ACE Answers

Investigation 1

Applications

1. a. Answers will vary. Possible answers:

The seventh-grade goal is twice the fifth-grade goal.

Each grade's goal is \$60 more than the previous grade's goal.

The sixth-grade goal is $1\frac{1}{2}$ times the fifth-grade goal.

b. Answers will vary. Possible answers:

The teachers' goal is $\frac{3}{4}$ of the eighth-grade goal.

For every \$75 the teachers plan to collect, the eighth graders plan to collect \$100.

The teachers' goal is \$75 less than the eighth graders' goal.

2. $\frac{24}{32}$ or $\frac{3}{4}$

3. a. This is true. If the teacher made groups of 2 boys and 4 girls, there would be six of these groups with no children left out of a group.

b. Answers will vary. Possible answers:

There are twice as many girls as boys.

There are 12 more girls than boys.

4. There could be 3 boys and 2 girls. There could be 6 boys and 4 girls, 9 boys and 6 girls, etc. If the class is going to be close in size to the one in ACE Exercise 3, there could be 21 boys and 14 girls. In each of these possibilities, you can think about making groups of 3 boys and 2 girls. The ratio does not tell you how many of these groups there are, so there are many possibilities.

5. Possible answers: eighths, twelfths and sixteenths (multiples of 4)

6. halves, fourths, twelfths

7. $\frac{1}{4}$

8. $\frac{3}{4}$

9. $\frac{2}{3}$

10. a. Shown are $\frac{3}{4}$, $\frac{6}{8}$, $\frac{12}{16}$.

b. Another equivalent fraction would be $\frac{15}{20}$.

11. a. $\frac{5}{5}$ is the same as 1.

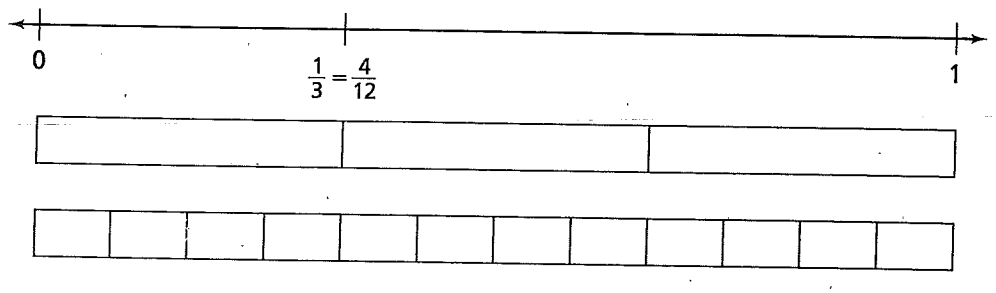
b. Sally is correct. Any two segments are $\frac{2}{5}$ of a whole. She is concentrating on a fraction as a *part of a whole*. However, if you took any two segments and lined them up to start with 0, you would arrive at a *location* of $\frac{2}{5}$ on the number line.

c. $\frac{1}{5}$ would now be marked with $\frac{2}{10}$, $\frac{2}{5}$ with $\frac{4}{10}$, $\frac{3}{5}$ with $\frac{6}{10}$, $\frac{4}{5}$ with $\frac{8}{10}$, and 1 with $\frac{10}{10}$. These are equivalent fractions. For every one fifth there are two tenths, so for two fifths there are four tenths, etc.

d. Possible answers: For every one half, there would be 5 tenths. For every one whole, there would be 10 tenths.

12. Correct. (See Figure 1 for possible picture of number line and fraction strips.)

Figure 1



13. Correct. (See Figure 2 for possible picture of number line and fraction strips.)

14. Incorrect. (See Figure 3 for possible picture of number line and fraction strips.)

15. Incorrect. (See Figure 4 for possible picture of number line and fraction strips.)

16. (See Figure 5.)

17. (See Figure 6.)

18. Possible answer: You could draw a fraction strip and divide it into five equal parts. Shade three of these parts to represent $\frac{3}{5}$. Then divide each of the five parts into two equal parts. You would then have ten equal parts, and six of the parts would be shaded. Therefore, $\frac{3}{5}$ is the same as $\frac{6}{10}$, so is equivalent to $\frac{3}{5}$.

Figure 2

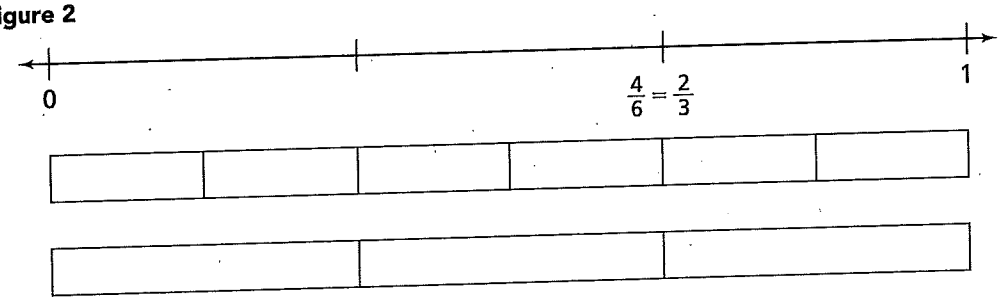


Figure 3

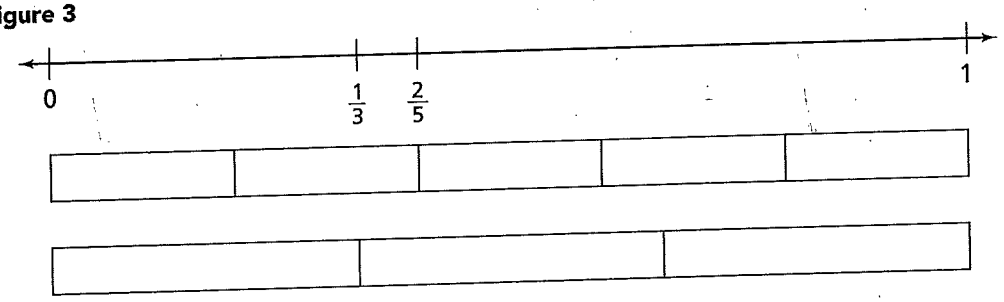


Figure 4

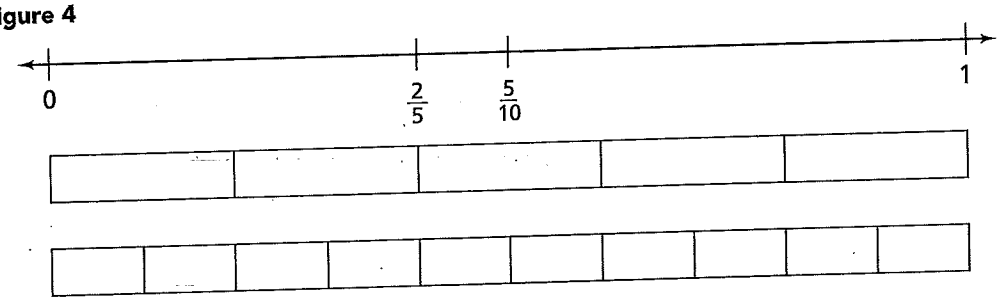
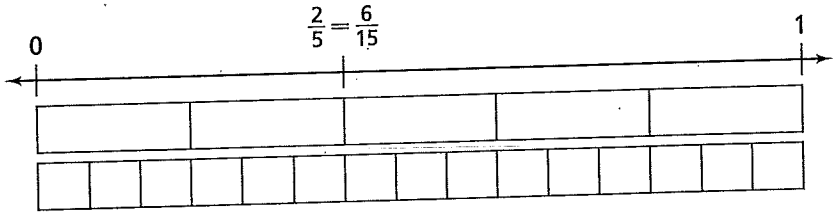


