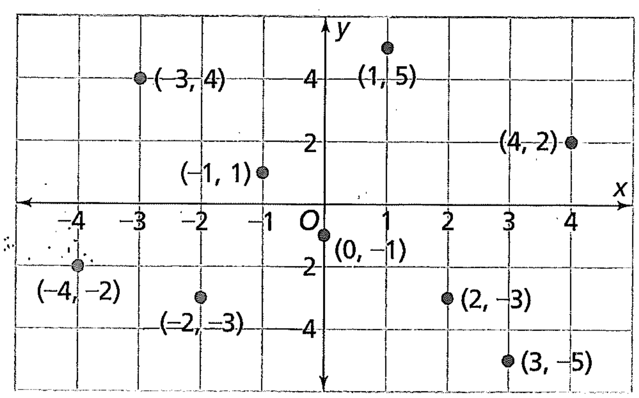
e. To reach (−1.5, 2) from (3, 4), move 4.5 units to the left and 2 units down.

f. To reach (−2.5 − 3.5) from (3, 4), move 5.5 units to the left and 7.5 units down.

D. 1. Jakayla is correct. (−3, 4) is a reflection of (3, 4) across the y-axis, since the x-coordinate changes from 3 to 3. Similarly, (3, −4) is a reflection of (3, 4) across the x-axis. (−3, −4) is a reflection of (3, 4) across the origin.

2. Mitch is correct. The x-axis and y-axis are like mirrors. The image, or reflection, of (−3, 4) over the y-axis is (3, 4). The same is true for the other points. The mirror image is also a reflection.

Figure 3



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Assignment Guide for Problem 2.4

Applications: 10–16 | Extensions: 24

Answers to Problem 2.4

A. Graph 2: The number of students who go on a school trip depends on the price of the trip for each student. The pattern in graph 2 seems most likely to represent the relationship of those variables because it shows number of students going on the trip declining as price increases.

B. Graph 6: When a skateboard rider goes down one side of a half pipe ramp and up the other side, her speed increases until she reaches the bottom and then decreases as she rides up the other side. Despite the appeal of graph 5 (it shows a half-pipe shape), the graph best showing speed over time in this case is graph 6.

C. Graph 1 is probably best for showing how water level changes over time when someone fills a tub, takes a bath, and empties the tub. Graph 1 shows water entering the tub at a constant rate for a time; then a short interval when the bather allows the water to cool a bit; then a jump in water level when the bather enters the tub; then a period of constant depth; then a quick drop when the bather gets out of the tub; and finally a steady decline as water is drained out of the tub.

D. Graph 4: The waiting time for a popular ride at an amusement park is related to the number of people in the park. Graph 4 shows that waiting time increases as the number of people increases.

E. Graph 8: The daily profit or loss of an amusement park increases as the number of paying customers increases. Graph 8 shows this. It starts with a negative value for profit because there will still be expenses if there are no customers.

F. Graph 3: As the seasons change, the number of hours of daylight changes in a periodic pattern illustrated best by Graph 3.

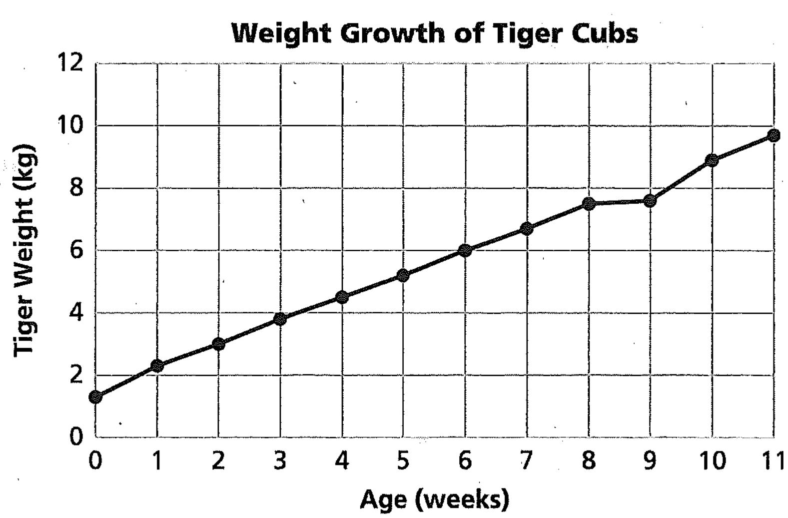
G. Graph 7: The daily profit or loss of an outdoor skating rink depends on the daytime high temperature. For temperatures that are either too high or too low, the number of customers and thus the daily profit will decline and even become a loss. For temperatures in the cool-but-not-too-cold range, the rink will make a profit. This pattern is shown best by Graph 7.

H. Graph 2: Weekly attendance at a popular movie declines as time passes from the date the movie first appears in theaters. The most common pattern is that shown in Graph 2.

Applications

1. a. 2.3 kg
   b. 8.9 kg
   c. between 7 and 8 weeks
   d. It makes sense to connect points on a coordinate graph, because the weight growth occurs all throughout each month. (See Figure 1.)
   e. The tiger cubs' weight increases fairly steadily at a rate of about 0.75 kg per week.
   f. The rate of change is seen in the successive differences of weights from one week to the next in the table.
   g. The relatively steady growth rate shows up in the graph as a linear increasing pattern of data points.
2. a. 50 laps will cost $105 at Kartland and about $95 at Thunder Alley.
   b. 20 laps will cost $45 at Kartland and about $50 at Thunder Alley.
   c. 35 laps will cost $75 at Kartland and about $70 at Thunder Alley.
   d. It looks as if the pricing plan at Kartland is $5 for the group and then $2 per lap; at Thunder Alley it is $20 for the group and then $1.50 per lap. The table shows an increase in cost of $20 for each 10 laps ($2 per lap), but the first 10 laps cost $25 (suggesting the $5 group fee). The graph starts at $20 for 0 laps (suggesting the $20 group fee) and increases as a steady slope of over 10 and up 15 (suggesting the per-lap charge of $1.50). The slope is clearest for the points (0, 20) and (40, 80), which shows that for 40 laps the increase in cost is $60.
   e. Kartland is cheaper than Thunder Ally if Desi and his friends want at most 30 laps (where both charge $65).
3. a. (See Figure 2.)
   b. It doesn't make sense to connect the points because campers come in whole numbers only.
   c. The camping fee appears to be $12.50 per campsite.

Figure 1



d. The fee rule is shown in the steady (linear) upward trend of points on the graph—increase of one camper leads to rise of $12.50 in total cost.

4. a. It looks as if each increase of $5 in the price will lead to a loss of 10 shirt sales. As price increases, fewer people will be willing to pay for the shirts, so yes, this pattern is expected.

b. (See Figure 3.)

c. As price increases, the income increases to a maximum of $450 at a price of $15 per shirt; however, the projected income then begins to decrease because the loss of customers overcomes the gain from higher prices. Students might find this at first surprising, but not unbelievable.

d. (See Figure 4 and 5.)

e. The first graph shows steady decline in sales as price increases; the second graph shows increase to a maximum and then decrease of income as price increases.

5. a. (See Figure 6.)

b. (See Figure 7.)

c. The pattern of change in the table and the graph is very similar to that of Exercise 3. The only difference is that the rate of change in camping gear cost is double that of the rate of change for campsite usage.