Figure 5



19. **a.** $\frac{3}{4}, \frac{6}{8}, \frac{12}{16}$

b. 2.1 GB

20. The diagram below shows that the distance between these fractions is $\frac{1}{8}$. (See Figure 7.)

21. $\frac{1}{4}$; other estimates are acceptable

22. $\frac{3}{8}$; other estimates are acceptable

23. **a.** about two thirds $\left(\frac{2}{3}\right)$

b. about 80 cups

c. about one third $\left(\frac{1}{3}\right)$

d. about 40 cups

24. A

25. J

26. $\frac{37}{120} \approx \frac{1}{3}$, $\frac{10}{120} = \frac{1}{12}$

27. **a.** $\frac{5}{6}$ of a dispenser is almost full. (See Figure 8.)

b. $\frac{3}{12}$ of a dispenser is almost empty. (See Figure 9.)

c. $\frac{5}{8}$ of a dispenser is about half full. (See Figure 10.)

28. $\frac{155}{755}$ or $\frac{1}{5}$

29. The MathCast: $\frac{45}{60}$ or $\frac{3}{4}$ of the podcast has been downloaded.

The Fraction Podcast: $\frac{20}{30}$ or $\frac{2}{3}$ of the podcast has been downloaded.

Figure 6

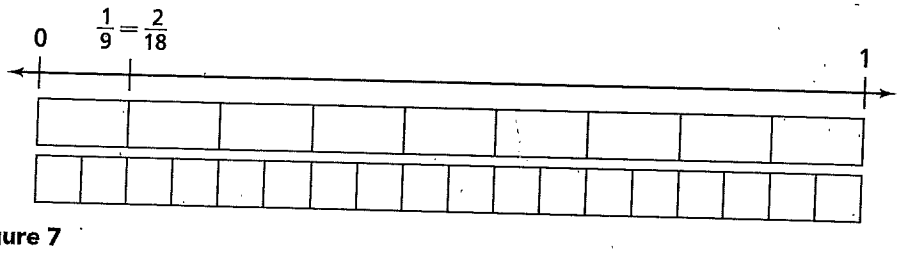


Figure 7

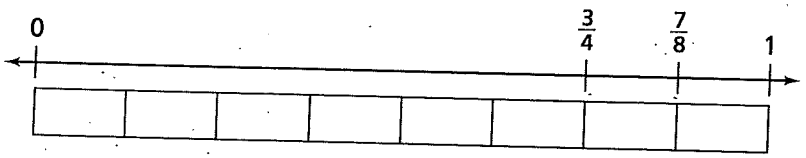


Figure 8

Figure 9

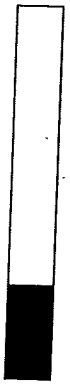


Figure 10

30. Answers will vary. Possible answer: The MathCast is twice the size of the Fraction Podcast.

31. Answers will vary. Possible answer: The downloaded part of The MathCast is more than twice the downloaded part of the Fraction Podcast.

It is possible that some students will take the directions to mean to compare the fractions from part (a). In this case, the downloaded fraction of The MathCast is only a little bit larger than the downloaded fraction of the Fraction Podcast.

32. Assuming a constant download rate, the MathCast takes 88 seconds from beginning to end. The Fraction Podcast takes 3 minutes.

33. a. Answers will vary. Possible answers: Dan 8 miles, Karim 4 miles; Dan 3 miles, Karim $1\frac{1}{2}$ miles, etc.

b. Answers will vary. Possible answers: Karim 4 miles, Shawn 3 miles; Karim 8 miles, Shawn 6 miles; Karim 1 mile, Shawn $\frac{3}{4}$ mile, etc.

c. Dan ran further than Karim, who ran further than Shawn. So Dan ran furthest.

34. a. Answers will vary. Possible answers: Kate could have scored 6 points, Sue 4 points. Kate could have scored 12 points, Sue 8 points, etc. Fractional numbers of points are not possible. The ratio of Kate's points to Sue's points is always 3 to 2.

b. Lisa could have made only free throws, which are worth 1 point.

c. Kate scored the most points because she scored more than Sue, who scored the same number as Lisa.

d. Lisa made the most baskets because she made more than Sue, who made the same number as Kate.

Connections

35. Yes, because 450 can be divided evenly into groups of 5, 9, and 10 with no remainders.

36. Yes, because $12 \times 4 = 48$.

37. No, not evenly. $150 \div 4 = 37.5$

38. Yes, because $3 \times 17 = 51$.

39. C

40. J

41. Mr. Chan: one third or $\frac{1}{3}$

Mr. Will: one fourth or $\frac{1}{4}$

Ms. Luke: one fourth or $\frac{1}{4}$

42. Orange juice was the most popular in Mr. Chan's class because $\frac{1}{3}$ is greater than $\frac{1}{4}$.

43. a. Mr. Will: about 7 cans of orange juice

Ms. Luke: about 8 cans of orange juice

b. Mr. Chan: 30 cans of juice

Mr. Will: about 28 cans of juice

Ms. Luke: about 32 cans of juice

44. a. Miguel is correct. If a number is divisible by 2, you can separate it into two equal halves.

b. Manny is also correct. If a number is divisible by 3, you can separate it into 3 groups of equal size, or into thirds.

c. Lupe is correct. If a number is divisible by *n*, you can separate it into n groups of equal size, or into *nths*.

45. a. Possible answer: You can measure with a twelfths strip all fractions with denominators that are factors of twelve (halves, thirds, fourths, sixths, and twelfths). You can also measure with a twelfths strip some fractions that have denominators that are multiples of twelve. For example, you can measure with a twelfths strip $\frac{12}{24}$, which is equivalent to $\frac{6}{12}$, but you cannot measure $\frac{13}{24}$. (Note to teacher: Actually you can measure any fraction with a twelfths strip but you will not get a whole number numerator. This answer should not be excluded, but it is not expected.)

b. Possible answer: If you start with a fraction strip folded into 2, 3, 4, or 6 parts of equal size, you can repartition the strip to make a twelfths strip. You can repartition strips that are factors of 12 to make a twelfths strip.

46. a. Possible answer: You can measure with a tenths strip all fractions with denominators that are factors of ten (halves, fifths, and tenths). You can also measure with a tenths strip some fractions that have denominators that are multiples of ten. For example, you can measure with a tenths strip $\frac{12}{20}$, which is equivalent to $\frac{6}{10}$, but you cannot measure $\frac{11}{24}$. **(Note to teacher:** Actually you can measure any fraction with a tenths strip but you will not get a whole number numerator. This answer should not be excluded, but it is not expected.)

b. Possible answer: If you start with a fraction strip folded into 2 or 5 (factors of 10) parts, you can repartition the strip to make a tenths strip.

47. a. 4 beetles

b. 12 beetles

c. $3\frac{1}{4}$ fraction strips long

48. a. 1 and 5 are the common factors of 25 and 30.

b. 1, 2, 5, 10, 25 and 50 are the common factors of 250 and 300.

c. Assuming the two numbers in the ratio are whole numbers, they will always have a common factor of 1. No other common factors are guaranteed. For example, the ratio 25 : 30 is equivalent to 5 : 6. The only common factor of 5 and 6 is 1.

49. a. The common factors of 25 and 250 are 1, 5 and 25.

b. The common factors of 30 and 300 are 1, 2, 3, 5, 6, 10, 15 and 30.

c. Assuming all of the numbers in the ratios are whole numbers, the first numbers in two equivalent ratios will always have the common factor of 1.

Other common factors will depend on the "simplest form" of the ratio. The simplest form of a ratio is the equivalent ratio with the smallest whole numbers. In the case of the ratio 25 : 30, the simplest form is 5 : 6. The first number in the simplest form of the ratio (here 5) will be a common factor of the first numbers in any other equivalent ratios.

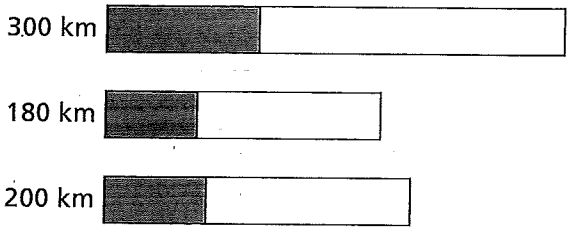
50. about $\frac{1}{7}$

51. about $\frac{5}{7}$

52. a. (See Figure 11.)

b. 100 km, 60 km, about 67 km. Possible explanation: Divide each of the numbers by 3 and that will represent the distance that is $\frac{1}{3}$ the total distance

Figure 11



53. a. Brett (See Figure 12.)

Jim (See Figure 13.)

b. Brett – 3 kilometers (See Figure 14.)

Jim – 6 kilometers (See Figure 15.)

c. Brett $\frac{4}{5}$ (See Figure 16.)

Jim $\frac{4}{10}$ or $\frac{2}{5}$ (See Figure 17.)

For every kilometer Brett runs, Jim needs to run two kilometers.

54. a. Since 12.63 : 100, scaling up would produce 1,263 : 10,000. This means it would take the sprinter 1,263 seconds, or 21 minutes, 3 seconds.

b. Note: The following is used as time, no a ratio.

37:30 − 21:03 = 16:27

The difference between the long-distance runner's actual time and the sprinter's hypothetical time is 16 minutes and 27 seconds.

Figure 12

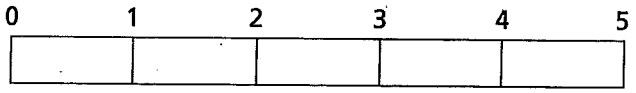


Figure 13

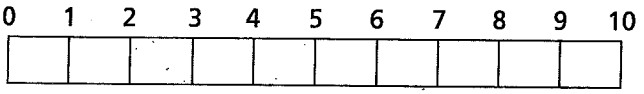


Figure 14

Figure 15

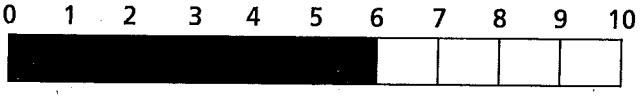
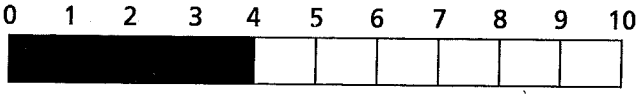


Figure 16

Figure 17



55. C

56. $\frac{20}{60}, \frac{15}{60}, \frac{12}{60}, \frac{10}{60}, \frac{6}{60}, \frac{4}{60}$

57. 12

58. 3

59. 24

60. 9

61. $\frac{1}{4}$

62. $\frac{2}{3}$

63. $\frac{1}{3}$

64. $\frac{2}{5}$

71. $1\frac{1}{2}$

72. $1\frac{2}{3}$

73. $2\frac{1}{4}$

74. $3\frac{1}{2}$

75. (See Figure 18.)

76. (See Figure 19.)

77. (See Figure 20.)

78. (See Figure 21.)

79. (See Figure 22.)

80. (See Figure 23.)

Figure 18

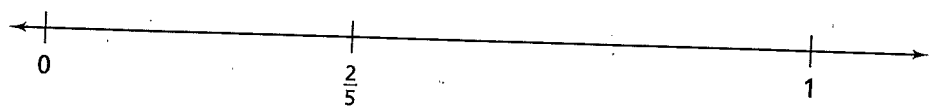


Figure 19

Figure 20

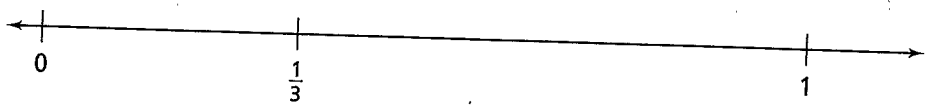


Figure 21

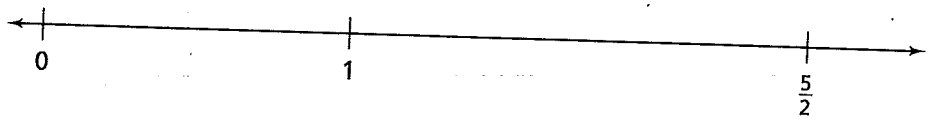


Figure 22

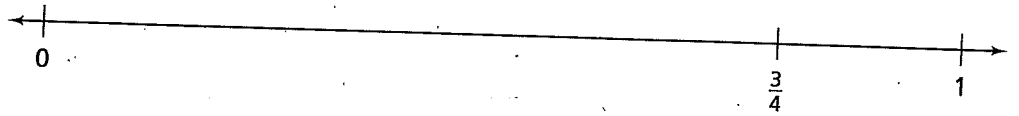
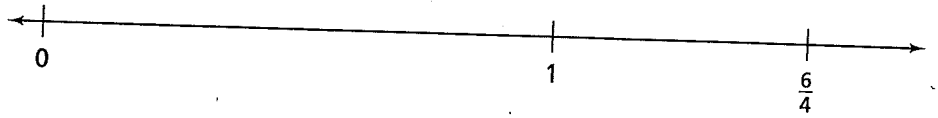