Figure 2

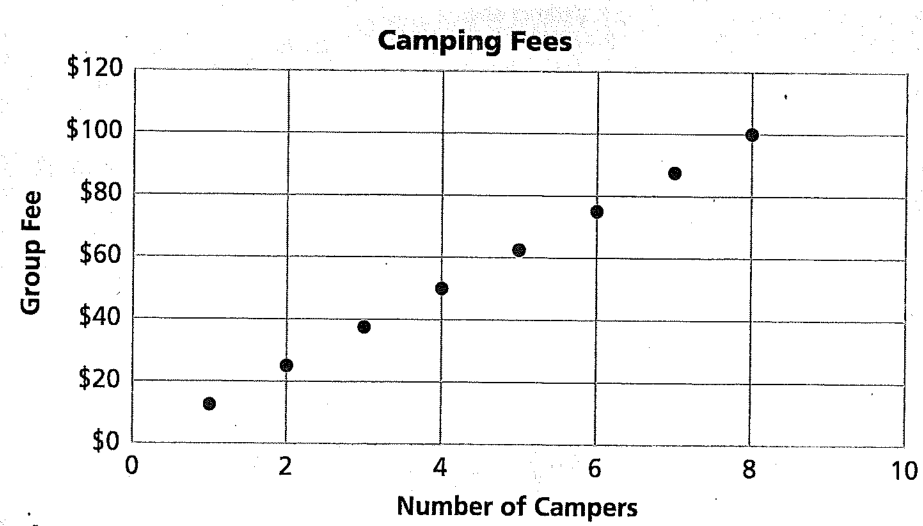


Figure 3

Projected Shirt Sales

Price per Shirt	$5	$10	$15	$20	$25
Number of Shirt Sales	50	40	30	20	10
Value of Shirt Sales	$250	$400	$450	$400	$250

Figure 4

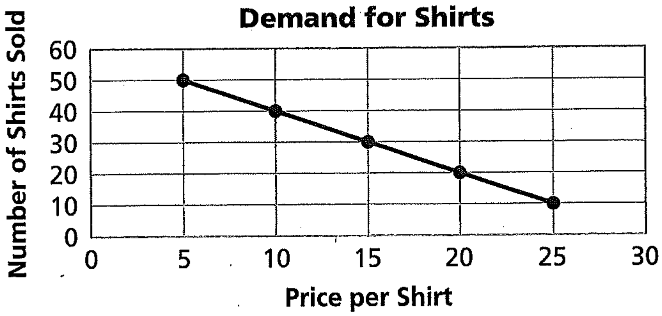


Figure 5

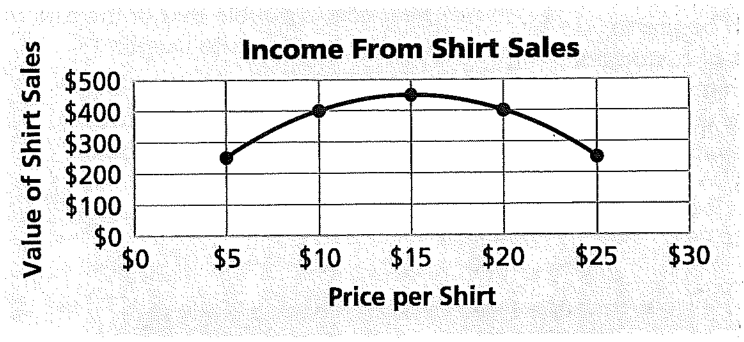
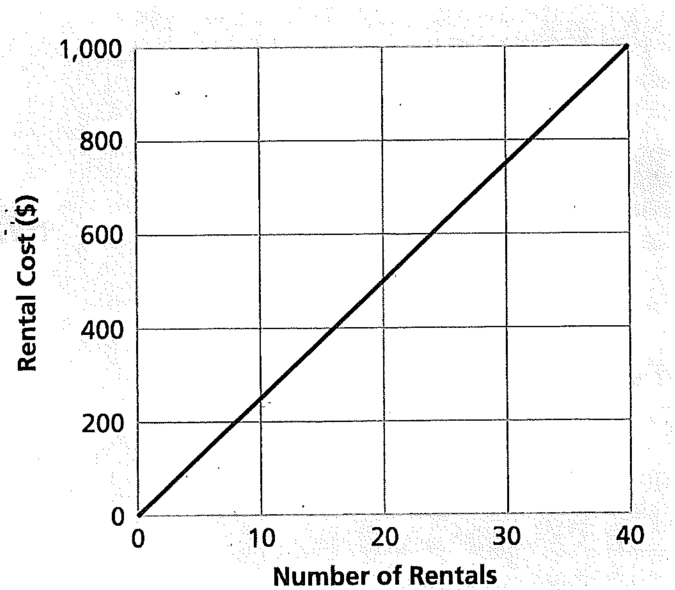


Figure 6

Miles	0	5	10	15	20	25	30	35	40
Charge ($)	0	125	250	375	500	625	750	875	1,000

Figure 7



6. a. East Coast Trucks (See Figure 8.)

   b. Philadelphia Truck Rental (See Figure 9.)

   c. East Coast graph starts at (0, 0). (See Figure 10.)

   d. The tables and graph show equal charges for 200 miles, with East Coast Trucks cheaper for fewer miles and Philadelphia Truck Rental cheaper thereafter.

7. a. (See Figure 11.)

   b. Since camping fees appear to be charged by the whole day, it does not make sense to connect the dots in this graph.

   c. It appears that there is a $10 registration fee, then $10 per day for the first six days and $5 per day thereafter.

Figure 8

Miles	0	100	200	300	400	500	600	700	800
Charge ($)	0	400	800	1,200	1,600	2,000	2,400	2,800	3,200

Figure 9

Miles	0	100	200	300	400	500	600	700	800
Charge ($)	200	500	800	1,100	1,400	1,700	2,000	2,300	2,600

Figure 10

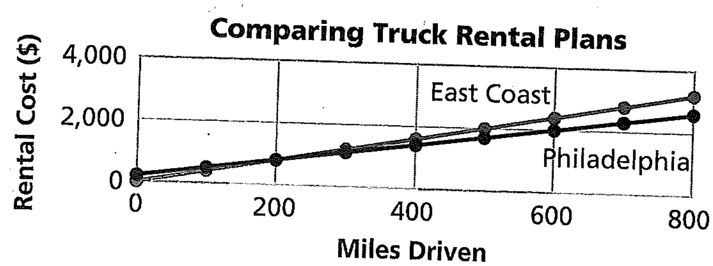
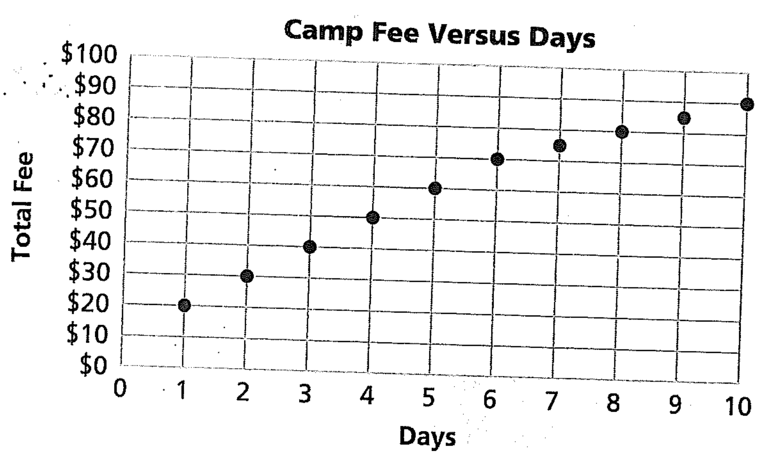


Figure 11



8. a. (See Figures 12 and 13.)

   b. It appears from the plotted points that the indoor water park makes a profit if the temperature is not too cold or not too hot, with maximum profit when the outdoor temperature is about 50 degrees Fahrenheit. That temperature would be too cold for outdoor swimming but not so cold that it discourages people from getting out and around. This overall pattern is shown by the pattern of points that rise from left to right up to the point representing (50, 7,500) and then falling again to the point corresponding to (100, 0).

9. a. City Hall (0,0)
   b. hospital (−1, 4)
   c. stadium (4, 1)
   d. police station (3, −2)
   e. fire station (−2, −4)
   f. middle school (−3, −1)
   g. high school (2, 3)
   h. shopping mall (−4, 2)

10. a. Graph 3
    b. Graph 4
    c. Graph 1
    d. Graph 5
    e. Graph 2

Figure 12

Temperature (°F)	−10	−5	0	10	30	50	70
Profit ($)	−5,000	1,000	1,500	2,500	5,000	7,500	4,500

Figure 13

Temperature (°F)	80	90	100
Profit ($)	3,000	1,000	0

11. Answers will vary.

   a. (See Figure 14.)

   b. (See Figure 15.)

   c. (See Figure 16.)

   d. (See Figure 17.)

   e. (See Figure 18.)