Figure 23



Extensions

65. Possible answers:

close to $\frac{1}{2}$: $\frac{10}{22}$ or $\frac{12}{22}$

close to but greater than 1: $\frac{23}{22}$

66. Possible answers:

close to $\frac{1}{2}$: $\frac{21}{43}$ or $\frac{22}{43}$

close to but greater than 1: $\frac{44}{43}$

67. Possible answers:

close to $\frac{1}{2}$: $\frac{8}{17}$ or $\frac{9}{17}$

close to but greater than 1: $\frac{18}{17}$

68. Possible answers:

close to $\frac{1}{2}$: $\frac{22}{43}$ or $\frac{22}{45}$

close to but greater than 1: $\frac{22}{21}$

69. Possible answers:

close to $\frac{1}{2}$: $\frac{43}{85}$ or $\frac{43}{87}$

close to but greater than 1: $\frac{43}{82}$

70. Possible answers:

close to $\frac{1}{2}$: $\frac{17}{33}$ or $\frac{17}{35}$

close to but greater than 1: $\frac{17}{16}$

81. a. Yes, two people can have half if "half" means half of the three complete pizzas or $1\frac{1}{2}$ pizzas each.

b. Yes, six people can have half if "half" means half of one pizza, making 6 halves.

c. Yes, twelve people can have half if "half" means half of one half of a pizza or one fourth of a pizza.

82. Check students' work to see if the thermometers are drawn to be the same length as the sixth- and seventh-grade thermometers. The thermometers should be partitioned and shaded to show that $\frac{3}{4}$ of the goal has been met.

ACE Answers Investigation 2

Applications

1. Students may write the answers in fraction form. (**Note:** Fraction forms are covered later.) Each person gets $\frac{1}{3}$ of the worm. The first picture below shows that this is $\frac{5}{15}$; the second shows that this is $\frac{1\frac{2}{3}}{5}$, or $1\frac{2}{3}$ segments.
(See Figure 1 and Figure 2.)

2. Each person gets $\frac{1}{5}$. The first picture below shows that this is $\frac{3}{15}$ of a worm; the second shows that this is $\frac{1\frac{1}{5}}{6}$ of the worm, or $1\frac{1}{5}$ segments per person.
(See Figure 3 and Figure 4.)

3. **a.** There could be 12 people in Sharon's group, or any factor of 12: 6, 4, 3, 2 or 1.

   b. If there are 12 people, each person gets $\frac{1}{3}$ of a segment. Different ways to write this rate include: 12 people : 4 segments, 1 person : $\frac{1}{3}$ segment, 3 people : 1 segment.

4. Students can write the original ratio as 48 oz : 6 people or equivalently 8 oz : 1 person. There are $3 \times 48 = 144$ inches of licorice lace total, so the ratio is 144 in. : 6 people, or as a unit rate, 24 inches of licorice lace per person.

5. Answers will vary. Three sandwiches can be cut into 9, 18, or 27 pieces since each sandwich can be cut into 3, 6, or 9 pieces.

6. Answers will vary. 24 : 3 or 8 : 1.

Figure 1

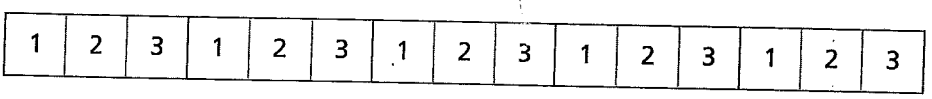


Figure 2

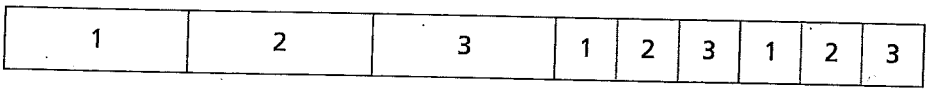


Figure 3

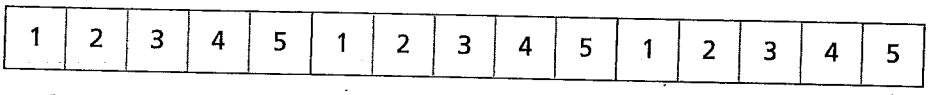
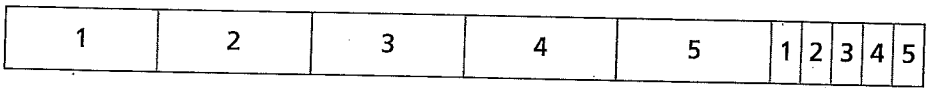


Figure 4



7. Ara's age : Frank's age is 4 : 12 = 1 : 3. Frank is 3 times as old as Ara. Their possible ages include: Ara 2, Frank 6; Ara 4, Frank 12; etc.

Pat's age : Geno's age is 6 : 10 = 3 : 5. Their possible ages include: Pat 3, Geno 5; Pat 6, Geno 10; etc.

Kerri's age : Misty's age is 11 : 5. Their possible ages include: Kerri 11, Misty 5; Kerri 22, Misty 10; etc.

8. The ratio of their ages is 2 : 1. Alexa runs 50 yards or half as much as Crystal. Together they run 150 yards.

9. The ratio of their ages is 2 : 3. Jared runs 60 yards and Peter runs 90 yards. Together they run 150 yards.

10. The ratio of how far they ran is 3 : 2 which is also the ratio of their ages. Their possible ages including: 6 : 4, 9 : 6, 12 : 8, etc.

11. Yes. There are many possibilities. For example, the parent could be 54 and the child 27.

12. Yes. There are many possibilities. For example, the parent could be 54 and the child 18.

13. Yes. There are many possibilities. For example, the parent could be 81 and the child 54.

14. This is unlikely. A parent would have to give birth at a very young age and live to be very old. For example, a parent who gave birth at 13 would have to live to 130, and the child would have to live to 117.

15. a. The ratio of Crystal's age to Alexa's age is 2 : 1. Any pair where the first person is twice the age of the second person will have their chewy fruit worms divided in the same ratio, 2 : 1. Possible answers are: Alan to Lisa (48 : 24), Maren to Dale (42 : 21), Brad to Kari (36 : 18), Lisa to Crystal (24 : 12). (**Note** Students might focus more on the additive difference between Crystal's age and Alexa's age, a difference of 6 years. If you notice that your students focus on differences, consider exploring the example of Alan to Maren (48 : 42). Their ages differ by 6 years, but their chewy fruit worm would be divided almost in half.

b. All the ratios involve pairs of people where the first person is twice as old as the second person.

16. **Note:** Once students find both unit rates, they can find the value for 7 segments for Alan by multiplying.

Both unit rates are given in the table: Alan 2 : Lisa 1; Alan 1 : Lisa $\frac{1}{2}$.

(See Figure 5.)

Figure 5

Segments for Alan	48	12	16	1	2	7
Segments for Lisa	24	6	8	$\frac{1}{2}$	1	$3\frac{1}{2}$

17. Both unit rates are given in the table: Lisa 1 : Alexa $\frac{1}{4}$; Alexa : Lisa 4.

(See Figure 6.)

18. Both unit rates are given in the table: Alan 1 : Alexa $\frac{1}{8}$; Alexa 1 : Alan 8.

(See Figure 7.)

Note: Students might use their results in problems 16 and 17 to find some of the values.