12. a. Graph 2

   b. Graph 5

   c. Graph 1

   d. Graph 4

   e. Graph 3

13. Graph B is the best match for the situation.

14. a. Maximum temperature of 30°C occurred at points 2, 2.5, and 3.5 hours into the hike.

   b. Temperature seems to be rising most rapidly between 1 and 1.5 hours into the hike.

   c. Temperature seems to falling most rapidly between 3.5 and 4.0 hours into the hike.

   d. Temperature was about 24°C at points about 1.25 and 3.75 hours into the hike.

   e. The thunderstorm (and likely cooling of the temperature) probably occurred about 3.5 hours into the hike and lasted about half an hour.

15. H

16. C

Figure 14

Time (seconds)	1	2	3	4	5	6	7	8
Distance (feet)	5	10	15	20	25	30	35	40

Figure 15

Time (seconds)	1	2	3	4	5	6	7	8
Distance (feet)	1	3	6	11	17	24	31	40

Figure 16

Time (seconds)	1	2	3	4	5	6	7	8
Distance (feet)	8	14	18	20	22	26	32	40

Figure 17

Time (seconds)	1	2	3	4	5	6	7	8
Distance (feet)	$6\frac{2}{3}$	$13\frac{1}{3}$	20	20	20	$26\frac{2}{3}$	$33\frac{1}{3}$	40

Figure 18

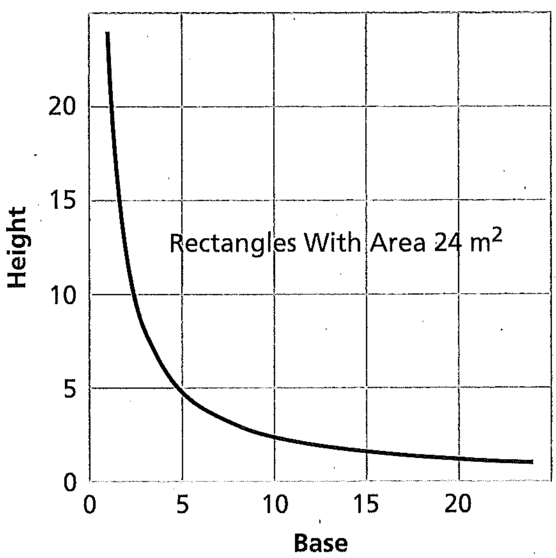
Time (seconds)	1	2	3	4	5	6	7	8
Distance (feet)	9	16	23	29	34	37	39	40

Connections

17. a.

Length	1	2	3	4	6	8	12	24
Width	24	12	8	6	4	3	2	1

b.



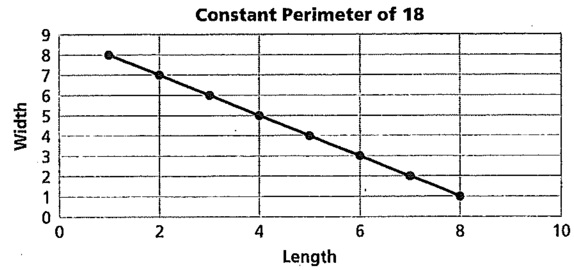
c. It makes sense to connect the points because rectangles can have any length and width with product 24 (e.g., 1.5 and 16).

d. As length increases, the corresponding width decreases rapidly at first and then more slowly to keep the constant product of 24. This is an example of an inverse variation relationship that students will encounter in future CMP units. Such relationships have the algebraic form $xy = k$ or $y = \frac{k}{x}$.

18. a.

Length	1	2	3	4	5	6	7	8
Width	8	7	6	5	4	3	2	1

b.



c. It makes sense to connect the points on the graph because lengths and widths can be any parts of a meter (e.g., 3.5 and 5.5).

d. As length increases at a constant rate, the corresponding widths decrease at a constant rate. This is shown by the downward slope of the graph from left to right and by the patterns of change in the rows of the table.

19. **a.** (See Figure 19.)

b. After dropping rapidly from 1968 to 1976, the times have been quite constant. In fact, the record was set in 1996 (perhaps before testing for performance-enhancing drugs became much more rigorous).

20. **a.** (See Figure 20.)

b. After a fairly successful opening week, interest in the movie peaked in the second week and then began to decline (probably as the potential customers all saw the film). The decline is a bit more rapid in weeks 3–6. In the table, you see the weekly earnings decreasing after the peak in week 2. In the graph, you see the points going down to the right after week 2.

c. Total earnings in the eight weeks for which data are given were $83 million.