19. Crystal : Alexa, Lisa : Krystal, Alan : Lisa, Brad : Kari, Maren : Dale.

20. Lisa : Alexa, Alan : Crystal.

21. Kari : Lisa, Brad : Alan.

22. Kari : Crystal, Brad : Lisa.

23. a. (See Figure 8.)

b. 56 oz of macaroni.

c. 11 cups of cheese.

24. a. (See Figure 9.)

b. $\frac{4}{3}$: 1

c. 1 : $\frac{3}{4}$

Figure 6

Segments for Lisa	24	12	8	1	4	6
Segments for Alexa	6	3	2	$\frac{1}{2}$	1	$1\frac{1}{2}$

Figure 7

Segments for Alan	48	24	16	1	8	12
Segments for Alexa	6	3	2	$\frac{1}{8}$	1	$1\frac{1}{2}$

Figure 8

Macaroni and Cheese

Ounces of Macaroni	Cups of Cheese
8	1
16	2
24	3
32	4
40	5
48	6

Figure 9

Spaghetti and Sauce

Ounces of Spaghetti	Ounces of Tomatoes
12	16
6	8
3	4
2	$\frac{8}{3}$
1	$\frac{4}{3}$

Connections

25. Ursula's, Ubaldo's, and Dora's strategies work. Students may argue that Ulysses's strategy of using a spinner makes dividing up the extra piece "fair." If the spinner is used, one person will get more than the others, i.e., the worm will not be divided equally.

26. Prime numbers have only two factors, the number and 1. This makes breaking up the worm evenly difficult using the segment marks. For example, a worm with 11 marks requires 11 people in order to use the segment marks, but a worm with 12 marks could be divided with 2, 3, 4 or 6 people using the segment marks.

27. **a.** The ratio of concentrate to water is 1 to 3.

b. At least 48 oz. This is more than a quart, but less than a half-gallon.

c. She needs $\frac{1}{4}$ of a gallon of concentrate, or one quart, or 32 oz.

28. **a.** The ratio of concentrate to water is 1 to $4\frac{1}{3}$.

b. 64 oz., or a half-gallon.

c. Based on the answer in part (b), she wil need 24 oz. of concentrate, or 2 cans.

29. **a.** Betsy is incorrect. She is not considering the relative sizes of the worms. For example, one large worm could be the same size as three small worms. John has the correct answer bu for the wrong reason. Emily is correct. You need to compare by a fixed dollar amount the quantities of the candy by size.

b. Unit rates could make the comparisons easier, the large worms are \$.10 per worm, the medium worms are \$.11 per worm, and the small worm is \$.107 per worm.

30. Johann is mostly correct. If one unit rate does not have a fraction in it, for example, $n : 1$, where n is a whole number, then the corresponding unit rate will be $1 : \frac{1}{n}$. Johann would only be wrong if $n = 1$, or when the unit rate is 1 : 1.

Extensions

31. This statement is true. If you begin by giving one segment to each person, there will not be enough segments to go around. To share equally, those with a segment must give part of their segment to those without segments.

32. This is true. See the solutions to Problem 2.1 for different ways to share a chewy fruit worm.

33. This is not true. The ratio 1 : 2 means each person gets two segments.

34. The ratio will never be 1 : 1 because their ages will never be the same. The ratio however will get closer and closer to 1 : 1 as both people get older.

35. The sum of the swimmers ages is 109 years, which is close to 100. As an estimate, students might multiply each of the swimmers' ages by 4 to find out how far each team member would swim. Using this estimate, the 25-year-old would swim 100 meters, the 21-year-old 84 meters, the 22-year-old 88 meters, and the 41-year-olc 164 meters. For a more accurate answer, divide each swimmer's age by 1.09, then multiply by 4: the 25-year-old $\approx$ 92 m, the 21-year-old $\approx$ 77 m, the 22-year-old $\approx$ 81 m, the 41-year-old $\approx$ 150 m.

36. Marriette gets $35\frac{1}{2}$ worms by dividing \$3.55 by \$.10, the cost per worm. $\$3.55 \div \$.10 = 35.5$.

Melissa does the same thing but reasons that you can't buy half a worm.

Michelle says that you have to buy a box of 30 worms, and you can't buy the worms individually.

37. a. (See Figure 10.)

b. You can use unit rates, or scale the ratios to convert between two different types of money. For example \$20 US ≈ 16 Euros, so 16 Euros ≈ 19 Australian Dollars; or using a unit rate, 0.80 Euro : \$1 US, and \$1 US : 0.95 AUD, so 0.80 Euro : 0.95 AUD.

Figure 10

a. \$20 US ≈ 19 Australian Dollars	\$1 US ≈ 0.95 AUD	\$1.05 US ≈ 1 AUD
b. \$5 US ≈ 4 Euros	\$1 US ≈ 0.80 Euros	\$1.25 US ≈ 1 Euro
c. \$50 US ≈ 49 Swiss Francs	\$1 US ≈ 0.98 SF	\$1.02 US ≈ 1 SF
d. \$3 US ≈ 2 Pounds (UK)	\$1 US ≈ 0.67 Pounds	\$1.50 US ≈ 1 Pound
e. \$4 US ≈ 5 Singapore Dollars	\$1 US ≈ 1.25 SGD	\$0.80 US ≈ 1 SGD

ACE Answers — Investigation 3

Applications

1. $\frac{7}{3}$ and $2\frac{1}{3}$ are equivalent. It takes 3 thirds to make 1 whole, so 7 thirds is equivalent to 2 wholes and one third.
2. B
3. (See Figure 1.)
4. Answers will vary. (See Figure 2.) Pay attention to whether students understand the meaning of *greater* and less in the context of negative numbers (i.e., do they know that $-2 > -3$, or do they think $-3 > -2$?)
5. $\frac{5}{3}$
6. $\frac{27}{4}$
7. $-\frac{88}{9}$
8. $-\frac{30}{7}$
9. $5\frac{2}{4}$ or $5\frac{1}{2}$
10. $1\frac{4}{6}$ or $1\frac{2}{3}$
11. $-3\frac{2}{5}$
12. $-4\frac{4}{8}$ or $-4\frac{1}{2}$
13. $2\frac{1}{2}$ or $-2\frac{1}{2}$
14. Answers will vary. Possible answers include 3, −3, 100, −100, $2\frac{3}{4}$ and $-2\frac{3}{4}$.
15. **a.** They moved 14 yards.
$|7| + |-2| + |4| + |-1| =$
$7 + 2 + 4 + 1 = 14$
   b. No. It is because they only got 8 yards forward. Gain 7 and lost 2 makes a gain of 5. Then 5 gain 4 makes a gain of 9. Finally, 9 lost 1 makes a total of 8 yards gained forward.
16. $15°F > 5°F$
17. $-8°F < 6°F$
18. $-10°F > -25°F$
19. $-5°F < -3°F$
20. The Blue team is the winner, −50 points is the highest point value. You might remind students that numbers on the right of the number line are greater than numbers to the left. −50 is the number furthest to the right of all of them.
21. **a.** Ordering #1 is ordered from least to greatest.
   b. Ordering #2 is ordered least to greatest by absolute value.
   c. Ordering #3 is ordered from greatest to least.

Figure 1

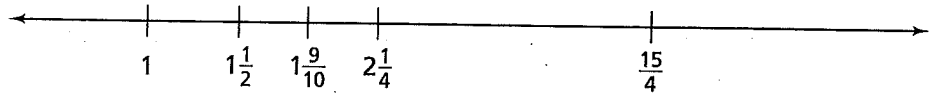
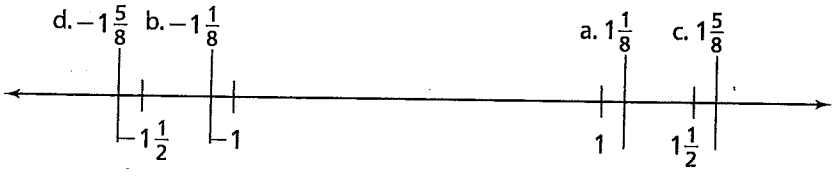


Figure 2



22. Ordering #1 (least to greatest): −9, −2, 1, 5, 7; Ordering #2 (least to greatest by absolute value): 1, −2, 5, 7, −9; Ordering #3 (greatest to least): 7, 5, 1, −2, −9; One other ordering that could be done is to reverse ordering #2 (greatest to least by absolute value): −9, 7, 5, −2, 1

23. Jeremiah is correct, because the winner is the person closest to zero. They should be concerned about the absolute values of the numbers.

24. **a.** Order from lowest to highest: Death Valley, Indio, New Orleans, Denver, Wenzuan

b. Order from least to greatest distance from sea level: New Orleans, Indio, Death Valley, Denver, Wenzuan

c. I used absolute value in part b because the order of the cities is in terms of distance, not numerical value. Therefore, it doesn't matter whether the city's elevation level is above or below seal level. The order starts from the city closest to sea level, 0 feet, to the city farthest away from sea level.

25. The person owes $15.00, or has a debt of $15.00

26. The elevation is 20 feet below sea level.

27. The quarterback lost 8 yards.

28. The lemonade stand lost ten dollars.

Note: The term *par*, which is included in the phrase that Rosemary rewrote, is the number of strokes an expert golf player would take to complete a particular hole in the golf course.

29. $\frac{8}{10} > \frac{3}{8}$

30. $\frac{2}{3} > \frac{4}{9}$

31. $\frac{3}{5} > \frac{5}{12}$

32. $\frac{1}{3} < \frac{2}{3}$

33. $\frac{3}{4} > \frac{3}{5}$

34. $\frac{3}{2} > -\frac{7}{6}$

35. $-\frac{8}{12} < \frac{6}{9}$

36. $\frac{9}{10} < \frac{10}{11}$

37. $-\frac{3}{12} > -\frac{7}{12}$

38. $-\frac{5}{6} < -\frac{5}{8}$

39. $-\frac{3}{7} = -\frac{6}{14}$

40. $-\frac{4}{5} > -\frac{7}{8}$

For Exercises 41–44, answers may vary:

41. $\frac{3}{16}$

42. $\frac{3}{24}$

43. $-\frac{3}{12}$

44. $\frac{1}{4}$

45. between $\frac{1}{2}$ and 1; nearer benchmark: $\frac{1}{2}$

46. between 1 and $1\frac{1}{2}$; nearer benchmark: $1\frac{1}{2}$

47. between 1 and $1\frac{1}{2}$; nearer benchmark: 1

48. between 0 and $\frac{1}{2}$; nearer benchmark: 0

49. between $1\frac{1}{2}$ and 2; nearer benchmark: 2

50. between $1\frac{1}{2}$ and 2; nearer benchmark: 2

51. F

52. D

53. Each person gets $\frac{1}{20}$ or $\frac{5}{100} = 0.05$ of a pan. (5 servings)

54. Each person gets $\frac{1}{40}$ or $\frac{2\frac{1}{2}}{100} = 0.025$ of a pan. ($2\frac{1}{2}$ servings)

55. Each person gets $\frac{1}{30}$ or $\frac{3\frac{1}{3}}{100} = 0.33$ of a pan. ($3\frac{1}{3}$ servings)

56. $\frac{8}{10}$

57. $\frac{4}{10}$

58. $-\frac{4}{100}$

59. $-\frac{84}{100}$

60. $\frac{75}{100} = 0.75$

61. $\frac{14}{100} = 0.14$

62. $-\frac{52}{100} = -0.52$

63. $-\frac{7}{10} = -0.7$

64. Possible answer: 0.60 is greater than 0.45 because it has a larger value in the tenths place. Also, we can compare these as fractions: $\frac{45}{100} < \frac{60}{100}$.

65. Possible answer: 0.6 and 0.60 are equivalent because they both cover the same area of a hundredths grid. Also, we can compare these as fractions: $\frac{6}{10} = \frac{60}{100}$.

66. $\frac{40}{100} = \frac{4}{10} = 0.4$

67. $\frac{62.5}{100} = \frac{625}{1000} = 0.625$

68. $1\frac{6}{100} = \frac{106}{100} = 1.06$

69. Possible answers: $\frac{4}{10}, \frac{40}{100}, \frac{2}{5}, \frac{20}{50}$. The decimal value 0.40 means $\frac{40}{100}$.

If you divide 100 into 10 equal parts or tenths, each tenth will have 10 hundredths in it and 4 of those tenths will have 40 hundredths in it, so $\frac{4}{10}$ is equivalent to $\frac{40}{100}$.

70. The step is 0.01. (See Figure 3.)

71. The step is 0.001. (See Figure 4.)

72. The step is 0.1. (See Figure 5.)

73. $-\frac{1}{3}$

74. $-\frac{1}{2}$

75. $-\frac{1}{8}$

76. $-\frac{1}{6}$

77. $0.205 < 0.21$

78. $0.1 = 0.1000$

79. $-0.04 > -0.050$

80. $-1.03 < -0.03$

81. $\frac{5}{10} < 0.6$

82. $-\frac{3}{5} < -0.3$

83. $-0.127, -0.12, 0.2, 0.33, \frac{45}{10}$

84. $-\frac{45}{10}, -0.005, \frac{3}{1000}, 0.34$

85. G

86. Use Pilar's approximation for $\frac{1}{9}$.

a. $\frac{2}{9} = 2 \times 0.1111 = 0.2222$

b. $\frac{11}{9} = 11 \times 0.1111 = 1.2222$

c. $\frac{6}{9} = 6 \times 0.1111 = 0.6666$

d. $\frac{2}{3}$ is equivalent to $\frac{6}{9}$=, or 0.6666.

e. Answers will vary. Students may say that, when the fractions are written as either proper fractions or mixed numbers, the number that repeats in the decimal is the numerator of the fraction.

87. When Belinda entered $21 \div 28$ into her calculator, the display read 0.75. This is the decimal equivalent for $\frac{3}{4}$. Since $\frac{21}{28}$ is equivalent to $\frac{3}{4}$, the decimal equivalent is also 0.75.