Figure 19

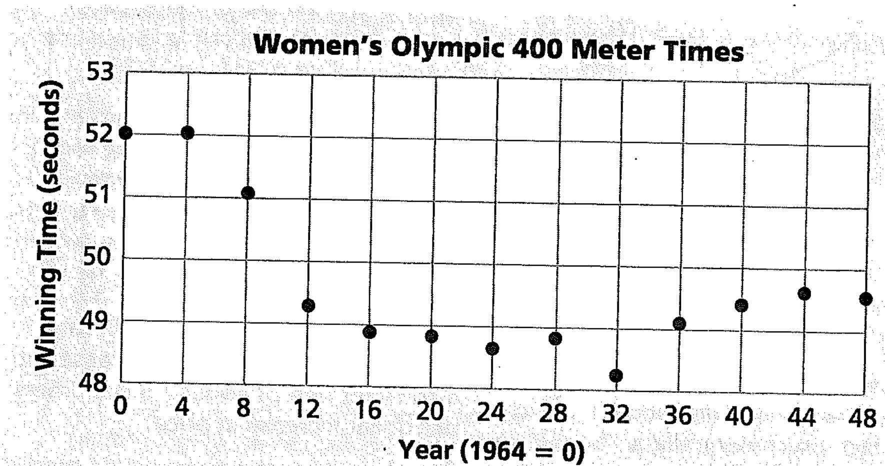
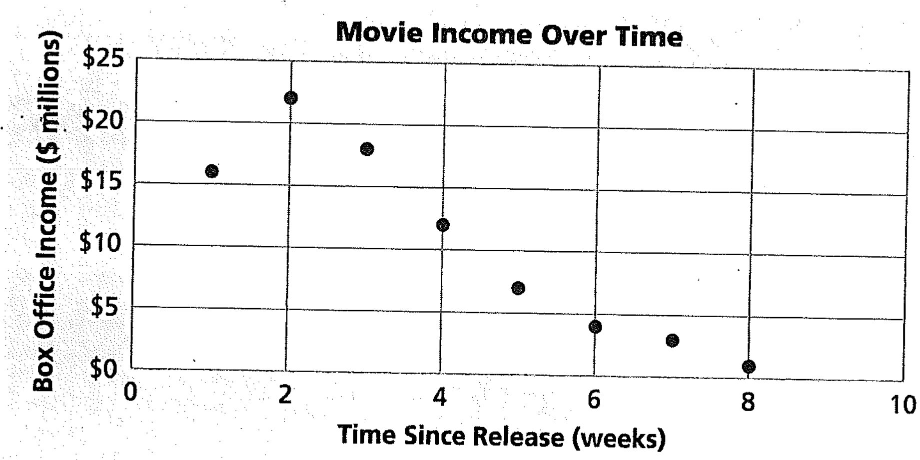
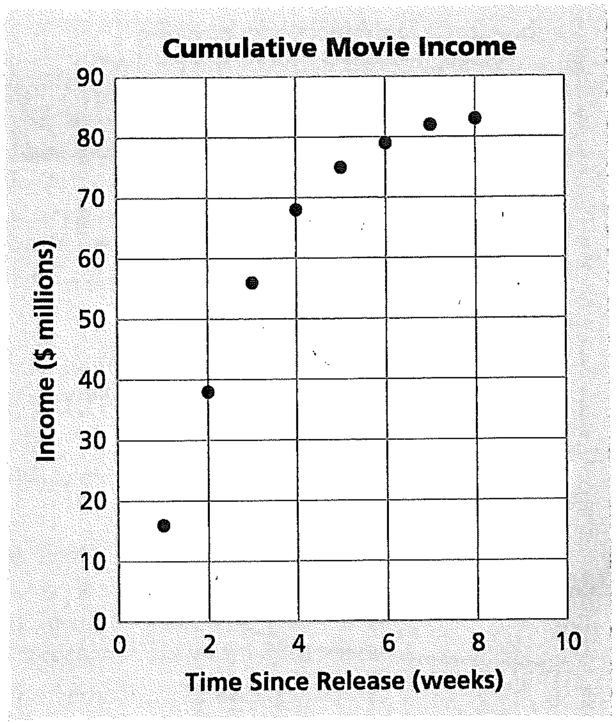


Figure 20



d.



e. The cumulative earnings increase rapidly at first, and then more slowly as the film's audience is tapped out. This pattern is shown by the rapid rise of the data points at first and then a slower rise from week to week for the later points.

21. In fact, the choice of variable to be labeled independent and that to be labeled dependent is often (but not always in applications) arbitrary. It depends on the way the person framing the problem views the variables.

Extensions

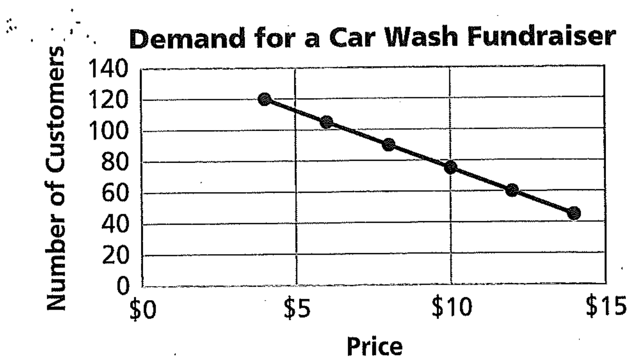
22. a. (See Figure 21.)

It makes some sense to connect the data points, since fractional dollar prices are possible. In fact, psychologically, a price of $5.99 might seem much lower than $6.00 to some people.

b. As price increases at a constant rate, the number of customers decreases at a constant rate of about 7.5 customers per dollar increase in price.

c. Based on the pattern, you would predict 30 customers if the price were $16, 0 customers if the price were $20, and 135 customers if the price were $2.

Figure 21



d. (See Figure 22.)

e.

Table 1: Linear Relationship

x	0	1	2	3
y	5	6	7	8

Table 2: Exponential Relationship

x	0	1	2	3
y	1	2	4	8

f. It makes sense to consider price to be the independent variable because that is the quantity that the car wash operators can manipulate as they choose in this situation. The number of customers will be determined by how the price is set.

g. In this graph it does make sense to connect the points because any dollar prices are quite feasible.

h. The pattern of change in projected car wash income is similar to income functions encountered in Problem 2.3—as the price increases from a low value, the income increases to a maximum value of $750 at the $10 price, and the income decreases as the price increases further. This happens because a higher price reduces the number of customers.

i. To calculate profit when the $1.50 cost per car is considered, one has to deduct that number from the price per customer before multiplying to get a total. For instance, a per-car price of $4 would yield 120 customers and income of $480. However, only $2.50 of that price would be profit, so the projected profit at a price of $4 per car would actually be $2.50 × 120, or $300.

23. Sample answers:

a. (1, 0.5), (2, 1), (3, 1.5), (4, 2), (5, 2.5)

b. (0.1, 0.05), (0.2, 0.1), (0.33, 0.15), (0.4, 0.2), (0.5, 0.25)

24. a. Graph 1 shows that *y* increases by 2 for every increase of 1 in *x*.

Graph 2 shows that y increases by 1.5 for every increase of 1 in *x*.

Graph 3 shows that *y* increases by about 0.75 for every increase of 1 in *x*.

Graph 4 shows that *y* increases by 1 for every increase of 1 in *x*.

b. Graph 1 shows the greatest rate of increase in the dependent variable (2 up for each 1 over). While Graph 4 looks equally steep, the dependent variable increases only from 0 to 5 over the interval shown.

c. While Graphs 2 and 3 seem to have the same slope, the scales on the *y*-axis are different. So Graph 3 shows a pattern of slower increase in the dependent variable over the interval shown. Graph 2 shows that y increases by 1.5 for every increase of 1 in *x*. Graph 3 shows that *y* increases by about 0.75 for every increase of 1 in *x*.

Figure 22

Price Customers Would Pay for a Car Wash

Car Wash Price	$4	$6	$8	$10	$12	$14
Number of Customers	120	105	90	75	60	45
Projected Income	$480	$630	$720	$750	$720	$630