88. Possible answer: I would tell the new student to divide the numerator by the denominator on the calculator. I would tell her to round to the nearest hundredth. To show her this makes sense, I would write this decimal as a fraction with a power of ten in the denominator and show her that the original fraction and this fraction are nearly the same amount. I could do this by finding a number to multiply the numerator and the denominator of the original fraction by that gives a denominator close to a power of ten.

Figure 3

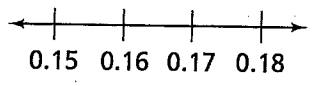


Figure 4

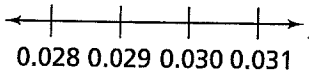
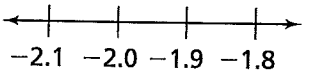


Figure 5



Connections

89. $3\frac{1}{3}$ miles

90. If four days at this rate, the Drama Club can clean $\frac{20}{3}$ miles. So it is correct that they can clean *at least* $\frac{19}{3}$ miles, but not that they will clean *exactly* $\frac{19}{3}$ miles.

91. They are both correct. $\frac{28}{4} = 7$

92. **a.** $\frac{8}{10}$ or $\frac{4}{5}$; 0.8

b. Possible answer: If every student were to receive one piece of every pizza, each pizza would have to be divided into ten equal pieces, with each piece being $\frac{1}{10}$ of a pizza. Each student would receive eight pieces, giving each $\frac{8}{10}$ of a pizza. The picture below shows one student's share of the pizza. (See Figure 6.)

93. **a.** Each part represents $\frac{1}{10}$, 0.1, or one tenth of a centimeter. (See Figure 7.)

b. Each part represents $\frac{1}{100}$, 0.01, or one hundredth of a centimeter.

c. Each part represents $\frac{1}{1{,}000}$, 0.001, one thousandth of a centimeter.

94. $\frac{1}{5}, \frac{2}{10}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}, \frac{8}{40}, \frac{9}{45}$

95. $\frac{8}{3}, \frac{16}{6}, \frac{24}{9}, \frac{32}{12}, \frac{40}{15}, \frac{48}{18}, \frac{56}{21}, \frac{64}{24}, \frac{72}{27}, \frac{80}{30}, \frac{88}{33}, \frac{96}{36}, \frac{104}{39}, \frac{112}{42}, \frac{120}{45}, \frac{128}{48}$

96. Written as improper fractions, $\frac{5}{3}, \frac{10}{6}, \frac{15}{9}, \frac{20}{12}, \frac{25}{15}, \frac{30}{18}, \frac{35}{21}, \frac{40}{24}, \frac{45}{27}, \frac{50}{30}, \frac{55}{33}, \frac{60}{36}, \frac{65}{39}, \frac{70}{42}, \frac{75}{45}, \frac{80}{48}$

97. $-\frac{8}{10} = -\frac{4}{5}$, so there are no fractions between them.

98. $\frac{4}{5}$ is the largest part of a whole. This is a difficult comparison. One way to compare these fractions is based on knowing that if we multiply or divide the numerator and denominator of a fraction by a common factor, we get an equivalent fraction. In comparing $\frac{17}{23}$ to $\frac{51}{68}$, multiply its numerator and denominator by 3. The equivalent fraction is $\frac{51}{69}$. Now that they have the same numerator, we see that $\frac{51}{68}$ is larger (because sixty-eighths are bigger than sixty-ninths). Next, compare $\frac{51}{68}$ to $\frac{4}{5}$ by dividing its numerator and denominator by 17. $\frac{51}{68}$ reduces to the equivalent fraction, $\frac{3}{4}$. $\frac{4}{5}$ is greater than $\frac{3}{4}$ so it's the largest part of the whole.

Figure 6

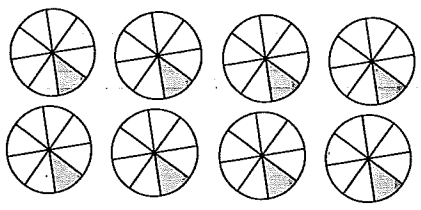


Figure 7

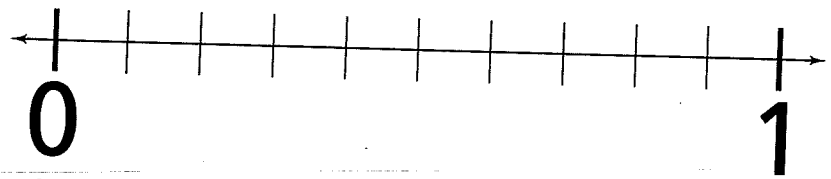
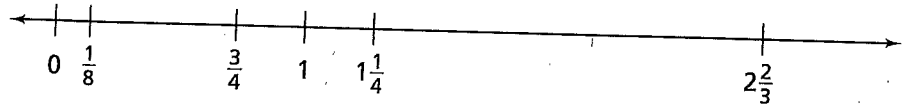


Figure 8

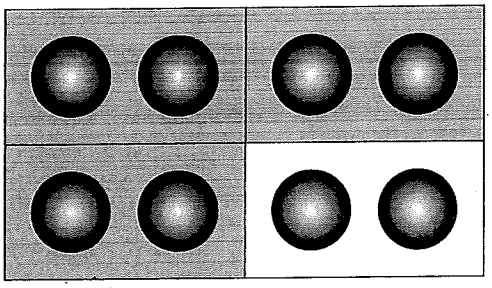


Connections

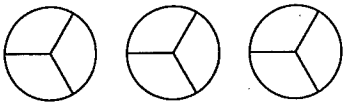
99. (See Figure 8, previous page.)

100. 3. Possible explanation: If you break 12 into fourths, or 4 groups of equal size, one group or $\frac{1}{4}$ of 12 will be 3. (See Figure 9.)

101. 6. Possible explanation: If you partition 8 into fourths or 4 groups of equal size, one group or $\frac{1}{4}$ of 8 is 2. Three of those groups or $\frac{3}{4}$ of 8 is 6.

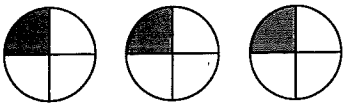


102. $\frac{2}{3}$. Possible explanation: If you partition 3 wholes into 9 parts of equal size, each part will be $\frac{1}{3}$ of 1 whole and 2 of the nine parts or $\frac{2}{9}$ of 3 will be 2 of the one-third size parts or $\frac{2}{3}$ of one of the wholes.



103. Possible explanation: If you partition 18 into ninths or 9 groups of equal size, one group or $\frac{1}{9}$ of 18 is 2. Two of those groups or $\frac{2}{9}$ of 18 is 4. (See Figure 10.)

104. $\frac{3}{4}$. Possible explanation: If you have 3 wholes and partition each whole into fourths, you can take $\frac{1}{4}$ from each of the 3 wholes. You will have 3 one-fourth-size parts, which is $\frac{3}{4}$ of one whole.



105. $2\frac{1}{4}$ or $\frac{9}{4}$ or 2.25. Possible explanation: If you have 3 wholes and partition each whole into fourths, you can take 3 fourth-size pieces or $\frac{3}{4}$ from each of the 3 wholes. You will have 9 one-fourth-size parts or $\frac{9}{4}$ which is equivalent to $2\frac{1}{4}$.

is the same as

Figure 9

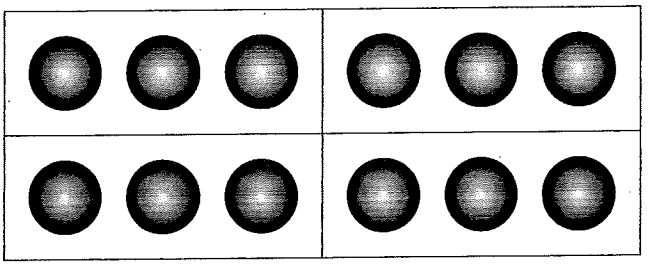
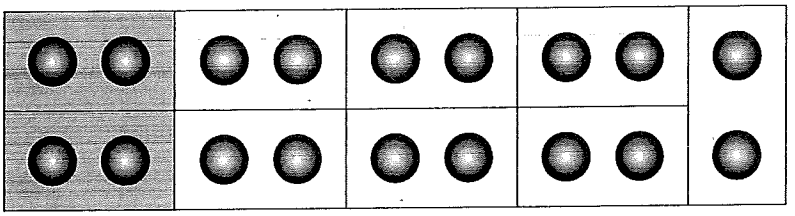


Figure 10



ACE Answers Investigation 4

Applications

1. **a.** Answers will vary. Possible answers include $\frac{3}{4}$ and $\frac{5}{6}$ for Team 1, and $\frac{5}{8}$ and $\frac{2}{3}$ for Team 2.

   b. Team 1's free-throw percentage is approximately 80%. Team 2's free-throw percentage is about 66%.

   c. At a rate of 80%, Team 1 will make 160 of the next 200 free throws, and 16 of the next 20 free throws.

2. A; Strategies may vary. $\frac{15}{25} = \frac{60}{100}$ or 60%. $\frac{27}{50} = \frac{54}{100}$ or 54%. Using percent bars divided into 10% intervals for $\frac{8}{14}$ and $\frac{25}{45}$, we find both 8 out of 14 and 25 out of 45 located below the 60% mark.

3. F; Strategies may vary. $\frac{150}{250} = \frac{15}{25} = \frac{60}{100}$ or 60%. $\frac{75}{100} =$ 50%. Dividing numerator by denominator for $\frac{75}{135}$ we get 55%. Similarly for $\frac{24}{42}$ we get about 57%.

4. C; Strategies may vary. $\frac{14}{20} = \frac{70}{100}$

5. F; Strategies may vary. $26 \div 60$ is about 0.43.

6. **a.** $\frac{54}{100}$

   b. $\frac{46}{100}$

   c. $\frac{54}{100} = 0.54 = 54\%$ and $\frac{46}{100} = 0.46 = 46\%$

7. **a.** $\frac{17}{100}$

   b. $\frac{83}{100}$

   c. $\frac{17}{100} = 0.17 = 17\%$ and $\frac{83}{100} = 0.83 = 83\%$

8. **a.** $\frac{10}{17}$

   b. $\frac{10}{17} \approx 0.59$ or 59%

9. 27%

10. 20%

11. 2%

12. 41%

13. 0%

14. 11%

15. Human food only. $\frac{75}{150} = 0.5 = 50\%$

16. Pet food only. $\frac{116}{200} = 0.58 = 58\%$

17. About 50 dog owners would have said human food only. About 30 would have said pet food only and about 20 dog owners would have said human and pet food.

18. About 9 cat owners would have said human food only. About 29 cat owners would have said pet food only and about 12 would have said human and pet food.

19. 104%

20. Little Neck is more in favor than Elmhurst. More than half (82%) of the Little Neck residents are in favor, while fewer than half (43%) of the Elmhurst residents are in favor. In spite of the fact that fewer Little Neck residents are in favor overall, a greater fraction (or percent) of them are in favor.

Neighborhood	Yes	No
Elmhurst	43	57
Little Neck	41	9

21. Angela made $\frac{12}{15}$ or 80%. Angela's ratio is 80% : 20%. Emily made $\frac{15}{20}$ or 75%. Her ratio is 75 : 25. Christina made $\frac{13}{16}$ or about 81%. Christina's ratio is 81 : 19.

22. It means that they want each new product to be preferred by 60% of consumers in taste tests.

23. The wide part is 36 inches; the narrow part is 24 inches.

24. According to the rule, 90% of your success in life comes from how you react to what happens. 10% comes from what happens to you.

25. (See Figure 1).

Connections

26. $\frac{7}{10} > \frac{5}{8}$

27. $\frac{11}{12} < \frac{12}{13}$

28. $\frac{12}{15} < \frac{12}{14}$

29. $\frac{3}{8} < \frac{4}{8}$

30. $\frac{3}{5} < \frac{4}{6}$

31. $\frac{4}{3} > \frac{15}{12}$

32.

Fraction	Mixed Number
$\frac{13}{5}$	$2\frac{3}{5}$
$\frac{37}{7}$	$5\frac{2}{7}$
$\frac{39}{4}$	$9\frac{3}{4}$
$\frac{23}{3}$	$7\frac{2}{3}$

33. (See Figure 2.)

Figure 1

Percent	Decimal	Fraction
62%	0.62	$\frac{62}{100}$ or equivalent
~44%	~0.44	$\frac{4}{9}$
123%	1.23	$1\frac{23}{100}$
80%	0.8	$\frac{12}{15}$
265%	2.65	$2\frac{65}{100}$ or equivalent
55%	0.55	$\frac{55}{100}$ or equivalent
48%	0.48	$\frac{48}{100}$ or equivalent
120%	1.2	$\frac{12}{10}$

Figure 2

Percent	10%	$12\frac{1}{2}$%	20%	25%	30%	$33\frac{1}{3}$%	50%	$66\frac{2}{3}$%	75%
Fraction	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Decimal	0.1	0.125	0.2	0.25	0.3	$0.3\overline{3}$	0.5	$0.6\overline{6}$	0.75

Extensions

34. $\frac{3}{6} = \frac{1}{2}$

35. $\frac{5}{8}$

36. $\frac{11}{60}$

37. $\frac{3}{8}$ of the square is shaded. If we continued to subdivide the square into congruent triangles, there would be 8 triangles. Three of these are shaded.

38. 0.375 of the square is shaded. Each of the two unshaded large triangles is $\frac{1}{4}$ of the square. The small unshaded triangle is $\frac{1}{8}$ of the triangle. Altogether, $\frac{5}{8}$ of the square is unshaded, so $\frac{3}{8} = 0.375$ of the square is shaded.

39. 50% of the square is shaded. The square can be partitioned into 16 congruent triangles as shown below. Eight of these are shaded. (See Figure 3.)

40. 25%

41. $112\frac{1}{2}$% This represents $112\frac{1}{2}$ out of 100.

42. $137\frac{1}{2}$% This represents 275 out of 200.

43. $212\frac{1}{2}$% This represents 425 out of 200.

44. **a.** \$3.60

b. \$10.50

c. \$20.00

Figure 3